NOTAS DE FÍSICA

VOLUME XIII

Nº 18

PHOTOPRODUCTION AND ELECTROPRODUCTION OF PIONS IN POLARIZED PROTON TARGETS

by

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CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Av. Wenceslau Braz, 71

RIO DE JANEIRO

1968

Notas de Física - Volume XIII - Nº 18

PHOTOPRODUCTION AND ELECTROPRODUCTION OF PIONS IN POLARIZED PROTON TARGETS *

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(Received May 29, 1968)

ABSTRACT: The angular distribution of pions for photoproduction and electroproduction of pions in polarized proton targets is given. Using explicit values for the multipoles we have calculated this angular distribution in the case where the energy of the π-N system in their center of mass is 1236 MeV and for momentum transfers equal to zero (photoproduction) and -3, -6 and $-10f^{-2}$ (electroproduction).

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* Supported in part by Conselho Nacional de Pesquisas (Brazil).

I. INTRODUCTION

As was pointed out by Christ and Lee [1], inelastic electron scattering in polarized targets may be used as a possible test of invariance of the electromagnetic current under time reversal. In this test one has to detect an asymmetry of the total cross-section, for fixed electron energy and momentum transfer, with respect to the plane of the incident and final electrons. This asymmetry [1] comes from an interference between the longitudinal and transverse parts [2] of the T matrix and one would not expect a large contribution of this term in the region of the first resonance, since experimental results [3, 4] indicate that the longitudinal part of the cross section is much smaller than the transverse part.

According to the results of Christ and Lee [1] for detecting a time-violating term in the cross section one has to sum over all final states. In this paper, we will assume that the time reversal in this process, if any, is small and analyse the angular distribution of pions produced by inelastic electron scattering and by polarized photons in polarized proton targets, in the region of the first nucleon resonance for both π^+ and π^0 production.

Photoproduction [5, 29] and electroproduction of pions [27,36] in the region of the first resonance have been analysed using dispersion relations technique, with relative success, by several authors. It is found that the main experimental results can be explained by the Born terms and the resonant amplitude in the 3-3 state. Here we will use the results for the amplitudes obtained by one of us [28, 36] (N.Z.).

The cross-section can be decomposed in a part that does not depend on the nucleon polarization $\vec{P}$ and another which is linear in $\vec{P}$. The unpolarized cross-section has been already studied by one of the authors in II, and we shall concentrate our attention in the term linear in $\vec{P}$. This term can be written as a linear combination of functions that depend only on the electron momentum transfer λ^2, energy W and the nucleon scattering polar angle in the π-N center of mass system. These functions can be separated out by varying the azymuthal nucleon scattering angle and the electron scattering angle, for fixed λ^2 and W. Assuming that the Fermi-Watson [37] theorem is valid in this region one can show that the term proportional to P is a linear combination of sin $(\delta_{J\ell T} - \delta_{J'\ell' T'})$ where $\delta_{J\ell T}$ is the phase shift for pion-nucleon scattering in a given total angular momentum J, orbital angular momentum ℓ and isotopic spin T. Therefore, one should look at regions where this difference in phase shifts is large. In particular, in the region near the first resonance one should expect a large effect. Of course, these effects will be atenuated by the effective low polarization which is available presently [38] but we hope that this paper will help in showing up which terms of the angular distribution are sufficiently large in order to be detectable with the present techniques.

In Section II we state some general results on photoproduction and electroproduction of pions. In Section III we write down the cross section and in Section IV we present results for the cross-section in the case where the total energy of the pion nucleon system in their center of mass is 1236 MeV and for several momentum transfers.

II. GENERAL RESULTS

Let $k_{1\mu}$, $k_{2\mu}$, $p_{1\mu}$, $p_{2\mu}$ and q_μ be the 4-momenta of the initial and final electron, of the initial and final nucleon and of the pion, respectively. m and m_e are the nucleon and electron masses, respectively. The square of the 4-momentum transfer given up by the electron is $\lambda^2 = (k_1 - k_2)^2$. In the case of electroproduction the S matrix is given by

$$S_{fi} = \delta_{fi} + (2\pi)^4 i \;\; \delta^4 (k + p_1 - p_2 - q) \times \left\{ \frac{m^2}{2q_0 \, p_{10} \, p_{20}} \right\}^{\frac{1}{2}} \cdot \left\{ \frac{m_e^2}{k_{10} \, k_{20}} \right\}^{\frac{1}{2}} T_{fi} \,, \tag{2.1}$$

where $k_\mu = k_{1\mu} - k_{2\mu}$ is the electron 4-momentum transfer and T_{fi} is the invariant T matrix. For photoproduction k_μ is the incident 4-momentum and the second parenthesis should be substituted by $(2k_o)^{-\frac{1}{2}}$. To first order in the electromagnetic coupling electroproduction is described by the diagram of Fig. 1 and we can write

$$T = \varepsilon_\mu \; j^\mu \,, \tag{2.2}$$

where

$$\varepsilon_\mu = \frac{1}{\lambda^2}\,\bar{u}\,(k_2)\,\gamma_\mu\; u(k_1) \tag{2.3}$$

is the leptonic current times the photon propagator and j_μ is the hadronic current. For photoproduction we can also write the expression (2.2) for T. In this case ε_μ is the photon polarization vector. Current conservation implies

$$k_\mu\,\varepsilon^\mu = 0 \tag{2.4}$$

and

$$k_\mu\; j^\mu = 0\ . \tag{2.5}$$

The T matrix can be written in terms of the π-N center of mass amplitudes $\mathcal{F}$'s as [28]

$$T_{fi} = \frac{4\pi W}{m}\,\chi_f^\dagger \Big\{ i\,\mathcal{F}_1\,\vec{\sigma}\cdot\vec{\varepsilon} + \mathcal{F}_2\;\vec{\sigma}\cdot\hat{q}\;\;\vec{\sigma}\cdot\hat{k}\times\vec{\varepsilon} + i\,\mathcal{F}_3\;\;\vec{\sigma}\cdot\hat{k}\;\;\hat{q}\cdot\vec{\varepsilon}\;+$$

$$+\, i\,\mathcal{F}_4\;\;\vec{\sigma}\cdot\hat{q}\;\;\hat{q}\cdot\vec{\varepsilon} + i\,\mathcal{F}_5\;\;\vec{\sigma}\cdot\hat{k}\;\;\hat{k}\cdot\vec{\varepsilon}\;+$$

$$+\, i\,\mathcal{F}_6\;\;\vec{\sigma}\cdot\hat{q}\;\;\hat{k}\cdot\vec{\varepsilon} - i\,\mathcal{F}_7\;\;\vec{\sigma}\cdot\hat{q}\;\;\varepsilon_0 - i\,\mathcal{F}_8\;\vec{\sigma}\cdot\hat{k}\;\varepsilon_0 \Big\}\chi_i\ , \tag{2.6}$$

where the χ's are the nucleon Pauli spinors, $\hat{q}$ and $\hat{k}$ are unit vectors in the directions of the pion momentum $\vec{q}$ and of the electron momentum transfer $\vec{k}$ (electroproduction) or of the photon direction $\vec{k}$ (photoproduction). W is the total energy of the π - N system in their center of mass. For photoproduction the last four terms do not contribute since the physical photon has only two independent components, both spatial and transverse to $\vec{k}$.

Conservation of the hadronic current (2.5) implies

$$k(\mathcal{F}_1 + \cos\theta\, \mathcal{F}_3 + \mathcal{F}_5) - k_o \mathcal{F}_8 = 0 \quad \text{and}$$
$$k(\mathcal{F}_6 + \cos\theta\, \mathcal{F}_4) - k_o \mathcal{F}_7 = 0 \,, \tag{2.7}$$

where $\cos\theta = \hat{q}\cdot\hat{k}$. Defining

$$\vec{\varepsilon}_T = \vec{\varepsilon} - (\hat{k}\cdot\vec{\varepsilon})\,\hat{k} \tag{2.8}$$

and using (2.4) and (2.7) we can write (2.6) as

$$T_{fi} = \frac{4\pi W}{m}\chi_f^\dagger \Big\{ i\, \mathcal{F}_1\ \vec{\sigma}\cdot\vec{\varepsilon}_T + \mathcal{F}_2\ \vec{\sigma}\cdot\hat{q}\ \ \vec{\sigma}\cdot\hat{k}\times\vec{\varepsilon}_T + i\mathcal{F}_3\ \vec{\sigma}\cdot\hat{k}\ \ \hat{q}\cdot\vec{\varepsilon}_T +$$
$$+ i\,\mathcal{F}_4\ \vec{\sigma}\cdot\hat{q}\ \ \hat{q}\cdot\vec{\varepsilon}_T + \frac{\lambda^2}{k^2}\,\varepsilon_o\ (i\,\mathcal{F}_7\ \vec{\sigma}\cdot\hat{q} + i\,\mathcal{F}_8\ \vec{\sigma}\cdot\hat{k}\Big\}\chi_i \,. \tag{2.9}$$

In the case of photoproduction ($\lambda^2 = 0$) the last two terms do not contribute.

In I and II one of the authors has calculated the $\mathcal{F}$'s as the sum of four contributions:

$$\mathcal{F} = \mathcal{F}_{Born} + \mathcal{F}^{3/2,\,3/2} - \mathcal{F}^{3/2,\,3/2}_{Born} + \mathcal{F}_{cor} \tag{2.10}$$

where $\mathcal{F}_{Born}$ corresponds to the contribution of the full Born terms in Fig. 2, $\mathcal{F}^{3/2,\,3/2}$ is the resonant contribution in the $J = 3/2$, $T = 3/2$ state, $\mathcal{F}^{3/2,\,3/2}_{Born}$ is the 3/2, 3/2 contribution of the Born term which should be subtracted off the full $\mathcal{F}_{Born}$ and $\mathcal{F}_{cor}$ is the correction to the E_{0+} and M_{1-} multipoles [28] due to the dispersion integrals. All these terms were evaluated in I and we shall refer to those values in the calculations that we will perform in this paper.

III. CROSS-SECTION

Let us assume that the nucleon target has a mean polarization $\langle\vec{P}\rangle$. Starting from (2.y) and after some lengthy but straightforward calculation we obtain the expression for the cross-section. In the case of electroproduction we have

$$\frac{d\sigma}{dk_2^L \, d\Omega_{k_2}^L \, d\Omega_\pi} = \Gamma \left\{ V + \langle\vec{P}\rangle . \vec{Z} \right\} \tag{3.1}$$

with

$$\Gamma = \frac{e^2}{(2\pi)^3} \frac{k_2^L}{m \, k_1^L} \frac{Wk}{(-\lambda^2)} \frac{1}{1-\varepsilon} \tag{3.2}$$

where the subscript L means laboratory system. ε is given by

$$\varepsilon = \frac{-\dfrac{\lambda^2 \, m^2}{2W^2 \, k^2} \cot^2 (\alpha^L/2)}{1 - \dfrac{\lambda^2 \, m^2}{2W^2 \, k^2} \cot^2 (\alpha^L/2)} \tag{3.3}$$

where α^L is the electron scattering angle in the Lab. system. The unpolarized term V was given in II:

$$\begin{aligned} V = {} & W_1(\theta) + \varepsilon \sin^2\theta \cos 2\Phi \; W_2(\theta) + \\ & + \sqrt{\varepsilon(\varepsilon+1)/2} \; \sin\theta \cos\Phi \; W_3(\theta) + \varepsilon W_4(\theta) \end{aligned} \tag{3.4}$$

where θ and Φ are the polar and azymuthal pion momentum angle in the $\pi - N$ center of mass (see Fig. 3); W_1, W_2, W_3 and W_4 are functions of θ, W and λ^2 only. W_1 and W_2 contain only transverse contributions, W_4 only longitudinal contributions and W_3 contains the contribution from the interference between the longitudinal and transverse terms. At $\lambda^2 = 0$, W_3 and W_4 vanish and $W_1 + \varepsilon \sin^2\theta \cos 2\Phi \; W_2$ becomes the photoproduction cross-section by a γ-ray beam with a degree ε of linear polarization along the i_1 axis (see Fig. 3).

The W's are expressed in functions of the π-N center of mass amplitudes $\mathcal{F}$'s by [39]

$$W_1(\theta) = q/k \left\{ |\mathcal{F}_1|^2 + |\mathcal{F}_2|^2 - 2\cos\theta \, \mathrm{Re}(\mathcal{F}_1 \mathcal{F}_2^*) \right\} + \sin^2\theta \, W_2(\theta) \quad (3.5)$$

$$W_2(\theta) = (1/2)(q/k) \left\{ |\mathcal{F}_3|^2 + |\mathcal{F}_4|^2 + 2\mathrm{Re}(\mathcal{F}_1 \mathcal{F}_4^* + \mathcal{F}_2 \mathcal{F}_3^* + \cos\theta \, \mathcal{F}_3 \mathcal{F}_4^*) \right\}, \quad (3.6)$$

$$W_3(\theta) = -2(q/k)(-\lambda^2/k^2)^{\frac{1}{2}} \mathrm{Re} \left\{ (\mathcal{F}_1 + \cos\theta \, \mathcal{F}_3 + \mathcal{F}_4) \mathcal{F}_7^* + (\mathcal{F}_2 + \mathcal{F}_3 + \cos\theta \, \mathcal{F}_4) \mathcal{F}_8^* \right\} \quad \text{and} \quad (3.7)$$

$$W_4(\theta) = (q/k)(-\lambda^2/k^2) \left\{ |\mathcal{F}_7|^2 + |\mathcal{F}_8|^2 + 2\cos\theta \, \mathrm{Re}(\mathcal{F}_7 \mathcal{F}_8^*) \right\}. \quad (3.8)$$

The term $\vec{Z}$, which is proportional to the nucleon polarization, can be written as

$$\vec{Z} = \hat{i}_1 \sin\Phi \left\{ Z_1(\theta) + \varepsilon\left[Z_2(\theta) + \cos 2\Phi\ Z_3(\theta)\right] + \right.$$

$$\left. + \sqrt{\varepsilon(\varepsilon+1)/2}\ \cos\phi\ Z_4(\theta) \right\} +$$

$$\hat{i}_2 \left\{ - \cos\Phi\ Z_1(\theta) + \varepsilon\left[\cos\Phi\ Z_5(\theta) - \frac{1}{2}\cos 3\Phi\ Z_3(0)\right] + \right.$$

$$\left. + \sqrt{\varepsilon(\varepsilon+1)/2}\left[Z_6(\theta) - \frac{1}{2}\cos 2\Phi\ Z_4(\theta)\right]\right\} +$$

$$\hat{i}_3 \sin\Phi \left\{ \varepsilon\cos\Phi\ Z_7(\theta) + \sqrt{\varepsilon(\varepsilon+1)/2}\ Z_8(\theta)\right\} ; \tag{3.9}$$

$\hat{i}_1$, $\hat{i}_2$, $\hat{i}_3$ are unit vectors along the directions $(\vec{k}_1 \times \vec{k}_2) \times \vec{k}$, $\vec{k}_1 \times \vec{k}_2$ and $\vec{k}$, respectively. The Z_i's are functions of θ, W and λ^2 only and they are given by

$$Z_1(\theta) = - (q/k) \sin\theta\ \mathrm{Im}\left\{ - \mathcal{F}_1 \mathcal{F}_3^* + \mathcal{F}_2 \mathcal{F}_4^* - \right.$$

$$\left. -(\mathcal{F}_1 \mathcal{F}_4^* - \mathcal{F}_2 \mathcal{F}_3^*) \cos\theta + \mathcal{F}_3 \mathcal{F}_4^* \sin^2\theta \right\} , \tag{3.10}$$

$$Z_2(\theta) = - (q/k) \sin\theta\ \mathrm{Im}\left\{ 2\mathcal{F}_1 \mathcal{F}_2^* + \mathcal{F}_1 \mathcal{F}_3^* - \mathcal{F}_2 \mathcal{F}_4^* + \right.$$

$$\left. + (\mathcal{F}_1 \mathcal{F}_4^* - \mathcal{F}_2 \mathcal{F}_3^*) \cos\theta - 2(-\lambda^2/k^2)\ \mathcal{F}_7 \mathcal{F}_8^* \right\} , \tag{3.11}$$

$$Z_3(\theta) = - (q/k) \sin^3\theta\ \mathrm{Im}\left\{ \mathcal{F}_3 \mathcal{F}_4^* \right\} , \tag{3.12}$$

$$Z_4(\theta) = -2(q/k) \sin^2\theta\ (-\lambda^2/k^2)^{\frac{1}{2}}\ \mathrm{Im}\left\{ \mathcal{F}_4 \mathcal{F}_8^* - \mathcal{F}_3 \mathcal{F}_7^* \right\} , \tag{3.13}$$

$$Z_5(\theta) = - (q/k) \sin\theta\ \mathrm{Im}\left\{ 2\mathcal{F}_1 \mathcal{F}_2^* + \mathcal{F}_1 \mathcal{F}_3^* - \mathcal{F}_2 \mathcal{F}_4^* + \right.$$

$$+(\mathcal{F}_1 \mathcal{F}_4^* - \mathcal{F}_2 \mathcal{F}_3^*) \cos\theta - \tfrac{1}{2} \mathcal{F}_3 \mathcal{F}_4^* \sin^2\theta + 2(-\lambda^2/k^2) \mathcal{F}_7 \mathcal{F}_8^* \Big\} , \tag{3.14}$$

$$Z_6(\theta) = 2(q/k)(-\lambda^2/k^2)^{\frac{1}{2}} \operatorname{Im} \Big\{ (\mathcal{F}_1 \cos\theta - \mathcal{F}_2 - \tfrac{1}{2} \mathcal{F}_3 \sin^2\theta) \mathcal{F}_7^* +$$

$$+ (\mathcal{F}_1 - \mathcal{F}_2 \cos\theta + \tfrac{1}{2} \mathcal{F}_4 \sin^2\theta) \mathcal{F}_8^* \Big\} , \tag{3.15}$$

$$Z_7(\theta) = 2(q/k) \sin^2\theta \operatorname{Im} \Big\{ \mathcal{F}_1 \mathcal{F}_4^* + \mathcal{F}_2 \mathcal{F}_3^* \Big\} \quad \text{and} \tag{3.16}$$

$$Z_8(\theta) = -2(q/k) \sin\theta \, (-\lambda^2/k^2)^{\frac{1}{2}} \operatorname{Im} \Big\{ \mathcal{F}_1 \mathcal{F}_7^* + \mathcal{F}_2 \mathcal{F}_8^* \Big\} . \tag{3.17}$$

If $\lambda^2 = 0$ (photoproduction) Z_4, Z_6 and Z_8 vanish and $Z_5 = Z_2 - 1/2\, Z_3$. Therefore, $\vec{Z}$ reduces to

$$\vec{Z} = \hat{i}_1 \sin\Phi \left[Z_1 + \varepsilon (Z_2 + \cos 2\Phi \, Z_3) \right] +$$
$$\hat{i}_2 \cos\Phi \left[- Z_1 + \varepsilon (Z_2 - \cos 2\Phi \, Z_3) \right] +$$
$$\hat{i}_3 \, \varepsilon \sin\Phi \cos\Phi \, Z_7 , \tag{3.18}$$

which is the linear term in $\langle P \rangle$ of the cross-section for photoproduction by a γ-ray beam with a degree ε of linear polarization along the i_1 axis.

If one integrates (3.9) on Φ, only the term proportional to $Z_6(\theta)$ survives. That is

$$\int \vec{Z} (\theta, \Phi) \, d\Phi = \hat{i}_2 \, 2\pi \sqrt{\varepsilon(\varepsilon + 1)/2} \, Z_6(\theta) . \tag{3.19}$$

In I we have expressed the $\mathcal{F}$'s in function of the multipole amplitudes [40]. Using these relations and integrating (3.19) on the polar angle we obtain

$$\int \vec{Z}(\theta, \phi)\, d\phi\, d(\cos\theta) = \hat{1}_2\, 4\pi \sqrt{-\lambda^2/k^2}\, \sqrt{\varepsilon(\varepsilon + 1)/2}\, (q/k) \times$$

$$\mathrm{Im} \sum_{\ell} \left\{ (\ell+1) \left[\ell\, M_{\ell+} + (\ell+2)\, E_{\ell+} \right] S_{\ell+}^{*} + \ell \left[(\ell-1)\, E_{\ell-} - (\ell+1) M_{\ell-} \right] S_{\ell-}^{*} \right. \qquad (3.20)$$

where $M_{\ell\pm}$, $E_{\ell\pm}$ and $S_{\ell\pm}$ are the magnetic, electric and scalar amplitudes leading to a final state with $J = \ell \pm 1/2$.

In the region of energies around the first resonance, production of two pions is quite low. Therefore, we can use the Fermi-Watson theorem to show that the multipoles amplitudes for a given total angular momentum J, orbital angular momentum ℓ and isotopic spin T have the same phase as the π-N scattering amplitudes of the same J, ℓ and T.

Then one can easily show that if we have, for instance, a proton target and add the contributions of (3.20) for both π^+ and π^0 production, that is, add over the final isotopic spin states, the net result will be zero. This is a particular case of the elegant proof stated in Section II of the Christ and Lee [1] paper. In the case that we consider the production of only one kind of pions, (3.20) will be, in general, different from zero. However, we should expect that the values in (3.20) be approximately zero, in this case, since the T matrix is dominated by only one isotopic spin state (T = 3/2).

IV. RESULTS AND COMMENTS

As shown in I, the multipoles amplitudes are linear functions of the electromagnetic form factors. For the nucleon form factor we have used Harvard 4-pole fit [41], while we set the pion form factor equal to the isovector nucleon form factor, as some experimental results [42] indicate.

We have computed the cross-section for W = 1236 MeV and $\lambda^2 = 0$, $-3\ f^{-2}$, $-6f^{-2}$ and $\mp 10\ f^{-2}$. We set $\delta_{33} = \pi/2$ and neglect all the other phase shifts.

In Figs. 4 - 11 we have plot the calculated values for the angular distributions Z's for both π^+ and π^o productions.

In the case of photoproduction by unpolarized ($\mathcal{E} = 0$) photons, (3.18) reduces to

$$\vec{Z} = Z_1(\theta)\ \hat{n} \tag{4.1}$$

where $\hat{n}$ is the unit vector along the $\vec{q} \times \vec{k}$ direction. It is interesting to compare Z_1 with the unpolarized photoproduction cross-section W_1. The result is shown in Fig. 12. One can see that Z_1 is quite large and probably it can be detectable even for a low mean polarization $\langle \vec{P} \rangle$.

For π^o production only the low multipoles contribute. Assuming that only the multipoles with $J \leqslant 3/2$ are present, we can write $Z_1(\theta)$ as

$$Z_1(\theta) \approx 3\ (q/k)\ \sin\theta\ \mathrm{Im}\left[A + B\cos\theta + C\cos^2\theta\right] \ ; \tag{4.2}$$

keeping in mind that we are neglecting all the phase shifts but δ_{33}, we have

$$A = (E_{0+} - 2E_{2-}) \quad E_{1+}^{*} - (E_{0+} + 2M_{2-}) M_{1+}^{*} ,$$
$$B = (M_{1-} - 4E_{1+}) M_{1+}^{*} - M_{1-} E_{1+}^{*} \quad \text{and} \tag{4.3}$$
$$C = 6 (E_{2-} E_{1+}^{*} + M_{2-} M_{1+}^{*}) .$$

Experimental results [43] indicate that for J = 3/2 T = 3/2 $E_{1+} \ll M_{1+}$. Neglecting also the D-wave multipoles in A, we get

$$A \approx -E_{0+} M_{1+}^{*} \quad \text{and}$$
$$B \approx (M_{1-} - 4E_{1+}) M_{1+}^{*} . \tag{4.4}$$

Therefore measurements of the coefficients A and B can give valuable additional information for estimating the low multipoles at the resonance. In particular, if our prediction for $Z_1(\theta)$ is correct, the coefficient B would have a large value (see Fig. 12).

For $\lambda^2 \neq 0$ each one of the Z's may be separated experimentally. However, it will be a very difficult task to obtain each one of those terms, first due to the low polarization available and second due to the difficulties in vary ε in a wide range. As one of the goals of this paper is to show where it would be easier to look at experimentally, it is interesting to compare the results obtained for Z with those that one would get with an unpolarized target. We will consider, for instance that the proton is polarized along the i_2 axis (i.e. in a direction perpendicular to the electron scattering plane). We will also take $\varepsilon \approx 1$ as an example, and write down the cross-section 3.1 as

$$\frac{d\sigma}{dk_2^L \, d\Omega_{k_2}^L \, d\Omega_\pi} = \Gamma \Big\{ (A + PA') + (B + PB') \cos\phi + (C + PC') \cos 2\phi + PD' \cos 3\phi \Big\} \tag{4.5}$$

where

$$A = W_1 + W_4 \;, \qquad A' = Z_6 \;,$$
$$B = W_3 \sin\theta \;, \qquad B' = -Z_1 + Z_5 \;, \qquad (4.6)$$
$$C = W_2 \sin^2\theta \;, \qquad C' = -(1/2)\, Z_4 \quad \text{and}$$
$$D' = -(1/2)\, Z_3 \;.$$

The term in $\cos 3\Phi$ is peculiar of the polarized part. It is rather small for π^0-production. However in π^+-production it is quite large due to the presence of high multipoles. This term is proportional to Z_3 which has already been shown in Fig. 6. In Figs. 13 - 15 we compare the other terms in (4.5) for both π^+ and π^0-production, in the cases where $\lambda^2 = -3f^{-2}$ and $-10f^{-2}$.

Perhaps a more realistic approach from the experimental point of view is to consider only the angular distribution in θ without regard to the azymuthal angle Φ. As was shown in Section III, we get in this case

$$\frac{d\sigma}{dk_2^L \, d\Omega_{k_2}^L \, d(\cos\theta)} = 2\pi\Gamma \left\{ W_1 + \varepsilon W_4 + \langle \vec{P} \rangle \cdot \hat{i}_2 \sqrt{\varepsilon(\varepsilon+1)/2}\; Z_6 \right\} \qquad (4.7)$$

where $\hat{i}_2$ is a unit vector in direction $\vec{k}_1 \times \vec{k}_2$. For $\varepsilon = 1$, this reduces to

$$\frac{d\sigma}{dk_2^L \, d\Omega_{k_2}^L \, d(\cos\theta)} = 2\pi\,\Gamma \left[A + \langle \vec{P} \rangle \cdot \hat{i}_2 A' \right] . \qquad (4.8)$$

The distribution A and A' have already been shown in Fig. 13.

For completeness we reproduce in Figs. 16-19 the unpolarized terms W_1, W_2, W_3 and W_4 for energy W = 1236 MeV and momentum transfer $\lambda^2 = 0, -3, -6$ and $-10 f^{-2}$.

* * *

ACKNOWLEDGEMENTS:

The calculations were performed in the 1130 - IBM computer of the Pontifícia Universidade Católica and we are grateful for their staff assistance.

FIGURE CAPTIONS

Fig. 1 - One photon exchange approximation for electroproduction.

Fig. 2 - Born term: a) direct nucleon pole diagram; b) crossed nucleon pole diagram; c) meson pole diagram.

Fig. 3 - The pion-nucleon center of mass system: we take $\vec{k} = \vec{k}_1 - \vec{k}_2$ along the z axis, and $\vec{k}_1$ and $\vec{k}_2$ in the xz plane. Then θ and Φ are the polar and azymuthal angles of the pion momentum.

Fig. 4 - Angular distribution $Z_1(\theta)$. The heavy, dashed, broken and dotted lines correspond to momentum transfer $\lambda^2 = 0, -3, -6$ and $-10f^{-2}$, respectively. a) π^+ production; b) π^0 production.

Fig. 5 - Angular distribution $Z_2(\theta)$. The heavy, dashed, broken and dotted lines correspond to momentum transfer $\lambda^2 = 0, -3, -6$ and $-10f^{-2}$, respectively. a) π^+ production; b) π^0 production.

Fig. 6 - Angular distribution $Z_3(\theta)$. The heavy, dashed, broken and dotted lines correspond to momentum transfer $\lambda^2 = 0, -3, -6$ and $-10f^{-2}$, respectively. a) π^+ production; b) π^0 production.

Fig. 7 - Angular distribution $Z_4(\theta)$. The dashed, broken and dotted lines correspond to momentum transfer $\lambda^2 = -3, -6$ and $-10f^{-2}$, respectively. Note that $Z_4(\theta)$ reduces to zero when $\lambda^2 = 0$. a) π^+ production; b) π^0 production.

Fig. 8 - Angular distribution $Z_5(\theta)$. The heavy, dashed, broken and dotted lines correspond to momentum transfer $\lambda^2 = 0, -3, -6$ and $-10f^{-2}$, respectively. a) π^+ production; b) π^0 production.

Fig. 9 - Angular distribution $Z_6(\theta)$. The dashed, broken and dotted lines correspond to momentum transfer $\lambda^2 = -3, -6$ and $-10f^{-2}$, respectively. Note that $Z_6(\theta)$ reduces to zero when $\lambda^2 = 0$. a) π^+ production; b) π^0 production.

Fig. 10 - Angular distribution $Z_7(\theta)$. The heavy, dashed, broken and dotted lines correspond to momentum transfer $\lambda^2 = 0, -3, -6$ and $-10f^{-2}$, respectively. a) π^+ production; b) π^0 production.

Fig. 11 - Angular distribution $Z_8(\theta)$. The dashed, broken and dotted lines correspond to momentum transfer $\lambda^2 = -3$, -6 and $-10f^{-2}$, respectively. Note that $Z_8(\theta)$ reduces to zero when $\lambda^2 = 0$. a) π^+ production; b) π^o production.

Fig. 12 - Photoproduction; comparison between the unpolarized cross-section $W_1(\theta)$ and the term $Z_1(\theta)$ linear in $\langle\vec{P}\rangle \cdot \hat{n}$. The heavy lines represent π^+ production and the dashed lines correspond to π^o production.

Fig. 13 - Comparison between Φ -independent terms of the cross-section (4.5). $A(\theta)$ is the unpolarized part and $A'(\theta)$ is the term proportional to $\langle P\rangle$. The dashed lines and the dotted lines correspond to $\lambda^2 = -3f^{-2}$ and $-10f^{-2}$, respectively. a) π^+ production; b) π^o production.

Fig. 14 - Comparison between the terms of the cross-section (4.5) which are linear in $\cos\Phi$. $B(\theta)$ is the unpolarized part and $B'(\theta)$ is the term proportional to $\langle P\rangle$. The dashed lines and the dotted lines correspond to $\lambda^2 = -3f^{-2}$ and $= -10f^{-2}$, respectively. a) π^+ production; b) π^o production.

Fig. 15 - Comparison between the terms of the cross-section (4.5) which are linear in $\cos 2\Phi$. $C(\theta)$ is the unpolarized part and $C'(\theta)$ is the term proportional to $\langle P\rangle$. The dashed lines and the dotted lines correspond to $\lambda^2 = -3f^{-2}$ and $\lambda^2 = -10f^{-2}$, respectively. a) π^+ production; b) π^o production.

Fig. 16 - Angular distribution $W_1(\theta)$. The heavy, dashed, broken and dotted lines correspond to momentum transfer $\lambda = 0$, -3, -6 and $-10f^{-2}$, respectively. a) π^+ production; b) π^o production.

Fig. 17 - Angular distribution $\sin^2\theta\, W_2(\theta)$. The heavy, dashed, broken and dotted lines correspond to momentum transfer $\lambda = 0$, -3, -6 and $-10f^{-2}$, respectively. a) π^+ production; b) π^o production.

Fig. 18 - Angular distribution sin θ $W_3(\theta)$. The dashed, broken and dotted lines correspond to momentum transfers $\lambda^2 = -3$, -6 and $-10f^{-2}$, respectively. Note that $W_3(\theta)$ reduces to zero when $\lambda^2 \to 0$. a) π^+ production; b) π^o production.

Fig. 19 - Angular distribution $W_4(\theta)$. The dashed, broken and dotted lines correspond to momentum transfer $\lambda^2 = -3$, -6 and $-10f^{-2}$, respectively. Note that $W_4(\theta)$ reduces to zero when $\lambda^2 \to 0$. a) π^+ production; b) π^o production.

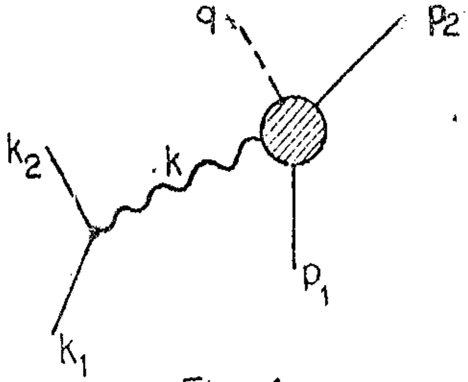
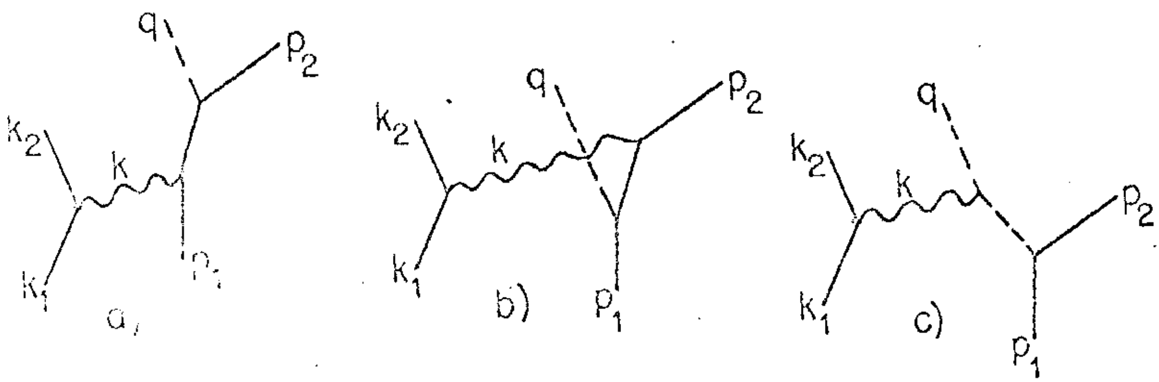


Fig. 1



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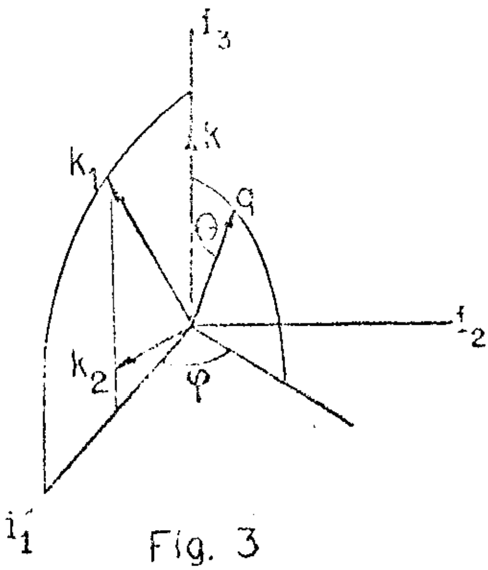
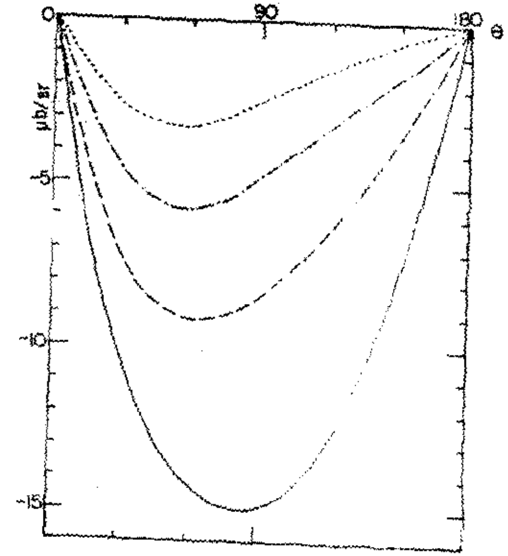


Fig. 3



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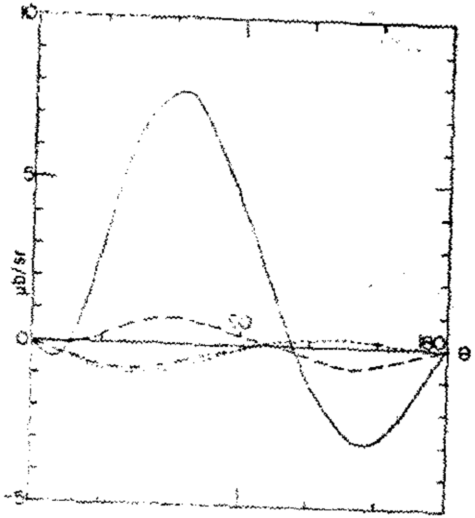

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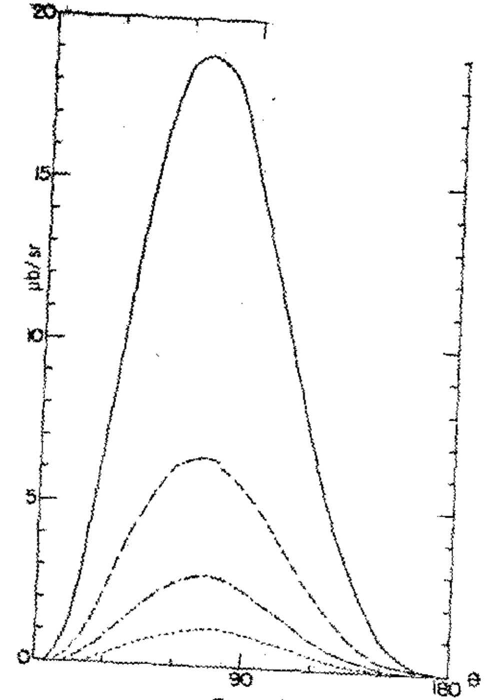

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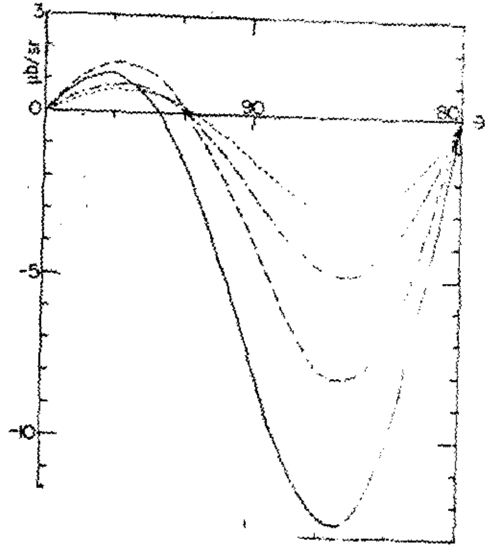

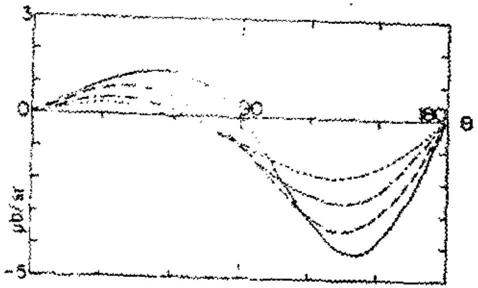

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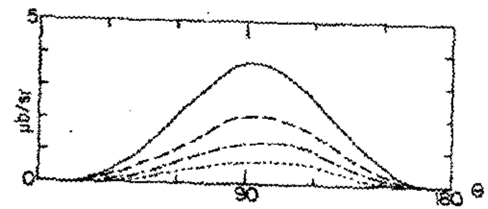

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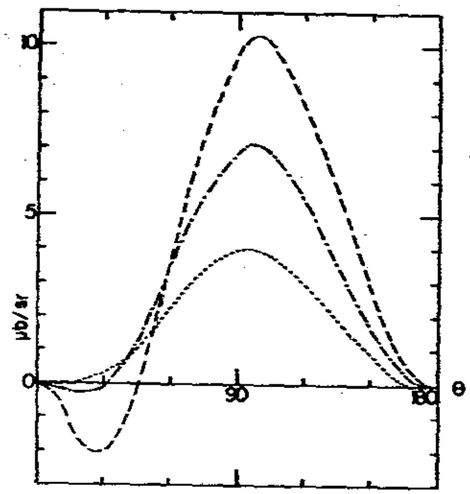

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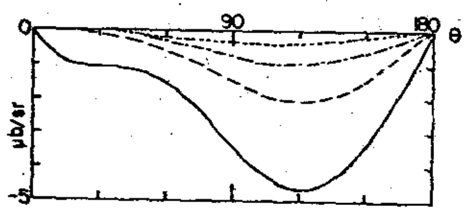

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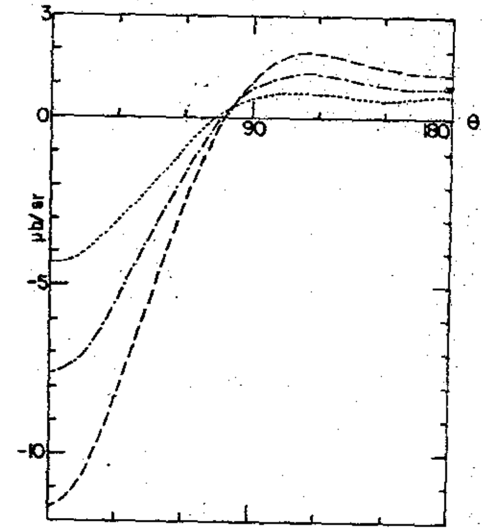

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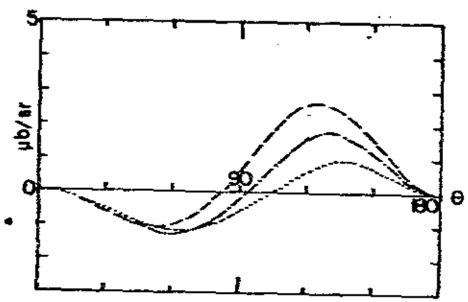

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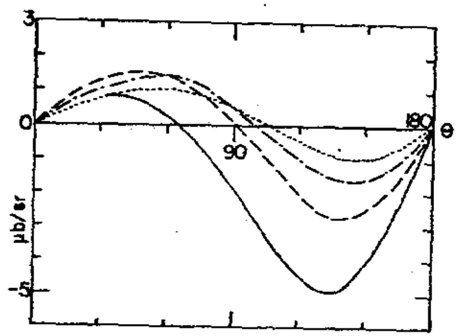

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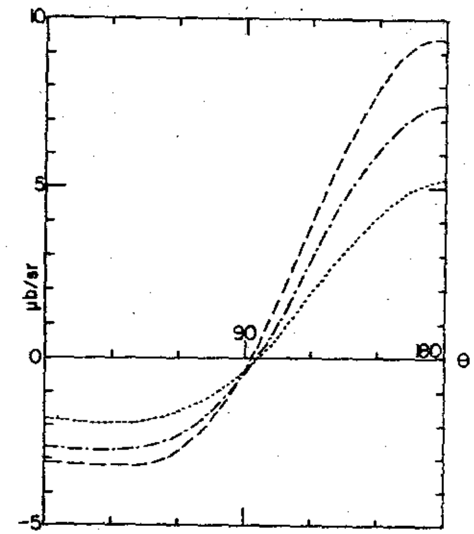

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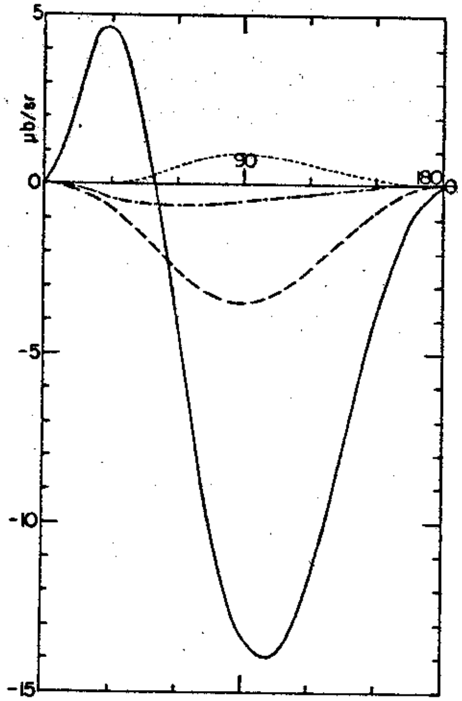

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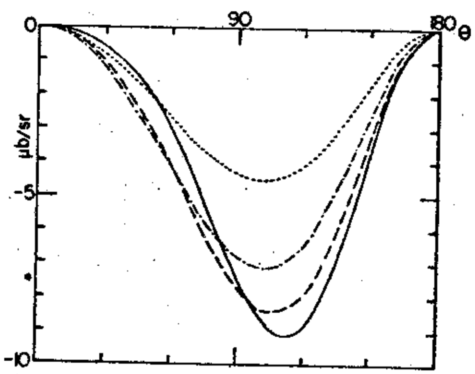

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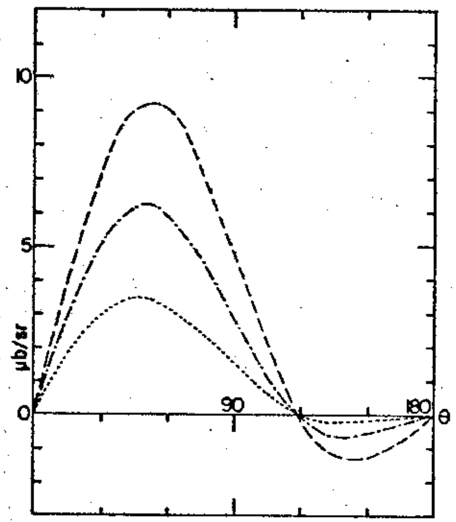

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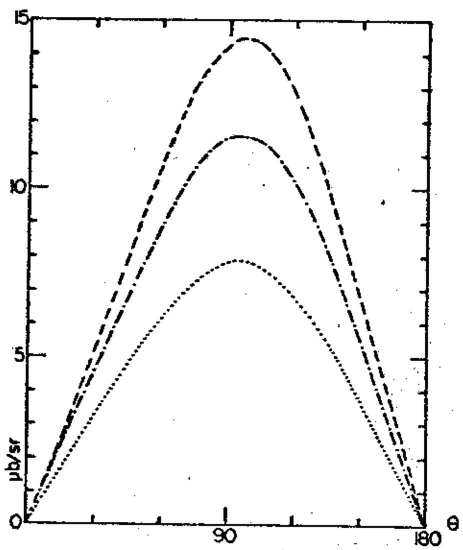

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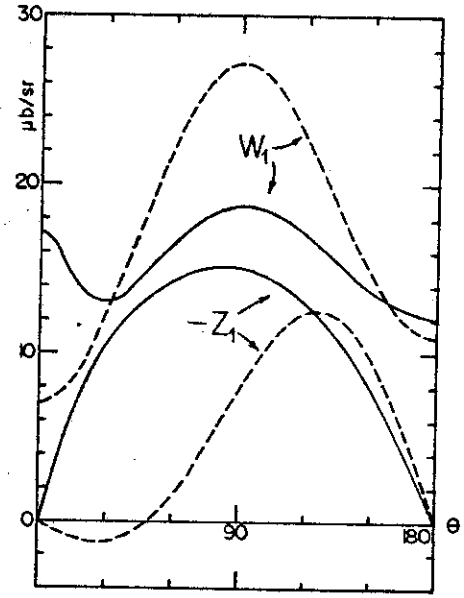
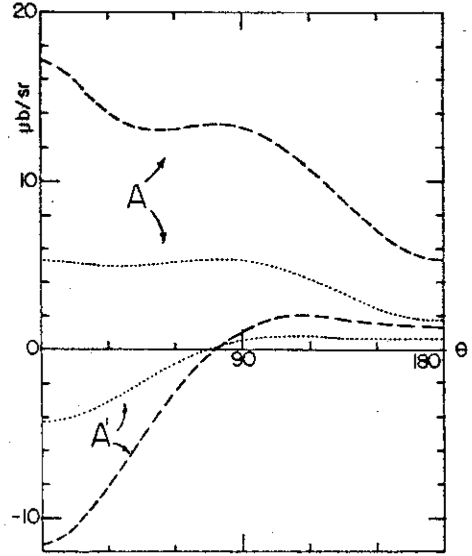


Fig. 12



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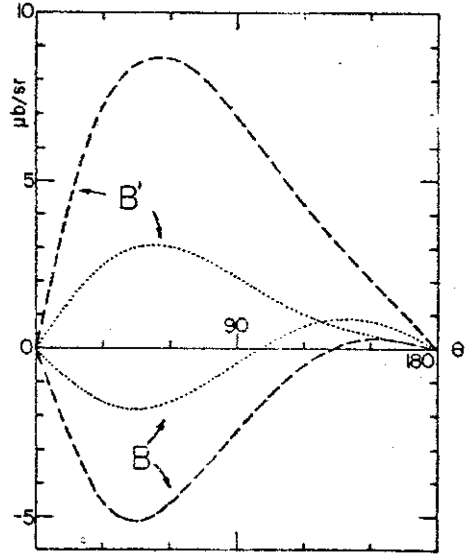

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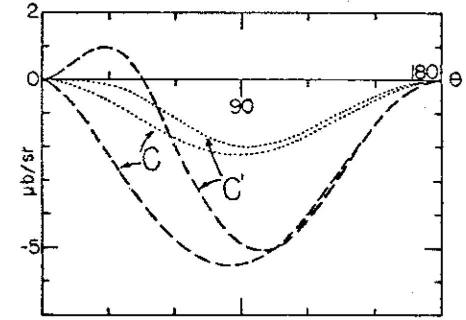

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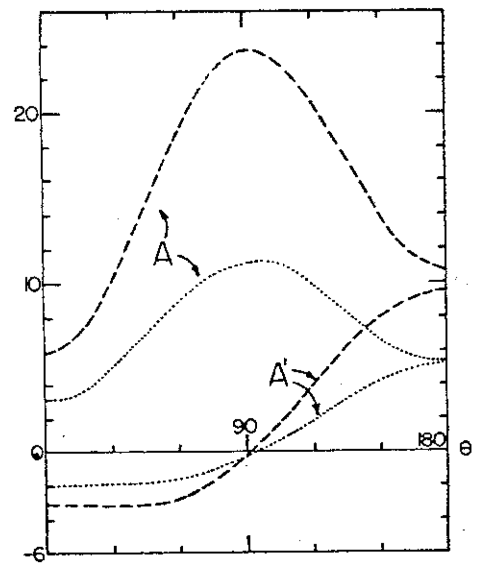

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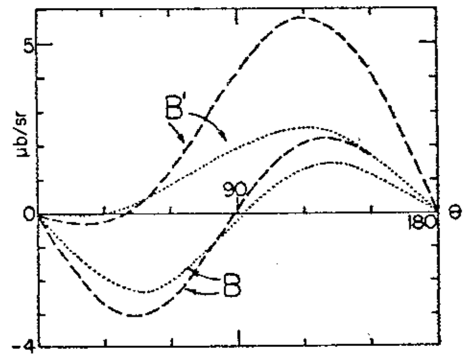

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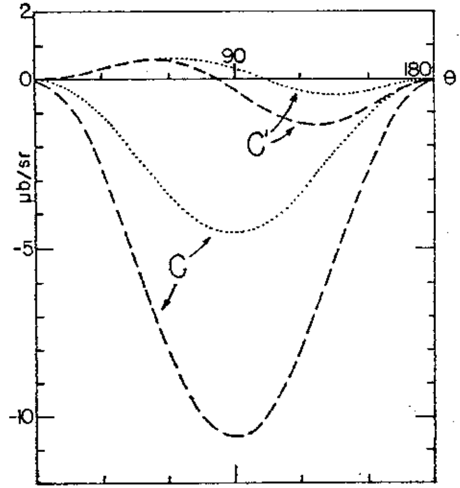

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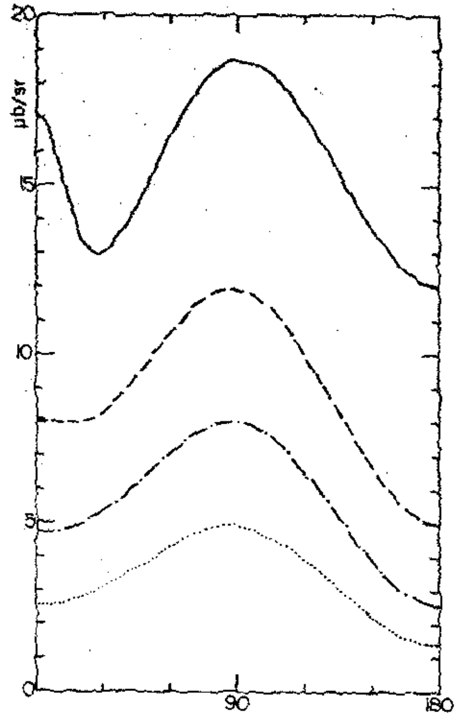

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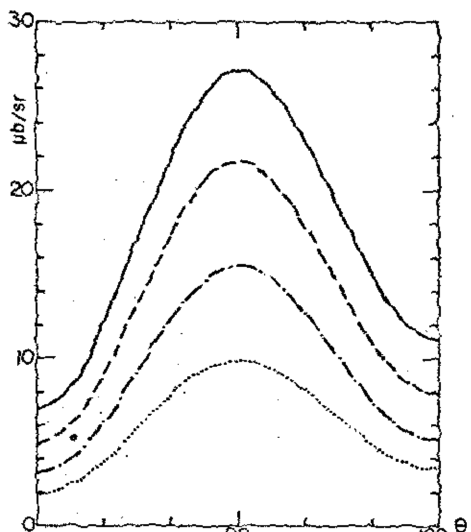

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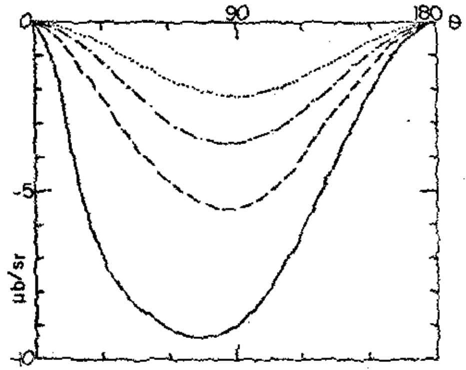

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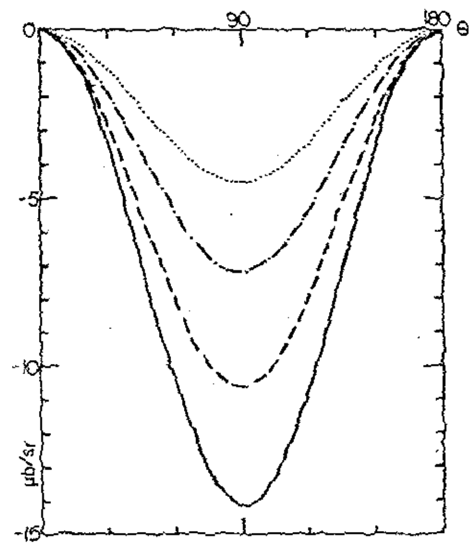

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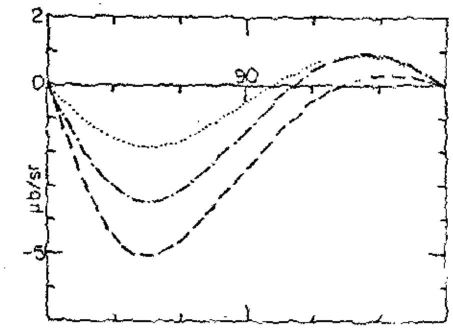

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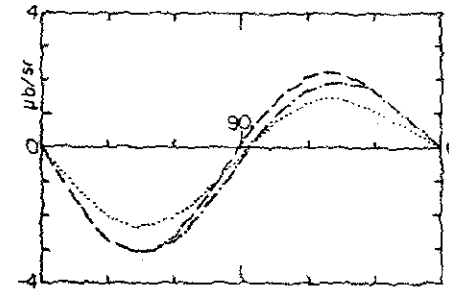

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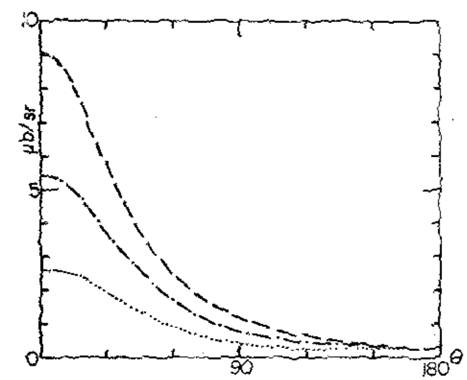

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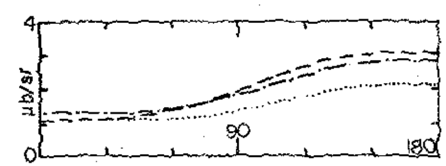

Fig. 19.b)

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