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ON BARYON ANOMALOUS MAGNETIC MOMENT CONTRIBUTION TO NEUTRAL PI MESON DECAY

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SUMMARY. The disintegration rate of the π^0 meson decay into two photons is calculated in a model in which the decay takes place via virtual baryon anti-baryon pair, including the effect of the anomalous magnetic moments of these baryons. The life time is found to depend appreciably on nucleons and Ξ hyperons virtual pairs contributions while less so on that of Σ if the mirror theorem for the magnetic moments of Σ is assumed. The branching ratio $\pi^0 \longrightarrow e^+ + e^- + \gamma / \pi^0 \longrightarrow \gamma + \gamma$ is not appreciably affected.

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1. INTRODUCTION.

The γ-instability of π^0 meson in the usual meson theory was first predicted by Sakita and Tanikawa[1]. Since its existence was definitely established[2] its mass[3] and lifetime[4] have been measured with increasing precision. The decay has been investigated theoretically by many authors in the lowest order perturbation theory[5] and recently, using dispersion theory[6]. Kinoshita[7] suggested the possible influence of intermediate hyperon pairs in lowering the theoretical decay rate obtained by using only the nucleon-antinucleon pair and thus bringing it close to the experimental value. A calculation taking into account all known baryons was made by Tiomno[8]. All these calculations, however, have ignored the effect of anomalous magnetic moments which are, at least for nucleons, known to be of the order of the intrinsic magnetic moment of a charged Dirac particle. This effect was first considered by Zimerman[9]. In the next section we will re-calculate* the decay rate including the effect of anomalous magnetic moments for hyperons which are being measured currently. The decay is appreciably influenced by them although unambiguous quantitative conclusions are hard to draw because of the appearence of logarithmic divergence.

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* In reference 9 there is a mistake in the relative signs of the electric charge interaction terms which gives a result in disagreement with the present calculations.

2. DISINTEGRATION RATE.

The strong interaction of the isovector π^0 with the third component of the isovector current due to baryons can be written as[10]

$$H_{\pi^0} = \Big[G_N(\bar{p}\, i\gamma_5\, p - \bar{n}\, i\gamma_5\, n) + G_\Xi\,(\bar{\Xi}^0 i\gamma_5\, \Xi^0 - \bar{\Xi}^- i\gamma_5\, \Xi^-)$$

$$+\ G_\Sigma(\bar{\Sigma}^+ i\gamma_5 \Sigma^+ - \bar{\Sigma}^- i\gamma_5 \Sigma^-) + G_\Lambda(\bar{\Sigma}^0 \eta \Lambda^0 + \bar{\Lambda}^0 \eta\, \Sigma^0)$$

$$+\ G'(\bar{\Sigma}^+ i\gamma_5\, \Sigma^+ + \bar{\Sigma}^- i\gamma_5 \Sigma^- - \bar{\Sigma}^0 i\gamma_5\, \Sigma^0 - \bar{\Lambda}^0\, i\gamma_5\, \Lambda^0)\Big]\ \pi^0$$

where $\eta = i\,\gamma_5$ or 1 for the same or opposite prities of Σ^0 and Λ^0 respectively. The usual charge independent interaction is obtained by putting $G' = 0$.

The electromagnetic field interacts both with the isoscalar and isovector baryon currents. The interaction with the electric charge is

$$H_{em} = -\,ie(\bar{p}\,\gamma_\mu\, p + \bar{\Sigma}^+ \gamma_\mu\, \Sigma^+ - \bar{\Sigma}^- \gamma_\mu \Sigma^- - \bar{\Xi}^- \gamma_\mu\, \Xi^-)\, A_\mu$$

while that with the anomalous magnetic moment is

$$-\sum_i \left[\lambda_i \left(\frac{e}{2M_i}\right) \frac{1}{2}\ \bar{\psi}_i\ \sigma_{\mu\nu}\, \psi_i\ F_{\mu\nu}\right] - \lambda\, \frac{e}{(M_{\Lambda^0} + M_{\Sigma^0})} \cdot \frac{1}{2} \cdot \bar{\Sigma}^0 \left\{ i\,\gamma_5 \right\} \sigma_{\mu\nu}\ \Lambda^0\, F_{\mu\nu} + \ + \text{h.c.}$$

where M_i is the mass of the particle (neutral or charged), λ_i the anomalous magnetic moment in units of $(e/2M_i)$, $\sigma_{\mu\nu} = (1/2i)[\gamma_\mu, \gamma_\nu]$, $F_{\mu\nu} = (\partial_\mu\, A_\nu - \partial_\nu\, A_\mu)$ and ψ_i is the field representing the baryon i. The last term is the $\Sigma^0\,\Lambda^0$ anomalous magnetic interaction responsible for the fast radiative decay of Σ^{o11}. It may be remarked that the contribution to the anomalous magnetic moment due to the isoscalar baryon current is in general much smaller as compared to the isovector current contribution.

The invariant matrix element for the decay of π^o into two photons with four momenta k_1 and k_2 and polarizations e_1 and e_2 respectively via baryon-antibaryon pair of mass M is given by*

$$M = (+ie^2G)\cdot\int d^4p\, Sp\left\{ \gamma_5[i(\not p+\not k_1)+M]^{-1}(r\not e_1+\alpha\not e_1\not k_1)[i\not p+M]^{-1} (r\not e_2+\alpha\not e_2\not k_2)[i(\not p-\not k_2)+M]^{-1} + \gamma_5[i(\not p+\not k_2)+M]^{-1}(r\not e_2+\alpha\not e_2\not k_2) [i\not p+M]^{-1}(r\not e_1+\alpha\not e_1\not k_1)[i(\not p-\not k_1)+M]^{-1}\right\}$$

where $\alpha = \lambda/2Mi$ and $r = +\cdot-$ or 0 according as the corresponding photon is emitted by a positively charged, negatively charge or neutral baryon. The two terms give equal contributions as can be easily verified. The numerator of the first term can be reduced (using $e\cdot k = 0$) to

$$M[r^2 - 2iM\alpha r - (p^2+M^2)\alpha^2]\ Sp\cdot(\gamma_5\not e_1\not k_1\not e_2\not k_2)$$

$$+\ ir\alpha(p^2+M^2)[Sp(\gamma_5\not e_1\not e_2\not k_2\not p) + Sp(\gamma_5\not e_1\not e_2\not k_1\not p)]$$

$$+\ (ir\alpha - 2\alpha^2 M)[k_1^2\ Sp(\gamma_5\not e_1\not e_2\not k_2\not p) + k_2^2\ Sp(\gamma_5\not e_1\not e_2\not k_1\not p)]$$

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* $\not a = a_\mu \gamma_\mu$.

while its denominators combined in the form

$$\frac{1}{[(p+k_1)^2 + M^2][(p-k_2)^2 + M^2][p^2+M^2]} = \int_0^1 2x\,dx \int_0^1 dy \frac{1}{[(p-\ell)^2+a^2]^3}$$

where $\ell = x[-k_1 y + k_2(1-y)]$ and $a^2 = [M^2 - m_\pi^2 x^2 y(1-y) + k_1^2 x(1-x)y + k_2^2 x(1-x)(1-y)]$

Making use of

$$\int \frac{p_\mu p_\nu p_\lambda}{[(p-\ell)^2+a^2]^3} d^4p = \int \frac{(p+\ell)_\mu (p+\ell)_\nu (p+\ell)_\lambda}{(p^2+a^2)^3} d^4p - \frac{i\pi^2}{12}$$

$$(\ell_\mu \delta_{\nu\lambda} + \ell_\nu \delta_{\mu\lambda} + \ell_\lambda \delta_{\mu\nu}) = \frac{1}{4}\int \frac{(\ell_\mu \delta_{\nu\lambda} + \ell_\nu \delta_{\mu\lambda} + \ell_\lambda \delta_{\mu\nu})}{(p^2+a^2)^3} d^4p +$$

$$+ \frac{i\pi^2}{2a^2} \ell_\mu \ell_\nu \ell_\lambda - \frac{5}{24} i\pi^2 (\ell_\mu \delta_{\nu\lambda} + \ell_\nu \delta_{\mu\lambda} + \ell_\lambda \delta_{\mu\nu}); \int \frac{p_\mu p_\nu}{[(p-\ell)^2+a^2]^3} d^4p =$$

$$= \frac{1}{4}\delta_{\mu\nu}\left(\int \frac{d^4 p}{(p^2+a^2)^2} - \frac{i\pi^2}{2}\right) + \frac{i\pi^2}{2}\ell_\mu \ell_\nu .$$

and neglecting the $(m_\pi/M)^2$ in relation to unity we obtain finally

$$M = i\, e^2\, G\, Sp(\gamma_5 \not{e}_1 \not{e}_2 \not{k}_1 \not{k}_2) \frac{i\pi^2}{2M} \cdot A$$

where

$$A = \left[D(M^2)\left(\frac{\lambda^2}{2} - \lambda r\right) + \left(r^2 - \frac{\lambda}{2} r\right)\right] - \frac{(k_1^2 + k_2^2)}{M^2} \left[\frac{4}{15} \lambda r + \frac{5}{48} \lambda^2\right]$$

Here $D(M^2)$ stands for the logarithmic divergent integral

$$D(M^2) = \frac{1}{i\pi^2} \int \frac{d^4p}{(p^2 + M^2)^2}$$

which can be evaluated, using a Feynman cut-off $\frac{\Lambda^2}{\Lambda^2 + p^2}$

$$D(M^2) = \ell n \left(\frac{\Lambda^2 + M^2}{M^2}\right) - \left(\frac{\Lambda^2}{M^2 + \Lambda^2}\right) .$$

The matrix element satisfies gauge condition with respect to the two photons independent of whether they are real or virtual as long as we assume $e_1 \cdot k_1 = 0$, $e_2 \cdot k_2 = 0$. Also the two photons can not be emitted with their four polarization vectors parallel to each other. It is also clear that when at least one of the photons is real the square of the four momentum of the other lies between $\pm m_\pi^2$ whence we conclude that the branching ratio[12] $\pi^o \to e^- + e^+ + \gamma / \pi^o \to \gamma + \gamma$ is not affected appreciably due to the inclusion of the anomalous magnetic moment effects.

For the real photons the matrix element in the rest frame of the pion is given by

$$|M = (8 i e^2 G) \frac{i\pi^2}{M} A \cdot \omega \cdot (\vec{e}_1 \times \vec{e}_2 \cdot \vec{k}_2)$$

where ω is the energy of the photon. The two photons are thus emitted in the rest frame with their polarizations perpendicular to each other. Summing the matrix element over the various intermediate baryon-antibaryon pairs and summing the square of the total matrix element over the polarization states of the two photons we obtain for the disintegration rate for two photons* ($\hbar = c = 1$):

$$\Gamma = \frac{1}{\tau} = \frac{1}{(4\pi)^2} \left(\frac{e^2}{4\pi}\right)^2 \frac{|L|^2}{4\pi} m_{\pi^o}^3$$

where $L = \Sigma\,(GA/M)$, the summation extending over various baryons.

The anomalous magnetic moment for isoscalar Λ^o and Σ^o - the third component of isovector ($\theta_3 = 0$) - is expected to be small as memtioned before**. Assuming the usual charge independent pion interaction we obtain for L

$$L = \frac{G_N}{M_N}\left\{\left(1 - \frac{\lambda p}{2}\right) + D(M_N^2)\left(\frac{\lambda_p^2 - \lambda_n^2}{2} - \lambda_p\right)\right\}$$

$$+ G_\Sigma\left\{\left(\frac{1}{M_{\Sigma^+}} - \frac{1}{M_{\Sigma^-}}\right) - \frac{1}{2}\left(\frac{\lambda_{\Sigma^+}}{M_{\Sigma^+}} + \frac{\lambda_{\Sigma^-}}{M_{\Sigma^-}}\right) + D(M_{\Sigma^+}^2)\left(\frac{\lambda_{\Sigma^+}^2}{2M_{\Sigma^+}} - \frac{\lambda_{\Sigma^+}}{M_{\Sigma^+}}\right)\right.$$

$$\left. + D(M_{\Sigma^-}^2)\left(-\frac{\lambda_{\Sigma^-}^2}{2M_{\Sigma^-}} - \frac{\lambda_{\Sigma^-}}{M_{\Sigma^-}}\right)\right\}$$

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* See Appendices I and II.

** In case Λ^o(or Σ^o) does have appreciable anomalous magnetic moment L will contain an extra term corresponding to the case in which π^o creates a $\Sigma^o\Lambda^o$ pair and one of the photons is emitted by the anomalous magnetic moment term (see ref. 11).

$$- G_{\Xi} \left\{ \frac{1}{M_{\Xi^-}} \left(1 + \frac{\lambda_{\Xi^-}}{2} \right) + \frac{D(M^2_{\Xi^-})}{M_{\Xi^-}} \left(\frac{\lambda_{\Xi^-}^{\ 2}}{2} + \lambda_{\Xi^-} \right) - \frac{D(M^2_{\Xi^o})}{2M_{\Xi^o}} \lambda^2_{\Xi^o} \right\}$$

If in addition we neglect the mass difference within a charge multiplet and use the "mirror theorem"[13] for magnetic moments viz.

$\lambda_{\Lambda^o}, \lambda_{\Sigma^o}, \frac{1}{2}(\lambda_{\Sigma^+} + \lambda_{\Sigma^-}), (\lambda_p + \lambda_n), (\lambda_{\Xi^-} + \lambda_{\Xi^o}), (\lambda_{K^+} + \lambda_{K^o})$ get vanishing contribution from the iso-boson (Λ, Σ, π) currents and a small contribution from the isofermion and anti-isofermion (K, N, Ξ) currents only we find that Σ almost do not contribute to the decay as compared to the N, and Ξ contributions. The decay does not thus provide a sensitive test to determine the relative sign of Σ and $\Lambda - \pi$ coupling constants in relation to that of nucleons coupling constant in a scheme in which all the coupling constants are equal in magnitude[8]. The contribution to the decay thus arises mainly from the intermediate nucleons and cascade hyperon terms.

In the following table we give the values of the decay rates for various values of the cut-off momentum Λ and the ratio G_{Ξ}/G_N for $\lambda_p = -\lambda_n = 1.8$, $\lambda_{\Xi^o} = -\lambda_{\Xi^-} \simeq (m_N/m_{\Xi})\lambda_p = 1.3$ and $G_N^2/4\pi = 15$.

<u>TABLE</u>: Decay Rates of $\pi^o \longrightarrow (\gamma + \gamma)$. (X 10^{16} numbers per second).

CUTT-OFF (Nucleon Masses) Λ	G_{Ξ}/G_N						
	0	1	-1	$\frac{1}{3}$	$-\frac{1}{3}$	3	-3
M_p	0.13	0.40	0.01	0.20	0.08	1.35	0.19
$2M_p$	3.96	3.09	4.94	3.66	4.28	1.67	7.24
$3M_p$	12.64	7.28	19.48	10.69	14.76	0.96	37.57
	Experiment[4]: 0.53						

3. DISCUSSION.

As in the case when only interactions with the electric charge is considered[8] we find that in the usual charge independent theory the decay of neutral pion into two photons is essentially due to the contributions from nucleons and cascade hyperons if the mirror theorem for the magnetic moments of sigma hyperons is assumed and the mass difference within a charge multiplet neglected even when the magnetic moment interactions are included. The anomalous magnetic moment terms, however, change appreciably the life time of π^0. The dependence on the cut-off is strong for the cut-off in the neighbor hood of the mass of the intermediate particles as is evident from the expression of $D(M^2)$ which does not allow us to draw unambiguous quantitative conclusions. Comparison with the experimental value of the lifetime[4] $\tau = 1.9 \pm 0.5 \times 10^{-16}$ s(or $\sim 0.526 \times 10^{16}$ disintegrations per second) shows that the same sign for G_Ξ and G_N is favoured in general. For a cut-off at one nucleon mass $G_\Xi/G_N = +1$ gives the result closest to the experimental one while for higher cut -off values $G_\Xi/G_N \simeq 3$ is the favoured case. The lifetime decreases with the increase in the value of the cut-off. If the anomalous magnetic moment of cascade particles is ignored the decay rate is 1.99×10^{16} or 0.47×10^{16} per sec. according as $G_\Xi/G_N = 1$ or -1 for $\Lambda \sim M_p$. Neglecting the anomalous magnetic moment effects[8] completely the decay rate is 0.18×10^{16} or 6.32×10^{16} per sec. according as G_Ξ/G_N is 1 or -1. If the cascade particles have no interactions with the pi-mesons and the anomalous magnetic moment interaction of nucleons is included agreement with the experimental result is found for a cut

-off $\Lambda \simeq 1.6\ M_p$.

The branching ratio $\pi^0 \longrightarrow e^- + e^+ + \gamma / \pi^0 + \gamma + \gamma$ is not affect appreciably.

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APPENDIX I

The third order S-matrix in the configuration space is given by ($\hbar = c = 1$)

$$S_3 = \frac{(-i)^3}{3!} \iiint d^4 X_1\, d^4 X_2\, d^4 X_3\, T(\mathcal{H}_I (X_1)\, \mathcal{H}_I (X_2)\, \mathcal{H}_I (X_3))$$

where the interaction Hamiltonian density $\mathcal{H}_I(X)$ is written for simplicity in illustration as

$$\mathcal{H}_I(X) = iG : \overline{\psi}\, \gamma_5\, \psi\, \varphi(X) : - ie : \overline{\psi}\, \gamma_\mu\, \psi\quad A_\mu :$$

where : : represents the "normal product" of operators. Suppressing the irrelevant indices and c-numbers the "time-ordered product" (T) relevant to our problem can be simplified using Wick's theorem[14] as follows:

$$T\Big[(:\bar{\psi}\psi\varphi(X_1):)(:\bar{\psi}\psi A(X_2):)(:\bar{\psi}\psi A(X_3):)$$

$$+ (:\bar{\psi}\psi A(X_1):)(:\bar{\psi}\psi\varphi(X_2):)(:\bar{\psi}\psi A(X_3):)$$

$$+ (:\bar{\psi}\psi A(X_1):)(:\bar{\psi}\psi A(X_2):)(:\bar{\psi}\psi\varphi(X_3):)\Big] = \Big[\dot{\bar{\psi}}(X_1)\dddot{\psi}(X_1)\dddot{\bar{\psi}}(X_2)\ddot{\psi}(X_2)\ddot{\bar{\psi}}(X_3)\dot{\psi}(X_3)$$

$$+ \dot{\bar{\psi}}(X_1)\ddot{\psi}(X_1)\ddot{\bar{\psi}}(X_2)\dot{\psi}(X_2)\dddot{\bar{\psi}}(X_3)\dddot{\psi}(X_3)\Big]\Big(A^{(-)}(X_2)\, A^{(-)}(X_3)\, \varphi^{(+)}(X_1)$$

$$+ A^{(-)}(X_1)\, A^{(-)}(X_3)\, \varphi^{(+)}(X_2) + A^{(-)}(X_1)\, A^{(-)}(X_2)\, \varphi^{(+)}(X_3)\Big) + \ldots$$

where $A^{\cdot} B^{\cdot} = \langle 0|T(AB)|0\rangle$ is the "contraction" of operators A and B.

Introducing the Fourier transforms (in interaction representation):

$$\varphi^{(+)}(X) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\, k}{\sqrt{2k_o}}\, C(k)\, e^{ik\cdot x}$$

$$A_\mu^{(-)}(X) = \frac{1}{(2\pi)^{3/2}} \int \frac{d^3\, k}{\sqrt{2k_o}} \left[\sum_{\lambda=0}^{3} a^{+(\lambda)}_{(k)}\ \ e^{(\lambda)}_{\mu(k)}\right] e^{-ik\cdot x}$$

$$\dot{\psi}_\alpha(X)\, \dot{\bar{\psi}}_\beta(y) = -\dot{\bar{\psi}}_\beta(y)\quad \dot{\psi}_\alpha(X) = -\frac{1}{2}\, S_{F_{\alpha\beta}}(x-y)$$

$$= \frac{-i}{(2\pi)^4}\left(\int d^4p\, \frac{e^{ip\cdot(x-y)}}{(i\not{p} + M)}\right)_{\alpha\beta}$$

the S-matrix in the momentum space reads after integration over X_i'S:

$$S_3 = \frac{i e^2 G}{\left[(2\pi)^{3/2}\right]^3} \iiint \frac{d^3 k d^3 k' d^3 q}{\sqrt{2k_o \, 2k_o' \, 2q_o}} (2\pi)^4 \delta^4(q-k-k') \, a_\mu^+(k) \, a_\nu^+(k') \, c(q).$$

$$(-1) \int \frac{d^4p}{(2\pi)^4} \, Sp \left[\gamma_5 \frac{-1}{(i\not p - i\not k) + M} \cdot \gamma_\mu \frac{-1}{(i\not p + M)} \cdot \gamma_\nu \frac{-1}{(i\not p + i\not k') + M} \right]$$

whence the required matrix element is

$$\langle k_1 \, e^{(\rho)}(k_1); \, k_2 \, e^{(\lambda)}(k_2) | S_3 | P \rangle = \langle 0 | a^{(\rho)}(k_1) \, a^{(\lambda)}(k_2) \, S_3 \, c^+(P) | 0 \rangle$$

$$= \frac{1}{\left[(2\pi)^{3/2}\right]^3} \frac{1}{\sqrt{2k_{1_o} \cdot 2k_{2_o} \cdot 2 P_o}} (2\pi)^4 \, \delta^4(P - k_1 - k_2) \cdot \mathbb{M}$$

$$\mathbb{M} = (-1) \int \frac{d^4p}{(2\pi)^4} \, Sp \left[\gamma_5 \frac{-1}{i(\not p - \not k_2) + M} \not e^{(\lambda)}_{(k_2)} \frac{-1}{(i\not p + M)} \not e^{(\rho)}_{(k_1)} \frac{-1}{i(\not p + \not k_1) + M} \right.$$

$$\left. + \gamma_5 \frac{-1}{i(\not p - \not k_1) + M} \not e^{(\rho)}_{(k_1)} \frac{-1}{(i\not p + M)} \not e^{(\lambda)}_{(k_2)} \frac{-1}{i(\not p + \not k_2) + M} \right] \cdot (-ie^2 G).$$

Here we have used the fact that

$$\langle 0 | a^{(\rho)}(k_1) \, a^{(\lambda)}(k_2) \, a_\mu^\dagger(k) \, a_\nu^\dagger(k') \, c(q) \, c^\dagger(p) | 0 \rangle$$

$$\equiv \delta(\vec{p} - \vec{q}) \left[\delta(\vec{k} - \vec{k}_2) \delta(\vec{k}' - \vec{k}_1) \, e_\mu^{(\lambda)}(k_2) \, e_\nu^{(\rho)}(k_1) \right.$$

$$\left. + \delta(\vec{k} - \vec{k}_1) \, \delta(\vec{k}' - \vec{k}_2) \, e_\mu^{(\rho)}(k_1) \, e_\nu^{(\lambda)}(k_2) \right]$$

since $\left[a^{(\rho)}(k),\ a_{\mu}^{+}(k')\right] = e_{\mu}^{(\rho)}(k)\ \delta(\vec{k}-\vec{k'})$

The decay rate is calculated in usual way; however, care should be taken that a (statistical) factor of (1/2) must be introduced to take into account of the fact that when integrating over all the directions of one of the photons we obtain each final state exactly twice.

* * *

APPENDIX II

Sum over the polarization states of two photons:

The dependence of the matrix element |M on the polarizations of the photons is contained in the factor

$$Sp(\gamma_5\, \not{e}^{(1)}\, \not{e}^{(2)}\, \not{k}_1\, \not{k}_2) = 4\,\varepsilon^{\mu\nu\lambda\rho} e_{\mu}^{(1)}\, e_{\nu}^{(2)}\, k_{1\lambda}\, k_{2\rho}$$

In the squared matrix element the polarization dependent factor is

$$16\,\varepsilon^{\mu\nu\lambda\rho}\,\varepsilon^{\mu'\nu'\lambda'\rho'}\, e_{\mu}^{(1)}\, e_{\mu'}^{(1)}\, e_{\nu}^{(2)}\, e_{\nu'}^{(2)}\, k_{1\lambda}\, k_{1\lambda'}\, k_{2\rho}\, k_{2\rho'}$$

The sum over the polarizations is conveniently done by summing covariantly over all four polarization vectors $e_{\mu}^{(\alpha)}(k)$, $(\alpha = 0, 1, 2, 3)$. The Lorentz condition will always assure that the timelike vector and the space-like vector in the propagation direction cancel exactly, so that effectively this covariant summation is equivalent to a summation over the two transverse polarizations of the free photon[15]. Using then

$$\sum_{\alpha=0}^{3} e_{\mu}^{(\alpha)}(k)\, e_{\nu}^{(\alpha)}(k) = \delta_{\mu\nu}$$

the squared matrix element summed over the polarizations of the two photons is

$$16\, \varepsilon^{\mu\nu\lambda\rho}\, \varepsilon^{\mu\nu\lambda'\rho'} k_{1_\lambda} k_{1_{\lambda'}} k_{2_\rho} k_{2_{\rho'}}$$

Noting that

$$\varepsilon^{\mu\nu\lambda\rho}\, \varepsilon^{\mu\nu\lambda'\rho'} = 2(\delta_{\lambda\lambda'}\, \delta_{\rho\rho'} - \delta_{\lambda\rho'}\, \delta_{\lambda'\rho})$$

it reduces to

$$32\left[k_1^2 \cdot k_2^2 - (k_1 \cdot k_2)^2\right] = -\,32(k_1 \cdot k_2)^2$$

which in the centre of mass frame of the two photons reduces to

$$-\,2^7\, \omega^4 \,.$$

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