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ORDER PARAMETERS IN A MULTI-PARTON THEORY*

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ABSTRACT

A multi-parton model with spin one-half partons is proposed. Clusterization similar to that contained in model by Kenneth Wilson is assumed. Predictions are made about the nature of the clusters using experimental data. The method of order parameters and their corresponding correlation functions is applied to the case of deep inelastic electron -proton scattering yielding information about clusterization in the proton. This paper serves as an introduction to the model and is thus fairly qualitative, succeeding papers will explore the model more quantitatively.

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I. THE MODEL

Since the name Parton was first suggested by Richard Feynman for the proposed constituents of the hadron, many different parton models have been proposed(1-5), some of them assuming the partons to be quarks. Most of these models could in some way be classified as being multi-parton models. Recently Kenneth G. Wilson proposed a multi-parton model which uses a cutoff Λ[5]; this article proposes another similiar version in that, it also, assumes clusterization of the partons.

As is the case with most multi-parton models, one views the proton as consisting of a very dense, very strongly interacting partonic gas. In the model proposed here, the only hard and fast assumption made about the quantum numbers of the parton is that it has spin one-half. Charge and other quantum observables are left for now as unspecified.

The usual approach in parton models is to use the impulse approximation at very high energy [1]. In this way the scattering can be thought of as incoherent scattering off the individual partons. The high energy and momentum of the scattering particle is required to insure that;

$$\tau_{INTERACTION} << \tau_B \qquad (1)$$

where here τ_B represents the lifetime of the bound state of the partons, i.e., the proton. It is assumed here that at low energies, the impulse approximation is no longer valid for the individual partons, but is still valid for the clusters formed by the partons. Thus in the case of e-p scattering in the inelastic region, the virtual photon scatters incoherently of the individual parton clusters present in the proton at that given time. At lower energies the individual partons are replaced by their clusters. In this way all the results obtained through the normal parton model can be translated over to the clusters for the low energy data. In this assumption of correspondence two things are assumed:

(i) The cluster-cluster interaction is much weaker than the basic parton-parton interaction. In this way one can assume incoherence of the scattering even though one considers lower energies.

(ii) The lifetime of the cluster is sufficiently long to satisfy the impluse approximation as applied to the clusters forming a proton in their bound state.

II. STATISTICAL APPROACH

A great handicap in trying to look at individual partons in a less than infinite momentum frame is that uncertainty enters the picture. Because of the small size of the proton, the internal momentum of the individual partons can be quite high. Uncertainty does not allow one to think of the parton as being localized inside the proton for even a short time. Thus, when one speaks of bound states or clusters, one must speak of them as being highly virtual and extremely short lived. Using an analogy from classical physics, that of a gas, a statistical approach seems to be the most reasonable. One could then consider the proton and its partons in the sense of a macroscopic-mircoscopic system; using the collective behaviour at the macroscopic level to infer the properties of the constituents at the partonic (microscopic) level.

But in this statistical approach the influence of uncertainty will still be felt. The density distribution of the partons can not be thought of as constant or uniform throughout the volume of the proton. One has at best a statistical equilibrium (the proton being stable) with a nonuniform density distribution giving rise to regions which can be labeled clusters that are constantly exchanging partons, but always in such a way that the statistical equilibrium (the proton) is preserved.

III. CLUSTERS

As the name used by Kenneth Wilson suggests, in a multi-parton model it becomes quite natural to think in terms of the interaction between the partons forming some sort of clusterization. If one indeed applies uncertainty and obtains a nonuniform density distribution for the partons in the proton, then quite naturally, one could label the areas of higher density as the clusters. Since the partons which make up the clusters have quantum numbers associated with them, the clusters themselves will possess quantum numbers which are just the appropriate summation of the quantum numbers of the partons contained in it.One then assumes that the quantum numbers of the clusters can be matched with those of some well known particle (e.g., a pion); the only difference in this case is that the particle thus formed is highly virtual.

In the more specializied case of deep inelastic electron-proton scattering where one assumes a single virtual photon is exchanged, one can examine those clusters more closely. As the energy and the momentum transferred squared (q^2) of the virtual photon are increased, the range of the photon will decrease, since the range r can be given by[6) ,

$$r \propto \frac{1}{m} \propto (|q^2|)^{-1/2} \tag{2}$$

Thus at higher energies the photon is able to discern more detail. Since the clusters are nonuniformities in the partonic distribution, it seems not too unreasonable, upon more detailed inspection by a more energetic photon, to find that these clusters themselves are formed by smaller clusters interacting with each other to form the larger clusters. These smaller or finer clusters have the same property as that of the larger ones, in that, they also are highly virtual copies of particles already known. Thus as the energy increases, finer or smaller clusters appear and the proton appears to be composed of more of these smaller clusters as the q^2 of the virtual photon increases. Thus when one considers the distribution function of the partons in terms of the e-p scattering data, then it will be necessary to take the range of the photon into account.

Theoretically this idea of seeing smaller clusters has its logical conclusion when the q^2 of the photon is so large that the photon views only the individual partons themselves and no longer even senses the clusters. This would then represent the same situation which is the starting point for the normal parton model. But there is a major reason why one might never see the "free" parton in the final state in an experiment. Since the partons are very strongly coupled, they communicate very quickly with their immediate neighbors. When a large q^2 virtual photon excites or interacts with a single parton, before the parton could be ejected singly, the parton would have via the interaction with its neighbors transmitted some of the energy. At best then, one could hope to isolate a "smallest" neighborhood which might be thought of as a type of fundamental cell. Further, since N, the number of partons present in the proton, is very large (or even diverges as some might suggest), and the partonic rest mass (if such a quantity makes physical sense) is also probably quite large; then the energy required to liberate a single parton is perhaps beyond the physically realizable. The energy required would not be simply that required to put the parton on the mass shell, but that needed to place all the members of the "fundamental cell" on the mass shell.

The net effect of thinking in terms of partonic clusters in the proton is to evolve a sort of bootstrap theory which ultimately does arrive at a truly elementary particle, but due to the reasons given above one never really sees this "elementary" particle, the parton, in the free state.

At this point it becomes useful to discuss the distribution function of the partons, which describes the proton in the terms of the partonic clusters. The complete discussion of this function will be left for a subsequent paper; for now the discussion will be limited to the determination of the variables of the distribution function. One natural variable is the x defined by ref. 1. Here x is the fraction of the total longitudinal proton momentum that the individual parton possesses in the infinite momentum frame; if $\vec{p}$ is the parton momentum and $\vec{P}$ that of the proton, then

$$\vec{p} = x_i \vec{P} \qquad (3)$$

In the case of this model the $\vec{p}_i$ and x_i now correspond to the i^{th} cluster in the proton and is no longer restricted to just the longitudinal part. At very high q^2 then one reverts back to the picture given in ref. 1.

The additional variable, since one is speaking of e-p scattering would be q^2. The number of clusters and/or partons the virtual photon sees are dependent upon the q^2 of the photon. Thus if one designates the function as $D(x,q^2)$, one immediate consequence is that:

$$\int_0^1 D(x,q^2)dx = N(q^2) \qquad (4)$$

with the auxiliary expression that,

$$\lim_{q^2 \to -\infty} N(q^2) = N_p \qquad (5)$$

where N_p represents the number of partons present in the proton.

IV. ORDER PARAMETERS

Now the method of order parameters will be used with the quantum numbers of the particle in question, the proton, being used as the order parameters. From the appendix one has that the correlation function for an order parameter is defined as:

$$g(\hat{M},\hat{M}) = < \sum_{i=1}^{N(q^2)} \hat{m}_i^2 > \quad < \hat{m} >^2 \qquad (6)$$

Applying this result to the case of deep inelastic electron-proton scattering, and viewing the equation in terms of the proton, the charge of the proton being plus one, this implies that $<\hat{M}>^2 = 1$. The $\hat{m}_i$'s represent the charges of the individual parton clusters or, at sufficiently high energy, the charges of the individual partons. Thus here $\hat{m}_i$ becomes the Q_i of ref. 1, and here denotes the charge of the i^{th} cluster. It should be noted at this point that the final functional form of the function g of equation (6) will probably have two additional variables for the following reasons. First, q^2 will have to be included since this parameter ultimately determines the Q_i's and the N used in the summation. Second, one will have to include s, the center of mass energy. The value of g is determined by its distribution W(M). As the energy s increases, one expects this function to change its shape. Alternately as the q^2 and the energy of the interaction increase, more and more clusters appear; thus, unless all the clusters would have charge equal to zero, one expects the first term of the right hand side of equation (6) to grow with increasing q^2 and s, since the likelyihood that the clusters are being broken up is greater. Thus one could define a function h such that:

$$h_p(Q_i,s,q^2) = \sum_N P(N) \frac{g(\hat{M},\overline{M})}{N(q^2)} \qquad (7)$$

where the dependence on the value one (the proton charge) has been dropped and just replaced by the subscript p denoting the proton. $N(q^2)$ is the quantity defined in equation (4). P(N) is the probability of finding N clusters. This function also contains the q^2 dependence, since one expects the probability to change with a change in q^2. The total expression one now obtains by substituting (6) into (7) is,

$$h_p(Q_i,s,q^2) = \sum_N P(N) \frac{<\Sigma_i\, Q_i^{\,2}>}{N(q^2)} - <N^{-1}(q^2)> \qquad (8)$$

The first term of the right hand side then represents the mean charge squared of the clusters per cluster. The second, the mean of the inverse of the number of clusters.

V. THE PROTON

One can now view the proton in two different situations and see what equation (8) predicts in terms of the charges of the clusters.

The first case consists of what one could call the quiet proton. In this case one considers a non-interacting proton. Here, then, the charge of the system is always plus one (one assumes that there are no fluctuations of the charge, so all measurements yield plus one). This then implies that the distribution or probability function $W(\hat{M})$ has the form,

$$W_p(\hat{M}) = \delta(\hat{M} - 1) \qquad (9)$$

implying then that

$$g(Q_i, 1) = 0 \qquad (10)$$

which in turn implies that

$$\left\langle \sum_{i=1}^{N(q^2)} Q_i^2 \right\rangle = 1 \qquad (11)$$

Since (11) contains the charge of the clusters squared, this implies that if one of the clusters has a charge of plus one then all the rest are required to have charge zero. If an additional cluster would have charge not equal to zero, then the rule (11) would be violated and one would expect fluctuations of the proton charge. Thus at rest the proton seems to be made up of only one charged cluster with the rest neutral (e.g. a virtual proton cluster, the rest virtual neutral pions).

Next one considers the case where the proton is undergoing a scattering. Here one looks at the case where it interacts with a virtual photon in deep inelastic e-p scattering. The first observation is that the equation (9) and (10) no longer hold. Since the final states of the deep inelastic scattering may contain many charged as well as uncharged particles, the $W_p(\hat{M})$ function will have a form dependent on the energy s. In this case the valid starting point for the discussion becomes equation (8). Thus one now attempts to link equation (8) with some other known or experimentally obtainable functions. One possible relation is a sum rule which has relevance in the parton model. this sum rule is

$$\frac{Q^2}{2M} \int_{\nu_{min}}^{\infty} \nu^{-1} W_2(\nu,q^2)d\nu = \sum_N P(N) \frac{\langle \sum_i Q_i^2 \rangle}{N} \; ; (Q^2 = -q^2) \qquad (12)$$

where the right hand side represents the mean squared value of the parton charges. Now if one again applies the basic assumption of this article, that at lower energy and q^2, the partons in the parton model can be replaced by their clusters in a multi-parton model, then equation (12) is equal to the first term of the right hand side of equation (8) , giving the relation,

$$h_p(Q_i,s,q^2) + \langle N^{-1}(q^2) \rangle = \frac{Q^2}{2M} \int_{\nu_{min}}^{\infty} \nu^{-1} W_2(\nu,q^2)d\nu \qquad (13)$$

Thus one links h_p and $\langle N(q^2) \rangle$ to an experimentally obtainable value. Although equation (13) has two different functions contained in it

(h_p and $< N^{-1}(q^2) >$), if one applies some simplifications, one might be able to ascertain the asymptotic behaviour of both.

The first simple case is where one looks at very high values of q^2. As has been assumed throughout this article, at very high q^2, $N(q^2)$ becomes very large. This would make the inverse of N very small;then one has asymptotically,

$$q^2 \to \infty \quad h_p(Q_i,s,q^2) = \frac{Q^2}{2M} \int_{\nu_{min}}^{\infty} \nu^{-1} W_2(\nu,q^2)d\nu \tag{14}$$

The other case would be in the q^2 still not too large. In this region one would expect h_p to be quite small yet, since the final state might be a proton plus some neutrals. Thus h_p could be considered ignorable. At the same time due to the low q^2, the photon will not see too much structure in the proton. Thus the value of $< N^{-1}(q^2) >$ will probably dominate in the region. This yields the relation,

$$N^{-1}(q^2) \cong \frac{Q^2}{2M} \int_{\nu_{min}}^{\infty} \nu^{-1} W_2(\nu,q^2)d\nu \quad (q^2\text{=low}) \tag{15}$$

Using the data given at the Kiev conference, one can then try to obtain a number for N at the given range available.

In this case the sum rule (13) was evaluated for values of ω from one to twelve, where ω is defined as,

$$\omega = \frac{2M_N\nu}{Q^2} \tag{16}$$

and in this variable, the sum rule (12) becomes,

$$\int_1^{\omega} \omega^{-2} F(\omega)d\omega \tag{17}$$

where one defines $F(\omega) = \nu W_2$. At the Kiev conference it [7] was reported that for the proton,

$$\int_1^{12} \omega^{-2} F(\omega) d\omega = 0.14 \qquad (18)$$

Applying the simplification which yield equation (15), one obtains from (15), that,

$$< N > \sim 7 \qquad (19)$$

The data used in obtaining (18) was data taken at 6^o and 10^o, with various values of $q^2 > 1$. But one can express q^2 as,

$$q^2 = - 4EE'\sin^2 (\theta/2) \qquad (20)$$

Thus if one keeps E and E'(the initial and final electron energies)fixed, then an increase in the angle means the same as increasing q^2.Evaluating the sum rule (12) for various angles, one could see whether the sum in fact decreases as the angle increases. Gilman[8] has in fact noted that there is a slight decrease, thus tending to confirm that the photon senses more structure at higher values of q^2. As q^2 grows the decrease may either lessen in terms of the rate of decrease, or the sum rule (12) may begin to grow since at high q^2 one expects to see the function h_p begin to grow and dominate.

A further example of possible experimental evidence, is when the sum rule (12) is evaluated for different values of q^2, q^2 being kept fixed over the integration. In a report by a SLAC-MIT group member [9], the sum rule is evaluated for ω = 1 to 10 for three different values of q^2. The values are as follows:

$$\int_1^{10} \omega^{-2}F(\omega)d\omega \quad \begin{aligned} &= 0.156 \pm 2\% \quad &\text{for } q^2 &= 1.5\,(\mathrm{Gev/c})^2 \\ &= 0.141 \pm 6\% & &= 4.0 \\ &= 0.130 \pm 10\% & &= 8.0 \end{aligned} \tag{21}$$

The errors are caused solely by the variation of the R ratio used to evaluate the sum rule, where the ratio is defined as the longitudinal cross section of the virtual photon dividied by the transverse cross section. The R's used ranged from 0.0 to 0.3. Thus since there seems to be a noticiable drop in the value of the sum rule, it would seem worth while to analyse the data in this fashion since there seems to be an indication of a weak q^2 dependence.

VI. OBSERVATIONS

The first observation deals with the case where the charge is equal to zero, e.g. the neutron. Then the second term on the right hand side of (6) is zero and likewise the term N^{-1} in equation (13)disappears. In cases like this application of the low energy approximation which yielded equation (15) would not be useful.

The next observation deals with the method of order parameters itself. In this paper, since the basic interaction was electromagnetic in nature, one linked the charge sum rule (12) with an order parameter expression using the charge as the order parameter. Theoretically one can do likewise with all the other quantum numbers,wherever one has an appropriate sum rule of the form of (12) involving a quantum observable. In such a case one can then apply the methods of this article to infer further properties of the partons and their clusters.

APPENDIX- ORDER PARAMETERS

In appying order parameters to a composite system, one considers the quantum operator M of the total system and the corresponding individ ual operators m_i of the particles forming the system. One assumes that,

$$\hat{M} = \sum_{i=1}^{N} \hat{m}_i \qquad (A.1)$$

Then one can define the average value of the operator as,

$$< \hat{M} > = \overline{M} = < \sum_{i=1} \hat{m}_i > \qquad (A.2)$$

To obtain the previous two equations one has to assume incoherence of the cluster wave functions. This assumption allows one to consider M as diagonal and yields the simple result of equation (A.2).But this assumpt ion is really nothing more than a restatement of the impulse approxima tion for the clusters.

Now one can define a distribution or probability function for $\hat{M}$ This function will be denoted by $W(\hat{M})$. Next one examines a quantity cal led the <u>fluctuation</u>, defined as,

$$< (\sum_{i=1}^{N} \hat{m}_i - < \hat{M} >)^2 >= < \sum_{i=1}^{N} \hat{m}_i^2 > - < \hat{M} >^2 \qquad (A.3)$$

When one has fluctuations, then one can define a correlation function $g(\hat{M},\overline{M})$ such that,

$$g(\hat{M},\overline{M}) = < \sum_{i=1}^{N} \hat{m}_i^2 > - < \hat{M} > \qquad (A.4)$$

This gives rise to two different cases. In the first case the probability function $W(\hat{M})$ is just,

$$W(\hat{M}) = \delta(\hat{M} - \bar{M}) \tag{A.5}$$

Then equation (A.3) is identically equal to zero and one has no correlation function. Another way of saying the same thing is just,

$$\frac{d}{dt} < \hat{M} > = 0 \tag{A.6}$$

The other case is when $W(\hat{M})$ has a non-delta function form; in that case a non-zero correlation function can be defined and accordingly one has that,

$$\frac{d}{dt} < \hat{M} > \neq 0 \tag{A.7}$$