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GRAVITATIONAL CORRECTION TO THE WEAK INTERACTION *

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ABSTRACT

By means of the operator of the evolution of the Clifford algebra set up in a previous work, we show that there is an interaction between spinor fields that assumes the form of the V-A leptonic weak interaction in the flat space case.

Some comments about the possibly effect of cosmology on leptonic weak interactions are set up.

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I. INTRODUCTION

The weak interaction between leptons may be described in a very approximate form by means of a field-theoretical way, the so-called V-A theory. In this paper we intend to investigate this interaction by an unusual approach, in a kind of geometrization of the theory.

The idea, as put earlier in a previous work, is to consider the existence of a sub-metric structure as given by means of a Clifford algebra (C-algebra). It is possible to show that we can introduce a very simple dynamics in this algebra by means of which we can elaborate Einstein's theory of gravitation as a consequence of some sort of self interaction between the basic Γ-field of the algebra.

In regarding the leptons as some spinor fields of the C-algebra, we can construct a non-trivial hamiltonian by means of the operator that gives the evolution of the sub-metric objects.

2. THE C-ALGEBRA

Let us start by summarizing certain results that we have obtained in [2]. We consider a vector $\Gamma_\alpha(x)$ that generates a C-algebra. This means that there is defined a product between the Γ's such that the anti-commutator of two Γ's is a multiple of the identity of the algebra. We write

$$\{\Gamma_\alpha(x), \Gamma_\beta(x)\} = 2\, g_{\alpha\beta}(x)\, \mathbb{1} \tag{1}$$

where $g_{\alpha\beta}(x)$ is the symmetric metric tensor of the vectorial space of the algebra. The universal C-algebra in a four-dimensional space has 2^4 linearly independent elements. Let us call then as

$$\Gamma^{\alpha}(x);\Sigma^{\alpha\beta}(x) = \frac{1}{2}(\Gamma^{\alpha}\Gamma^{\beta} - \Gamma^{\beta}\Gamma^{\alpha}) ;$$

$$\Gamma^{5}(x); \quad \mathbb{1} ; \quad \Gamma^{\alpha}(x)\Gamma^{5}(x)$$

The great indices represents the tensorial character. $\Gamma^5(x)$ is the pseudo-scalar of the algebra, that is, it anticommutes with all others members of the algebra. We will chose a representation of the algebra in such a way that each $\Gamma_\alpha(x)$ has a two-internal indice, that is

$$\Gamma_{\alpha}^{AB}(x) \quad (A, B = 1, 2, 3, 4)$$

The meaning of the word internal is that the Γ's may suffer in those indices a transformation of the kind

$$\tilde{\Gamma}_{\alpha}^{AB}(x) = M^{A}{}_{N}(x)\, \Gamma_{\alpha}^{NP}(x)\, M^{-1}{}_{P}{}^{B}(x) \tag{2}$$

If we look for the conditions that make possible a relation between the internal transformation (2) and an infinitesimal coordinate transformation, then we arrive at the expression

$$\Gamma_{\alpha\|\beta} = \sigma\left[U_{\beta}, \Gamma_{\alpha}\right] \tag{3}$$

where the bracket, as usually, means the commutator, U_β is a member of the algebra, σ is a constant and the symbol $\|$ means the covariant derivative defined by

$$\Gamma_{\alpha\|\beta} = \Gamma_{\alpha|\beta} - \left\{{\varepsilon \atop \alpha\beta}\right\}\Gamma_{\varepsilon} + \left[\tau_{\beta}, \Gamma_{\alpha}\right] \tag{4}$$

where

$$\Gamma_{\alpha|\beta} = \frac{\partial\Gamma_{\alpha}}{\partial x^{\beta}}$$

$\left\{{\varepsilon \atop \alpha\beta}\right\}$ are the Christoffel symbols

τ_β is the internal affinity [3].

In our deduction we have restricted the possible form of the internal transformation M(x) by a representation of the (local) Poincaré group. This gives us immediately the form of the operator U_λ as

$$U_\lambda = \Gamma_\lambda(\mathbb{I} + \Gamma^5) \qquad (5)$$

From the expression (4) it is possible to show [2] that there is a relation between the contracted curvature tensor $R_{\alpha\beta}$ and an arbitrary tensor of the second order $B_{\alpha\beta}$ that gives origin to Einstein's equation of gravitational interaction.

3. THE LEPTONS

Let us represent by $\psi(x)$ an element (spinor) of the C-algebra. In all this paper we will identify these $\psi(x)$ as leptonic fields.

We have seen that the evolution of the C-algebra is given by the U_α-operator. It seems reasonable to consider that there is some kind of interaction between the leptons that may be guided by this operator too. Let us consider the simplest vector-scalar of the algebra constructed with $U_\alpha^{AB}(x)$.

$$J_{\alpha rs} = \psi_r^+ \; U_\alpha \; \psi_s$$

where the labels r, s characterizes one of the leptons. Then, we can construct a scalar object and try to interpret it as the hamiltonian of interaction.

The simplest non-trivial object has the form

$$\mathcal{H} = J_\alpha \, J^\alpha \qquad (7)$$

Let us evaluate this expression in the case of a weak gravitational field, that is when the metric tensor assumes the form

$$g_{\alpha\beta}(x) \cong \overset{\circ}{g}_{\alpha\beta} + \varepsilon\, h_{\alpha\beta}(x) \tag{8}$$

where $\overset{\circ}{g}_{\alpha\beta}$ is the Minkowskian metric and ε is a small quantity. It is an easy matter to show that this approximation is a consequence of the approximation of the $\Gamma_\alpha(x)$ by means of the constant γ_α

$$\Gamma_\alpha(x) \sim \gamma_\alpha + \frac{\varepsilon}{2}\; h_{\alpha\beta}(x)\, \gamma^\beta \tag{9}$$

$$\Gamma^5(x) \sim \gamma^5 + \frac{\varepsilon}{2}\; \phi(x)\, \gamma^5 \tag{10}$$

where

$$\{\gamma_\alpha, \gamma_\beta\} = 2\, \overset{\circ}{g}_{\alpha\beta}$$

With these expressions we may write

$$J_\alpha \simeq j_\alpha + \frac{\varepsilon}{2} h_{\alpha\beta}\, j^\beta + \frac{\varepsilon}{2}\; \phi(x)\, \rho_\alpha \tag{11}$$

and then

$$\mathcal{H} \simeq j_{\alpha rs}\, j^\alpha_{r's'} + \frac{\varepsilon}{2}\; \phi(\rho_{\alpha rs}\, j^\alpha_{r's'} + j_{\alpha rs}\, \rho^\alpha_{r's'}) + \\ + \varepsilon\, h_{\alpha\beta}(j^\alpha_{rs}\, j^\beta_{r's'}) \tag{12}$$

where

$$j_{\alpha rs} = \psi^+_r\, \gamma_\alpha(\mathbb{1} + \gamma^5)\, \psi_s \tag{13}$$

$$\rho_{\alpha rs} = \psi^+_r\, \gamma_\alpha\, \gamma^5\, \psi_s \tag{14}$$

If we neglect the gravitational term ($\varepsilon = 0$) we can see that this interaction, that we have constructed by the U-operator of the C-algebra, is just the V-A interaction, So, we can say that expression (12) gives us a correction of this interaction in a gravitational field and that the weak leptonic interaction (as gravitation) has an intimate relation with the Γ's interaction.

4. COSMOLOGICAL CONSIDERATIONS

It has been argued by some authors that it is possible to consider a connexion between local theories and cosmology. We will try to prove here that in the usual cosmologies (Friedmann or steady-state models) there is no effect of the long-scale metric on the leptonic weak interaction, in the way we set up this theory in this paper. We start by considering that the common feature of these models is the so-called Robertson form of the metric, that is

$$ds^2 = dt^2 - R^2(t)\left\{\frac{dr^2}{1 - K_r^2} + r^2\, d\Theta^2 + r^2 \mathrm{sen}^2\Theta d\phi^2\right\} \qquad (15)$$

The crucial property of this metric is that there is a coordinate system such that the metric tensor has only diagonal terms. This property may be translated, in the C-algebra, by the relation

$$\Gamma_{\alpha}(x) = \phi(\alpha,x)\,\gamma_{\alpha} \qquad (16)$$

where γ_{α} is a constant.

Furthermore we can suppose that Γ^5 is normalized that is

$$\Gamma^5\,\Gamma^5 = \mathbb{1}$$

(This is by no way a restriction on the theory but only a choice of the form of the operator U_{α} to be (5). We could start with a non normalized Γ^5 and we could obtain the modified form $U_{\alpha} = \Gamma_{\alpha}\,(\mathbb{1} + \frac{\Gamma^5}{\sqrt{\phi}})$ where $\Gamma^5\,\Gamma^5 = \phi(x)\,\mathbb{1}$).

With these conditions, a straightforward calculation can show that

$$J_{\alpha}\, J^{\alpha} = j_{\alpha}\, j^{\alpha} \qquad (17)$$

with J_α and j_α defined by (16) and (13).

We se that (17) shows that there is no effect on the modification on the form of the hamiltonian (weak) leptonic interaction by the cosmological models treated above. This is essentially due to the diagonal form of the metric. This is not the case in Gödels model, for instance, that has a mixed term in the metric tensor. We can easily evaluate, in this case, what is the correction that gravitation may introduce in the interaction.

5. CONCLUSION

From these considerations we can see, for instance, that the annihilation process of electrons in a pair of neutrino may be influenced by the non-diagonal character of the metric. It is a very interesting question to look for the rate of modification of the neutrino creation process in a rotating star, for example, as it is known that the metric is also non-diagonal in this case.

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