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Pascual Jordan, his contributions to quantum mechanics and his legacy in contemporary local quantum physics

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Abstract

After recalling episodes from Pascual Jordan's biography including his pivotal role in the shaping of quantum field theory and his much criticized conduct during the NS regime, I draw attention to his presentation of the first phase of development of quantum field theory in a talk presented at the 1929 Kharkov conference. He starts by giving a comprehensive account of the beginnings of quantum theory, emphasising that particle-like properties arise as a consequence of treating wave-motions quantum-mechanically. He then goes on to his recent discovery of quantization of "wave fields" and problems of gauge invariance. The most surprising aspect of Jordan's presentation is however his strong belief that his field quantization is a transitory not yet optimal formulation of the principles underlying causal, local quantum physics. The expectation of a future more radical change coming from the main architect of field quantization already shortly after his discovery is certainly quite startling.

I try to answer the question to what extent Jordan's 1929 expectations have been vindicated. The larger part of the present essay consists in arguing that Jordan's plea for a formulation without "classical correspondence crutches", i.e. for an intrinsic approach (which avoids classical fields altogether), is successfully addressed in past and recent publications on local quantum physics.

Key-words: Scattering Theory; Quantum Field Theory; Particle Physics

1 Biographical Notes

There are not many physicists in whose biography the contradictions of human existence, the proximity of glorious scientific achievements and disturbing human weaknesses in the face of the great catastrophe of the 20th century, are as starkly reflected as in the personality of Pascual Jordan[1].

Born on October 18, 1902 in Hannover of mixed German-Spanish ancestry, he became (starting at age 22) a main architect of the conceptual and mathematical foundations of quantum theory and the protagonist of quantum field theory. Pascual Jordan owes his Spanish name to his great grandfather Pascual Jorda (probably unrelated to the biblical river), who came from the Alcoy branch (southern Spain) of the noble Jorda family with a genealogy which can be traced back to the 9th century. After the British-Spanish victory of Wellington over Napoleon, the family patriarch Pascual Jorda settled in Hannover where he continued his service to the British crown as a member of the "Koeniglich-Grossbritannisch-Hannoverschen Garde-Husaren Regiments" until 1833. Every first-born son of the Jordan (the n was added later) clan was called Pascual.

There is no doubt that Pascual Jordan took the lead in the formulation of the conceptual and mathematical underpinnings of "Matrix Mechanics" in his important paper together with Max Born [2] submitted on 27. September 1925 (3 months after the submission of Heisenberg's pivotal paper!) entitled "Zur Quantenmechanik". His mathematical preparation, particularly in the area of algebra, was superb. He had taken courses at the Göttingen mathematics department from Richard Courant and became his assistant (helping in particular on the famous Courant-Hilbert book); through Courant he got to know Hilbert before he met the 20 year older Max Born, the director of the Theoretical Physics department of the Göttingen university. By that time Jordan already had gained his physics credentials as a co-author of a book which he was writing together with James Franck [3].

After Max Born obtained Heisenberg's manuscript, he tried to make sense of the new quantum objects introduced therein. While he had the right intuition about their relation to matrices, he felt that it would be a good idea to look for a younger collaborator with a strong mathematics background. After Pauli rejected his proposal (he even expressed some reservations that Born's more mathematically inclined program could stifle Heisenberg's powerful physical intuition), Jordan volunteered to collaborate in this problem [4][5]. Within a matter of days he confirmed that Born's conjecture was indeed consistent. The Born-Jordan results made Heisenberg's ideas more concrete. Probably as a consequence of the acoustic similarity of pq with Pascual, the younger members of the physics department (the protagonists of the "Knabenphysik") in their discussions often called it the Jordan relation. Max Born became Jordan's mentor in physics. Jordan always maintained the greatest respect which withstood all later political and ideological differences.

The year 1925 was a bright start for the 22-year-old Jordan. After the submission of the joint work with Max Born on matrix mechanics, in which the p-q commutation relation appeared for the first time, there came the famous "Dreimaennerarbeit"[6] with Born and Heisenberg in November of the same year, only to conclude the year's harvest with a paper by him alone on the "Pauli statistics". Jordan's manuscript contained what is nowadays known as the Fermi-Dirac statistics; however it encountered an extremely unfortunate fate after its submission because it landed on the bottom of one of Max Born's suitcases (in his role as the editor of the Zeitschrift fuer Physik) on the eve of an extended lecture tour to the US, where it remained for about half a year. When Born discovered this mishap, the papers of Dirac and Fermi were already in the process of being published. In the words of Max Born [7][8] a quarter of a century later: "I hate Jordan's politics, but I can never undo what I did to him......When I returned to Germany half a year later I found the paper on the bottom of my suitcase. It contained what one calls nowadays the Fermi-Dirac statistics. In the meantime it was independently discovered by Enrico Fermi and Paul Dirac. But Jordan was the first"[2]. In Jordan's subsequent papers, including those with other authors such as Eugene Wigner and Oscar Klein, it was always referred to as "Pauli statistics" because for Jordan it resulted from a sraightforward algebraization of Pauli's exlusion principle.

From later writings of Born and Heisenberg we also know that Jordan contributed the sections on the

[1] The original title "Pascual Jordan, Glory and..." has been changed since although the birth of quantum theory represents one of the most glorious epochs in physics, Jordan's himself remained "the unsung hero" among the creators of that theory [1].

[2] In a correspondence with Stanley Deser, Stanley added a light Near East touch by remarking that without Max Born's faux pas the Fermions would have been called "Jordanions".

statistical mechanics (or rather kinetic gas theory) consequences to the joint papers on matrix mechanics; this is not surprising since the main point in his 1924 PhD thesis was the treatment of photons according to Planck's distribution whereas thermal aspects of matter were described according to Boltzmann. He continued this line of research by introducing the "Stosszahlansatz" for photons and using for electrons and atoms the Bose statistics [9][3] which brought praise by Einsteins and led to an unfortunately largely lost correspondence. In the following we will continue to mention his scientific contributions in the biographical context and reserve a more detailed account about their scientific content to the next section.

The years 1926/27 were perhaps the most important years in Jordan's career in which he succeeded to impress his peers with works of astonishing originality. The key words are Transformation Theory [11][12] and Canonical Anti-Commutation Relations [13]. With these discoveries he established himself as the friendly competitor of Dirac on the continental side of the channel and in its printed form one finds an acknowledgment of Dirac‘s manuscript[4]. As an interesting sideline, one notes that in a footnote at the beginning of the paper about transformation theory Jordan mentions a "very clear and transparent treatment" of the same problem in a manuscript by Fritz London, a paper which which he received after completing his own work and which was published in [14]. So it seems that the transformation theory was discovered almost simultaneously by three authors. Most physicists are more familiar with Dirac's notation (as the result of his very influential textbook whose first edition appeared in 1930). Jordan's most seminal contribution is perhaps his 1927 discovery of "Quantization of Wave Fields" which marks the birth of QFT. The reader finds a description about the chronology of this most important of of Jordan's discoveries, its relation to Dirac's radiation theory and its influence on the subsequent development of particle physics in the next section.

We are used to the fact that in publications in modern times the relation of names to new concepts and formulas should be taken with a grain of salt e.g. if we look at the original publication of Virasoro we are not terribly surprised that the algebra has no central term and consists only of frequencies of one sign (i.e. is only half the Witt algebra). But we trust that what the textbooks say about the beginnings of quantum mechanics can be taken literally. When we look at [11] page 811 we note that the relation between Born and the probability interpretation is much more indirect than we hitherto believed. Born in his 1926 papers was calculating the Born approximation of scattering theory and his proposal to associate a probability with scattering in modern terminology concerns the interpretation of the cross section. The generalization to the probability interpretation of the absolute square of the x-space Schrödinger wave function according to Jordan was done by Pauli[5] who was of course strongly influenced by Born. It happened frequently that new ideas were used freely in scientific discussions and that the later attachment of a person's name may represent the correct origin of that mode of thinking but cannot be taken prime facie when it comes to historical details.

His increasing detachment after 1933 from the ongoing conceptual development of QFT, and his concentration on more mathematical and conceptual problems of quantum theory whose investigation proceedes at a slower pace (and can be done without being instantly connected to the stream of new information) happens at the time of his unfortunate political activities, as he lets himself be sucked more and more into the mud of the rising Nazi-regime[6]. In trying to understand some of his increasing nationalistic and militaristic behavior it appears of some help to look at his background and upbringing, although a complete understanding would probably escape those of us who did not experience those times of great post-war turmoil.

Pascual Jordan was brought up in a traditional religious surrounding. At the age of 12 he apparently went through a soul-searching fundamentalist period (not uncommon for a bright youngster who tries to come to terms with rigid traditions) in which he wanted to uphold a literal interpretation of the bible against the materialistic Darwinism (which he experienced as a "quälendes Aergernis", a painful

[3] This paper was submitted simultaneously with another paper in which Jordan coined the term "Pauli-Principle" [10]; but the relation to statistics was only seen later.

[4] In those days papers were presented in a factual and very courteous style; however verbal discussions and correspondences were sometimes more direct and less amiable (e.g. see some published letters of Pauli [4][1]).

[5] This may partially explain why Pauli in his 1933 Handbuch article on wave mechanics introduced the spatial localization probability density without reference to Born.

[6] National Socialist=NS=Nazi. The colloquial expressions as Nazi and Sozi (socialist) originated from the times before Hitler's rise to power at the time of the big street fights between the rightwing and leftwing rowdies. The origin of the terminology "Nazi" is a bit like "Commie" (for communists) in the US, and for this reason not used by historians.

calamity), but his more progressive teacher of religion convinced him that there is basically no contradiction between religion and the sciences. This then became a theme which accompanied him throughout his life; he wrote many articles and presented innumerable talks on the subject of religion and science.

At the times of the great discoveries in QT many of his colleagues thought that the treaty of Versailles was unjust and endangered the young Weimar Republic, but Jordan's political inclination went far beyond and became increasingly nationalistic and right-wing. These were of course not very good prerequisites for resisting the temptation of the NS movement, in particular since the conservative wing of the protestant church (to which he adhered[7]) started to support Hitler in the 30's; in fact the behavior of both of the traditional churches during the NS regime belongs to their darkest chapters. Hitler presented his war of agression as a divine mission and considered himself as an instrument of God's predestination (göttliche Vorsehung), while almost all Christian churches were silent or even supportive.

Already in the late 20s Jordan published articles (under a pseudonym) of an agressive and bellicose stance in journals dedicated to the spirit of German Heritage; a characteristic ideology of right-wing people up to this day if one looks at the present-day heritage foundations and their political power in the US. It is unclear to what degree his more cosmopolitan academic peers in Göttingen knew about these activities. He considered the October revolution and the founding of the Soviet Union as extremely worrisome developments. One reason why Jordan succumbed to the NS-lure was perhaps the idea that he could influence the new regime; his most bizarre project in this direction was to convince the party leaders that modern physics as represented by Einstein and especially the new Copenhagen brand of quantum theory was the best antidote against the "materialism of the Bolsheviks". This explains perhaps why he joined NS organisations at an early date when there was yet no pressure to do so [15]. He of course failed in his attempts; despite verbal support[8] he gave to their nationalistic and bellicose propaganda and even despite their very strong anti-communist and anti-Soviet stance with which he fully agreed, the anti-semitism of the Nazis did not permit such a viewpoint since they considered Einstein's relativity and the modern quantum theory with its Copenhagen interpretations as incompatible with their anti-semitic propaganda; one can also safely assume that the intense collaboration with his Jewish colleagues made him appear less than trustworthy in the eyes of the regime.

Jordan's career during the NS times ended practically in scientific isolation at the small university of Rostock (his promotion to fill von Laue's position in Berlin in 1944 was too late for a new start); he never received benefits for his pro-NS convictions and the sympathy remained one-sided. Unlike the mathematician Teichmueller, whose rabid anti-semitism led to the emptying of the Göttingen mathematics department, Jordan inflicted the damage mainly on himself. The Nazis welcomed his verbal support, but he always remained a somewhat suspicious character to them. As a result he was not called upon to participate in war-related projects and spent most of those years in scientific isolation. This is somewhat surprising in view of the fact that Jordan, like nobody else, tried to convince the NS regime that fundamental research should receive more support because of its potential weapons-related applications; in these attempts he came closer to a "star wars" propagandist of the Nazis than Heisenberg who headed the German uranium program but never joined the party.

Jordan's party membership and his radical verbal support in several articles got him into trouble after the war. For two years he was without any work and even after his re-installment as a university professor he had to wait up to 1953 for the reinstatement of his full rights (e.g. to advise PhD candidates). When his friend and colleague Wolfgang Pauli asked him after the war: "Jordan, how could you write such things?" Jordan retorted: "Pauli, how could you read such a thing?" Without Heisenberg's and Pauli's help he would not have been able to pass through the process of de-nazification (in the jargon of those days Jordan got a "Persilschein", i.e. a whitewash paper) and afterwards to be re-installed as a university professor. In Pauli's acerbic way of dealing with such problems: "Jordan is in the possession of a pocket spectrometer by which he is able to distinguish intense brown from a deep red". "Jordan served every regime trustfully" is another of Pauli's comments. Pauli recommended Jordan for a position at the University of Hamburg and he also suggested that he should keep away from politics.

Jordan did not heed Pauli's advice for long; during the time of Konrad Adenauer and the big debates

[7] The oldest son of the family patriarch Pascual Jorda was brought up in the Lutheran faith of his foster mother, whereas all the other children born within that marriage were raised in the Catholic faith.

[8] In contrast to Heisenberg he did not directly work on any armament project but rather did most of his military service as a meteorologist.

about the re-armament of West Germany he became a CDU member of parliament. His speech problem (he sometimes fell into a stuttering mode which was quite painful for people who were not accustomed to him) prevented him from becoming a scientific figurehead of the CDU party. At that time of the re-armament issue there was a manifesto by the "Göttingen 18" which was signed by all the famous names of the early days of the university of Göttingen quantum theory, including Max Born. Jordan immediately wrote a counter article with the CDU parties blessing, in which he severely criticized the 18 and claimed that by their action they endangered world peace and stability. Max Born felt irritated by Jordan's article, but he did not react in public against Jordan's opinion. What annoyed him especially were Jordan's attempts to disclaim full responsibility for his article by arguing that some of the misunderstandings resulted from the fact that it was written in a hurry. But Born's wife Hedwig exposed her anger in a long letter to Jordan in which she blamed him for "deep misunderstanding of fundamental issues". She quoted excerpts from Jordan's books and wrote: "Reines Entsetzen packt mich, wenn ich in Ihren Büchern lese, wie da menschliches Leid abgetan wird" (pure horror overcomes me when I read in your books how human suffering is taken lightly). Immediately after this episode she collected all of Jordan's political writings and published them under the title: "Pascual Jordan, Propagandist im Sold der CDU" (Pascual Jordan, propagandist in the pay of the CDU) in the Deutsche Volkszeitung.

In the middle of the twenties the authors of the "Dreimaennerarbeit" were proposed twice for the Nobel prize by Einstein, but understandably the support for Jordan dwindled after the war. Nevertheless, in 1979 it was his former colleage and meanwhile Nobel prize laureate Eugen Wigner who proposed him. But at that time the Nobel committee was already considering second generation candidates associated with the second phase of QFT which started after the war with perturbative renormalization theory; there was hardly any topic left of the first pioneering phase which was not already taken into account in previous awards. Jordan did however receive several other honors, including the Max-Planck-medal of the German Physical Society.

Jordan did not only have to cope with political satirical remarks such as those from Pauli, but as a result of his neutrino theory of the photon [16] he also received some carnavalesque good-humored criticism as in the following song (the melody is that of Mack the Knife) [4]:

"Und Herr Jordan	"Mr. Jordan
Nimmt Neutrinos	takes neutrinos
Und daraus baut	and from those he
Er das Licht	builds the light.
Und sie fahren	And in pairs they
Stets in Paaren	always travel.
Ein Neutrino sieht man nicht."	One neutrino's out of sight".

Actually Jordan's idea is less "crazy" than it appears at first sight. In the third section we will return to some interesting points concerning the change of the concept of bound states in passing from quantum mechanics to quantum field theory.

Although Jordan took (along with the majority of German physicists) a strong position against those supporting the racist "German Physics"[9] and in this way contributed to their downfall, he defended bellicose and nationalistic positions and he certainly supported Hitler's war of aggression against the "Bolshevik peril". The fact that he was a traditional religious person and that several of the leading bishops in the protestant church were pro Hitler had evidently a stronger effect on him than his friendship with his Jewish colleagues, who by that time had mostly left Germany (in some cases he tried to maintain a link through correspondence).

In contrast to Pauli who contributed to the second post war phase of QFT and always followed the flow of ideas in QFT up to his early death, Jordan's active participation in QFT stopped around the middle of the 30s and it seems that he did not follow the development in that area. He turned his attention to more mathematical and conceptual problems as well as to biology [17] and psychology. His enduring interest in psychology was presumably related to the psychological origins of his stuttering handicap[10] which

[9] It was Jordan's opinion that nationalistic and racist views had no place in science; in his own bellicose style of ridicule (in this case especially directed against nationalistic and racist stance of the mathematician Biberbach): "The differences among German and French mathematics are not any more essential than the differences between German and French machine guns".

[10] One also should keep in mind that the interest in psychology became a "fashion" among the Copenhagen physicists (notably Bohr and Pauli).

prevented him from using his elegant writing style in discussions with his colleagues and communications with a wider audience; this perhaps explains in part why even in the physics community his contributions are not as well known as they deserve to be. In fact this handicap even threatened his Habilitation (which was a necessary step for an academic career) in Göttingen. Jordan was informed by Franck (with whom he had coauthored a book) that Niels Bohr had arranged a small amount of money for Jordan which was to be used for getting some cure of his speech problem. Wilhelm Lenz (whose assistant Jordan was for a short time after Pauli left) suggested to go to the famous psychologist Adler. Jordan went to Vienna, but we only know that he attended a lecture of Schrödinger and criticized his wave mechanics from the Göttingen point of view; there is no record of meetings with Adler.

His increasing withdrawal from the mainstream of quantum field theory and particle physics in the 30s may have partially been the result of his frustration that his influence on the NS regime was not what he had expected. After the defeat of Germany in 1945 his attempts to account for his membership in the Nazi party as well as the difficult task to make a living with the weight of his past NS sympathies (which cost him his position as a university professor for the first two years after the war) seriously impeded his scientific activities.

Unlike the majority of the German population, for which the early Allied re-education effort (which was abandoned after a few years) to rid society of aggressive militaristic and racist ideas was a huge success so that the subsequent change of US policy in favor of re-armament of West Germany ran into serious opposition during the Adenauer period, Jordan did not completely abandon his militaristic and rightwing outlook. In the 50s he joined the CDU party where he had to undergo the least amount of change [11], thus forgetting Pauli's admonitions in favor of political abstinence.

All the protagonists of those pioneering days of quantum physics have been commemorated in centennials except Pascual Jordan who, as the result of the history we have described, apparently remained a somewhat "sticky" problem despite Pauli's intercession by stating "it would be incorrect for West Germany to ignore a person like P. Jordan". His postwar scientific activities consisted mainly in creating and arranging material support (by grants from Academies and Industry as well as from the US Airforce) for a very successful group of highly motivated and talented young researchers in the area of General Relativity who became internationally known (Engelbert Schuecking, Juergen Ehlers,..) and attracted famous visitors especially from Peter Bergmann's group (Rainer Sachs,....). In this indirect way there is a connection between Jordan's post war activities in general relativity and the new Albert Einstein Institute in Golm (Potsdam). This somewhat meandering path leads from Jordan's Seminar in Hamburg through universities in Texas (where most of its members got positions), and then via the astrophysics in Garching (where Ehlers took up a position in 1971) to the AEI for Gravitational Physics of the MPI where Ehlers became the founding director in 1995.

Jordan died in 1980 (while working on his pet theory of gravitation with a time-dependent gravitational coupling); his post war work never reached the level of the papers from those glorious years 1925-1930 or his subsequent rather deep pre-war mathematical physics contributions. In the words of Silvan Schweber in his history of quantum electrodynamics, Jordan became the "unsung hero" of a glorious epoch of physics which led to the demise of one of its main architects.

It is however fair to note that with the exception of Max Born, Jordan's other collaborators, especially von Neumann and Wigner, shared the bellicose kind of anti-communism; Wigner later became an ardent defendor of the Vietnam war. Since both of them came from a cosmopolitan Jewish family background, their anti-communist fervor probably had its roots in their experience with the radical post World War I Bela Kuhn regime in the Hungarian part of the decaying Habsburg empire. Hence Jordan's right wing anti-communist views posed no friction during the time of his collaboration with Wigner and von Neumann.

The cultural and scientific achievements in a destroyed and humiliated Germany of the post World War I Weimar republic within a short period of 15 years belong to the more impressive parts of mankind's evolution and Jordan, despite his nationalistic political viewpoints is nevertheless part of that heritage.

[11] The leadership of the CDU recently supported Bush's war in Iraq (against the majority of its voters).

2 Jordan's contributions to Quantum Mechanics and to the first phase of Quantum Field Theory

In the following I will give a more detailed account of the content of some of Jordan's most seminal contributions to QM and QFT. Like the problem of appreciating the significance of the conceptual step from Lorentz to Einstein who share the same transformation formula, there are several instances where important progress is not primarily reflected in formulae but in paradigmatic changes of interpretations. Since the fashions of the last decades have led to an atrophy of the art of interpretation as compared to calculation, a careful presentation of the conceptual progress achieved in Jordan's work may be helpful.

The situation which Jordan was confronting, after he was called upon by Born to collaborate on the mathematical and conceptual underpinnings of Heisenberg's pivotal work, was as follows. Heisenberg had gone far beyond the somewhat vague correspondence principle by his proposal to substitute for the unobservable classical particle variables q,p a novel kind of in principle observable set of complex amplitudes obeying a non-commutative multiplication law. These quantities were supposed to satisfy a "quantum condition" which formally resembles the Thomas-Kuhn sum rule[12] for harmonic oscillators but which, in terms of the new quantities was to be regarded as a basic universal quantum law. Born immediately identified the new quantities as matrices. Moreover he found Heisenberg's quantum condition to assert that all diagonal elements of $pq - qp$ are equal to $\frac{\hbar}{i}$, whereupon he conjectured the famous commutation relation.

Jordan quickly succeeded to prove the vanishing of the off-diagonal elements as a consequence of the equations of motion and an ingenious algebraic argument [2]. From that time on the (matrix-form of the) commutation relation was the new principle of quantum mechanics. The subsequent "Dreimännerarbeit" [6] extended the setting of quantum mechanics to systems with many degrees of freedom.

Shortly after this paper, Jordan together with Heisenberg [18] demonstrated the power of the new formalism by presenting the first fully quantum-mechanical treatment of the anomalous Zeeman effect based on the new electron spin hypothesis of Goudsmit and Uhlenbeck. In this article the authors left no doubt that they considered this as temporary working hypothesis short of a relativistic description of the electrons intrinsic angular momentum; a problem which still had to wait three years before Dirac finally laid it to rest.

Most of Jordan's works after 1926 deal with "quantization of wave fields" i.e. with quantum field theory. There is however one important later contribution which addresses the foundations of quantum physics and led to an algebraic structure which bears his name, the so-called Jordan algebras. If one accepts von Neumann's axiomatic framework of quantum theory, which identifies observables with Hermitian operators acting in a Hilbert space, one would like to have at one's command a multiplication law which converts two observables into a composed observable. This is achieved by taking the anti-commutator and Jordan posed the problem of unravelling the algebraic structure which one obtains if one disposes of the Hilbert space setting and axiomatizes this abstract algebraic structure. The result was a commutative but not associative new structure, the so-called Jordan algebras [19]. This attracted the interest of von Neumann and Wigner and led to some profound mathematical results. In a joint paper [20] they proved that (apart from a very special exceptional case that for finite dimensional Jordan algebras without loss of generality one obtains ordinary matrix algebras with the anti-commutator composition law. This showed that the standard quantum theory setting of operator algebras in Hilbert space as axiomatized by von Neumann was more natural and stable against modifications than one had reasons to expect on the basis of Heisenberg's observable credo. This work was later extended to the infinite dimensional realm (by making suitable topological assumptions about Jordan algebras) without encountering any additional exception [21].

Works on Jordan algebras, to the extent that they are physically motivated, should be viewed in line with other attempts to obtain a better understanding of the superposition principle[13]. The latter is the fact that states in the sense of positive linear forms on algebras permit an (in general nonunique)

[12] In comparision with Lorentz versus Einstein where the significant conceptual difference did not necessitate a different notation for the transformation formula, the new Heisenberg world required a totally different notation as that in the papers of Thomas and Kuhn.

[13] The total Hilbert space of a quantum physical system decomposes into coherent Hilbert subspaces which are subject to the superposition principle (i.e. the von Neumann axiom that with two physically realizable vectors any vector in their linear span is also physically realizable) is valid.

interpretation as expectation values in vector states (or in terms of density matrices in case the states are impure) in a Hilbert space which carries an operator algebra realizations of the abstract algebra such that the linear combination of two vectors defines again a (pure) state. Since quantum states are conceptually very different from classical linear waves, the history of quantum physics up to this date is rich of attempts which try to make the quantum superposition principle physically more palatable. The dual algebra-state relation permits to explore this problem either on the algebraic side in the spirit of Jordan or on the side of the structure of the state space [22]. The farthest going results on the side of states was obtained by A. Connes [23] who succeeded to characterize operator (von Neumann) algebras in terms of the facial substructure of their convex state spaces (the predual of algebra).

In the remainder of this section we recall Jordan's most important and enduring discovery namely that of quantum field theory. The conceptual difference of what Jordan did as compared with Dirac's "second quantization" is somewhat subtle. The latter was an artful transcription (the Fock space formalism was not yet available) of the Schrödinger multi-particle setting into a form of what in modern terminology is called the occupation number representation; for the formal backup of this step Dirac used his version of the transformation theory.

Jordan had the bold idea to view the one-particle Schrödinger wave function as a classical wave equation to be subjected to the extended quantization rules which one would naturally impose on the canonical field formalism derived from classical Lagrangians. By a detailed calculation Jordan [24] showed that this procedure, projected onto the n-particle subspace, is equivalent to Dirac's occupation number description. At that time Jordan met Oscar Klein in Copenhagen, both of them were guests of Niels Bohr. In a widely acclaimed paper they jointly extended the field quantization formalism to the case with interactions [26].

Jordan considered it as a significant advantage that his new viewpoint allowed to incorporate "Pauli statistics" together with Bose-Einstein statistics into one and the same formalism; in fact some of the wave field quantization ideas were already contained in the concluding sections of [13] which delt with anti-commutation relations. He returned to this subject in a joint work with Wigner (submitted in Januaray 1928) which contains significant extensions and clarifications [25]. This paper is not only quoted as an alternative approach to Dirac's presentation of anti-commutation relations, the Jordan-Wigner method also received particular attention in connection with nonlocal transformations which are capable of changing commutation relations. In this paper Jordan and Wigner discovered an abstract (but highly ambiguous) method to write Fermions in terms of "Paulions" i.e Pauli matrices (which however as a result of Jordan's lifelong love for quaternions appear in a quaternionic camouflage). The ordering prescription which they need in order to write concrete Fermion formulas in the Hilbert space of a discrete array of Pauli spin matrices becomes physically unique in case of the presence of a natural spatial ordering as in the transfer matrix formalism of the 2-dim. Lenz-Ising model [28]. This kind of nonlocal formula involving a line integral (a sum in the case of a lattice model) became the prototype of statistics changing transformations (bosonization/fermionization) in d=!+1 models of quantum field theory [29][30] and condensed matter physics [31]

It is interesting to observe Dirac's reaction; different from the general enthusiastic acceptance of Jordan's approach he was somewhat disappointed that Jordan's version of second quantization[14] did not yield more results beyond the ones he himself had worked out. He also was not much impressed by the formal incorporation of the anti-commutation structure into Jordan's wave field quantization setting. His first complaint was of cause rendered unjustified as soon as the new field quantization approach was applied to the relativistic setting [27] were the interaction-caused vacuum polarization and real particle creation left no alternative. I think that his second point was more of an estetical and philosophical nature since it appears somewhat unnatural to invent a fictitious classical reality (namely classical Fermions, nowadays often equipped with a Grassmann structure) just in order to be able to grind halfinteger spin through the same quantization mill. As will be explained in the next section the new approach of Local Quantum Physics [46] avoids this problem by its dichotomy of local observables and local field systems as irreducible representation (superselection) sectors of these observables.

The continuation of the field quantization saga is well known. After the 1928 Jordan-Pauli paper on the spacetime treatment (overcoming the canonical formalism -aused separation into space and time) in which the free field Jordan-Pauli commutator functions appeared for the first time, the field quantization

[14] The present day usage of this terminology is in the sense of Jordan's field quantization approach.

torch was taken over by Heisenberg and Pauli. The subsequent study of properties of the vacuum problems in QFT brought the QFT train to a grinding halt. After the second world war the new locomotive of renormalization led to a remarkable recovery and a new faith in the underlying principles of QFT but the team running the train had changed. Instead of the meanwhile grown up members of the Knabenphysik of the Goettingen days, the physicists in the driving seat consisted to a large degree of young Americans; the physics language changed from German to English.

In 1929 at a conference in Kharkov[15], Jordan gave a remarkable plenary talk [32] (the conference language at that time was still German). In a way it marks the culmination of the first pioneering phase of QFT; but it also already raised some of the questions which were partially answered almost 20 years later in the second phase of development (i.e. renormalized perturbation theory, gauge theory). In his talk Jordan reviews in a very profound and at the same time simple fashion the revolutionary steps from the days of matrix mechanics to the subsequent formulation of basis-independent abstract operators (the transformation theory which he shares with Dirac) and steers then right into the presentation of the most important and characteristic of all properties which set QFT apart from QM: Locality and Causality as well as the inexorably related Vacuum Polarization. Already one year before in his Habilitationsschrift he identified the two aspects of relativistic causality namely the statistical independence for spacelike separations (Einstein causality, commutance of observables) and the complete determination in timelike directions (the causal shadow property) as playing a crucial role in the new quantum field theory. He ends his presentation by emphasizing that even with all the progress already achieved and that expected to clarify some remaining unsatisfactory features of gauge invariance (*Die noch bestehenden Unvollkommenheiten, betreffs Eichinvarianz, Integrationstechnik usw., duerften bald erledigt sein)*, one still has to confront the following problem[16]: *Man wird wohl in Zukunft den Aufbau in zwei getrennten Schritten ganz vermeiden muessen, und in einem Zuge, ohne klassisch-korrespondenzmaessige Kruecken, eine reine Quantentheorie der Elektrizitaet zu formulieren versuchen. Aber das ist Zukunftsmusik.* (In the future one perhaps will have to avoid the construction in two separated steps and rather have to approach the problem of formulating a pure quantum theory of electricity (a pure quantum field theory) in one swoop, without the crutches of classical correspondences. But this is part of a future tune.)

He returns on this point several times, using slightly different formulations (*....muss aus sich selbst heraus neue Wege finden)* for a plea towards a future intrinsic formulation of QFT which does not have to take recourse to quantization which requires starting with a classical analog.

These statements are even more remarkable if one realizes that they come from the protagonist of field quantization only two years after this pivotal discovery. When I accidentally came across the written account of this Kharkov talk, I was almost as surprised as I was many years before when it became known that Oscar Klein (with whom Jordan collaborated in the 30s in Copenhagen) had very advanced ideas about nonabelian gauge theory (which were published in the proceedings of the 1939 Warsaw international conference [33]). Apparently all of the classical aspects of nonabelian gauge theories and some quantum aspects were known before the second world war. Apart from QED the postwar interest developed away from gauge theories into pion-nuclear physics. The great era of gauge theory in connection with strong interactions had to wait 3 decades, and by that time Klein's work was completely forgotten and played no role in the birth of QCD.

In the following two section we will try to convince the reader that Jordan's expectations about a radically different realization of the underlying principles of QFT have meanwhile been realized in Local Quantum Physics (LQP) [46]. This approach has rejuvenated QFT by leading to a wealth of new questions which are presently being investigated by new methods. Again there is no direct historical connection between Jordan's ideas and modern LQP. Rather these episodes confirm the belief that although important ideas in the exact sciences may get lost in certain situations, sooner or later they will be rediscovered and expanded.

Jordan's expectation about a rapid understanding of the "imperfections of gauge theories" at the time of his Kharkov talk may have been a bit optimistic since the Gupta-Bleuler formalism only appeared 20 years later [34]. For Jordan gauge theory was an important issue already in 1929.

[15] Landau, after his return from Copenhagen, went to the university of Kharkov which for a short time became the "Mecca" of particle physics in the USSR.

[16] Here we have actualized in brackets the content of this sentence since "QED" (the only existing QFT in those days) was used in the same way as "QFT" in present days.

Finally one should add one more topic which shows that Jordan and Dirac had a very similar taste for what both considered the important problems of the times (in addition to: statistics and (anti)commutation relations, operator transformation theory, second quantization): magnetic monopoles. In this case Dirac is the clear protagonist of this idea, but Jordan found an interesting very different algebraic argument for the monopole quantization which he based on the algebraic structure of bilinear gauge invariants [35] (in the setting in which Mandelstam 30 years later attempted a gauge invariant formulation) adapted to the magnetic flux through a tetrahedron. The problem of how these kind of arguments have to be amended in order to take account of renormalization has according to my best knowledge not been satisfactorily answered.

Since Jordan left the area of QFT in the middle of the 30s and became disconnected from the discoveries thereafter, we do not know how he would have reacted to the amazing progress after the war brought about by renormalization theory. But contrary to expectations of the leading theorist during the 30s this progress was obtained by a careful conceptual distinction between formal and physical (observed) Lagrangian parameters and a systematic extension of the existing formalism i.e. it was achieved in a rather conservative manner [36]. Renormalization theory is too conservative as far as field quantization is concerned in order to serve as a candidate which could fit Jordan's.radical expectations. In strange contrast to his political stance, in physics Jordan was a visionary revolutionary.

3 Jordan's quantum field theoretical legacy in past and present achievements

In times of stagnation and crisis as the one we presently face in the post standard model era of particle physics, it is helpful to look back at how the protagonists of quantum field theory viewed the future and what became of their ideas and expectations. Perhaps the past, if looked upon with care and hindsight, may teach us where we possibly took a wrong turn and what alternative path was available.

Jordan's Kharkov talk marks a high point in presentations of developments of QFT and in particular his own participation in the shaping of physical concepts, but in a way it also can be viewed as rounding off the pioneering stage of QFT; in the nearly 20 years up to the beginnings of the second stage of perturbative renormalization theory due to Feynmann, Schwinger and Dyson (with important conceptual methodical and computational contributions by Kramers, Tomonaga, Bethe and many others), there was a kind of conceptional lull apart from some isolated but important contributions.

One significant exception whose full potential was appreciated only much later is Wigner's famous 1939 group representation-theoretical approach [37] to particles and their classification.

In fact Wigner accomplished in a very limited context to obtain the kind of intrinsic formulation that Jordan in his Kharkov talk was contemplating for a post quantized wave field approach. Instead of describing particles in terms of wave equations, which leads to many different-looking but nevertheless physically equivalent formulations, Wigner showed that one obtains a completely intrinsic and unique description of the possible wave function of relativistic particles without the necessity of quantizing classical structures by simply classifying irreducible positive energy ray representations of the Poincaré group[17]. One first determines the irreducible representations of the (universal covering of the) connected part and then extends to the full Poincaré group with the help of the parity and (anti-unitary) time reversal transformations which in some cases requires a doubling of the representation space. The standard way to come from the unique Wigner (m,s) representations to quantized free fields is well known [36]. The important step is the classification of u- and v- intertwiners which relate the unique (m, s) Wigner representation to covariant (spinor,tensor) representations; there is a countably infinite family of covariant representations originating from one (m, s) Wigner representation. The infinitely many pointlike fields are all singular operators (operator-valued distributions) in the same Hilbert space, in fact they turn out to be different "field coordinatizations" of a net of local algebras which is uniquely associated with the Wigner representation. However they do not exhaust the possibilities of pontlike coordinatizations of algebras but only constitute a linear subset; nonlinear coordinatizations are obtained by taking Wick

[17]Due to the projective nature of quantum mechanical states, one has to classify projective (ray) representations, but for many groups including the Poincaré group (depending on group cohomology) this can be encoded into vector representations of their universal covering.

polynomials in those fields.

The intrinsic field-coordinatization-independent description in the spirit of Jordan's Kharkov talk is that of the spacetime-indexed net of local algebras[18] [46]. If we had to construct first pointlike fields in order to find the net of algebras associated with the (m,s) Wigner representation, we would not have gained much as compared to the standard approach in [36]. There exists however a direct path from the unique (m, s) Wigner representation to the unique net of operator algebras via modular theory [38][39] without passing through nonunique field coordinatizations.

The idea is the following. The first step consists in characterizing a real Hilbert subspace $H_r(W) \subset H_{Wig}$ of the complex Wigner space which describes wave functions which are modular localized in a (Rindler) wedge region $x \geq |t|\,; y, z$ arbitrary. This is done with the help of an involutive closed antilinear Tomita S_T-operator (the +1 eigenstates of S_T) which is defined in terms of the Wigner representation theory (see next section). Being in the domain of S_T, the wedge-localized momentum space Wigner wave functions possess analyticity properties which permit a continuation to negative energies within the complex mass shell. This is reminiscent of the Klein paradox, which links positive with negative energies and drives the situation away from a standard quantum mechanical one-particle setting, except that in our case there is no x-space wave function and no coupling to external fields. The analytic continuation to negative energies in the $H_r(W)$ wave functions resembles a crossing operation [39]. In fact it turns out that these modular localization properties preempt among other things the spin-statistics connection and the TCP theorem which one usually derives in the full quantum field theory.

The remaining step in the construction of the net of operator algebras is quite simple. For integer spin the Weyl functor maps the wedge-localized real subspace into the wedge-localized operator (von Neumann) subalgebra $\mathcal{A}(W)$ of the algebra of all bounded operators $B(H)$ in Fock space, and the CAR functor accomplishes the analogous result for halfinteger spin[19]. The full net is obtained from the net of wedge-localized algebras by algebraic intersection. It is customary to consider the causally closed Lorentz-covariant family of double cone (diamond-shaped) regions $\mathcal{O}$ as the generating sets of Minkowski spacetime in which case the von Neumann algebras $\mathcal{A}(\mathcal{O})$ are the smallest building blocks of the net. The localization-core of double cones of arbitrary size is what one gets if one scales down the size to to zero, namely a spacetime point.

For the standard Wigner particle representations with s=(half)integer (in case of m=0, s is the helicity) this modular construction is mainly of methodological interest since the covariantization approach [36] leads to free fields which also provide a complete physical description. The main advantage of the operator algebra approach is that the somewhat singular nature (which unleashes its full "nastiness" only if one uses these singular objects for implementing interactions) of pointlike fields are avoided. The intersections of operator algebras whose localization regions intersect in a point consists of the trivial algebra (multiples of the identity) and hence pointlike fields are idealized singular objects associated to the algebras in the sense that after smearing with $\mathcal{O}$-supported test functions one obtains unbounded operators affiliated with $\mathcal{A}(\mathcal{O})$. This is what is meant by saying that pointlike fields constitute a "singular coordinatization" of the net of operator algebras. Jordan did of course obtain his quantum wave fields not as a singular coordinatization of algebras but by quantizing classical wave fields. The singular nature of these quantized fields (later mathematical research identified them as operator-valued distributions) may have been a motivation for contemplating an implementation of the physical principles underlying QFT without quantization "crutches".

The crucial question is whether such a viewpoint also exists in the presence of interactions. Before we address this problem (which will be the central issue of this and the last section), it is important to note that there are two special positive energy representations which do not belong to the above standard cases. One of these special cases is Wigner's famous zero mass "continuous spin" representation which has an internal structure described by a helicity spectrum which resembles those infinite towers which characterize the "stringyness" of string theory. The application of the modular localization theory reveals that their Wigner spaces *do not admit nontrivial compactly localized subspaces* $H_r(\mathcal{O})$ for e.g. $\mathcal{O}$ = double

[18] The impression that Haag was led to his intrinsic formulation as a result of Jordan's 1929 Kharkov remarks is of course wrong, inasmuch as Yang and Mills were unaware of the content of Klein's 1939 Warsaw talk on nonabelian gauge fields. But it shows that good ideas often have prophetic precursors.

[19] These functors map the category of real Hilbert spaces into von Neumann operator algebras in such a way that spatial inclusions pass into algebraic ones. It only exists in the absence of interactions and often refered to as "second quantization". In the words of E. Nelson: (first) quantization is a mystery, but second quantization is a functor.

cone; rather their smallest nontrivial localization regions are (arbitrarily narrow) *spacelike cones*[20] (i.e. $O =$ spacelike cone) *whose localization-cores are semiinfinite (straight) open "strings"* [38][39]. Although these "Wigner strings" have a vague resemblence to the classical Nambu-Goto string (the ur-version of string theory) in that they are objects which possess a rich internal spacetime related structure, the differences between them are much more important in the present context. *A Wigner string is a pure quantum string*, i.e. it can not be interpreted as the result of a quantization applied to a classical object and hence *does not permit a Lagrangian description.* This is the reason why in Weinberg's book [36] (whose main theme in the first section is the use of Wigner's theory as an additional support for Lagrangian quantization) the zero mass helicity tower representations play no role. The corresponding quantum fields are (as in the standard case) objects which are sharply localized on the localization core of the spacelike cone regions and have to be smeared in the localization-core in order to obtain Fock space operators whose one-fold application to the vacuum generate the localized Wigner subspaces. The standard u, v intertwiner construction [36] from Wigner- to covariant- representations has to be generalized in a nontrivial manner [40]. If Jordan would have known these consequences of the later theory of his colleague Wigner, he could have noticed that his farsighted post-quantization outlook would have been supported by this example which (aside from the avoidance of singular coordinatizations) does not admit a quantization description.

The other special case is that of massive particles in d=1+2; it does not appear in Wigner's d=1+3 list but was later analyzed by Bargman [41]. It turns out that in this case the localization-core of the smallest nontrivial localization region is also semiinfinite stringlike, but in this case the mechanism is not related to the rich internal structure but rather results from the fact that the d=1+2 Poincaré group has an infinite sheeted universal covering [42]. The projective nature of quantum theory always requires the use of the covering in order to include with the generic case. The latter leads to (m,s) representations with s=real, unquantized. For s=halfinteger the "anyonic" spin reduces to Bosons/Fermion Wigner spaces which admit nontrivial compact localization (with pointlike localization-cores).

The anyonic strings for $s \neq$ halfinteger are quite different from those of the helicity towers; they are more like those strings which Mandelstam envisaged for the description of charge-carrying objects in gauge theories except that their "living space" requires the topology of an auxiliary 2-dimensional de Sitter space in order to characterize the asymptotic string directions relative to a reference direction [42]. In contrast to the tensor product structure of the "string field theory" of the Wigner strings, the repeated applications of "free" anyonic string operators would not be compatible with tensor product factorization, i.e. they are not free fields in the usual technical sense. This is related to the fact that they obey braid group commutation relations which inevitably cause vacuum polarization clouds in their one-field states obtained from their application to the vacuum[21]; in fact their nonrelativistic limit would not allow a quantum-mechanical description if one insists upholding (as in case of Fermions/Bosons) the spin-statistics connection. The interesting question in the absence of a quantization procedure is of course whether one can generate the associated operator algebras by "smearing" singular field coordinates with covariance properties which reflect their stringlike nature. In the next section we will present powerful ideas from modular theory which indicate a solution for this kind of problem.

There are two important messages coming from the Wigner theory which illustrate the trans-quantization idea of Jordan in his plea for abandoning the quantization "crutches". The first message is that the non-unique singular pointlike free fields can be avoided in favor of a direct construction of the unique local net of algebras which they generate. The second message is that *the most general localization cores (associated to optimally localized subspaces of Wigner positive energy representations) are semiinfinite stringlike* and in this case these objects do not possess a classical counterpart. This is remarkable since this stringlike geometry is not there because some physicists set out to study what happens if one passes from point- to string-like extensions, but rather because the stability principle underlying the positive energy representations permit only point- and string-like realizations! Whereas in the stringlike cases the double cone localized one-particle subspaces are trivial (the nullvector), one expects that the associated field theory also admits double-cone localized observables (string/anti-strings), so that the string localiza-

[20] A spacelike cone is a noncompact causally complete region which consists of a conic neighbourhood of a semiinfinite spacelike line (the core) with the line starting at the apex of the cone.

[21] Braid group statistics can only be maintained in the presence of vacuum polarization clouds, even if there are no genuine interactions (zero cross sections), i.e. even in the case of "free anyons" [43].

tion refers to charge-carrying fields and the neutral local observables remain compactly localizable with pointlike localization-cores. The key question concerning local quantum physics is therefore: is semiinfinite stringlike localization the worst localization-core which can happen for charge-carrying optimally localized objects in LQP? Under somewhat more restrictive assumption (positive energy with mass gaps) the answer is affirmative; this is a fundamental result of Buchholz and Fredenhagen [46].

The generalization of this operator algebraic approach to the case of interaction has its beginnings in the late 50s when Haag formulated the idea that relativistic causality provides a very rich local substructure to the global operator algebra [44]. Other operator algebraic approaches (e.g. that of Irving Segal) failed to appreciate the significance of this local algebraic substructure . In the beginning of the 60s the first systematic accounts of these ideas appeared [45]. The local algebras had some very unfamiliar mathematical properties which were unknown in quantum mechanics and are (like all properties which distinguish QFT from QM) related to the physical phenomenon of vacuum polarization resulting from causality and leading to type III von Neumann algebras [47].

By the middle 60s the main mathematical and conceptual ingredients were in place (Haag-Kastler, Borchers). The new credo of local quantum physics as the intrinsic approach without quantization crutches was that the structural consequences of the physical principles of QFT are most naturally derived by a dichotomy between local observables and charge-carrying fields. In the setting of local nets this amounts to an inclusion of algebraic nets. The smaller net consists of local observables which fulfill obvious causality requirements which constitute the only firm link with classical physics[22]. Here the physical intuition about causality and stability in local quantum physics enters; however the somewhat mysterious concept of inner symmetries (additive charge quantum numbers and their isospin like generalizations) which attributes charge quantum numbers to particles and fields is not used because one of the reasons for this dichotomic approach is precisely to explain field and particle statistics and inner symmetries as a manifestation of spacetime-organized quantum matter.

The second step in the observable-field dichotomy is (in Wigner's representation theoretical spirit) the classification of all causally localizable representations of this observable net; for zero temperature particle physics this amounts to finding all positive energy representations of the observable net which are locally unitarily equivalent to the vacuum representation.

The last step is a mathematical conceptual one: organize these representations in such a way that they become the superselection sectors of the representation of a larger algebra, the so-called field algebra[23]. This step is achieved by introducing charge carrying intertwiners between the superselected (not coherently superposable) representations. There is some liberty in the choice of these intertwiners which leads to different relative spacelike commutation structures among fields which carry different charges; this can be settled by natural conventions (achievable by so-called Klein transformations). The end result is very beautiful; one obtains a field algebra i.e. a spacetime indexed net which has no further inequivalent localizable representations i.e. the dichotomy allows no further extension [49]. The increase of structural understanding is enormous.

On the one hand one rederived properties, as the connection between spin and statistics and the TCP theorem, which already were rigorously obtained in the general pointlike setting of Wightman, but now without making any assumptions about non-observable fields[24].

On the other hand one learns that the concept of inner symmetry in particle physics is preempted in the net structure of observables; the specific symmetry group for a given observable net is a compact subgroup of some SU(n) for large enough n which can be computed solely from the observable data [49]. There is no better verbal way to appreciate this intellectual achievement than by quoting Marc Kac's aphorism about the mathematical physics aspects of acoustics: "How to hear the shape of a drum?" The D-R construction is like a reconstruction of the full field theoretic data from their observable shadow.

[22] Instead of quantizing classical field theories, one adapts the causality principles to the setting of LQP and views the problem of classifying and constructing models as one which has to be handled inside LQP. Since the majority of particle physicists have a fair knowledge about chiral conformal theories, it may be helpful to point out that in many respects LQP may be viewed as a generalization of the algebraic ideas (representing algebras instead of quantizing Lagrangians) used in chiral theories.

[23] Here the meaning of "field" is not in the pointlike singular sense, but rather stands for charge-carrying operators as opposed to the neutral operators which generate the observable algebra.

[24] In Pauli's derivation which was later refined by Jost [55], one had to assume that the charge-carrying fields obey either spacelike commutation- or anticommutation-relations.

It is very revealing to compare this de-mystification of internal symmetry with the problems at the time its beginning in the middle 30s. Most particle physicists learned from textbooks that at that time Heisenberg had the idea that collecting the proton and neutron into a doublet facilitate the understanding of nuclear forces. What is not generally known is that even before Heisenberg wrote his second paper in which he added the group theoretic formalization of isospin, Jordan investigated the relation between the representations of the permutation group and internal SU(n) symmetry (multiplet structure) of his quantized wave field formalism in a remarkable paper [50]. Since the mathematically controllable part of the Lagrangian field formalism was restricted to bilinears (free fields), his point of departure was the algebraic structure of bilinears in momentum-space creation and annihilation operators. In this way he obtained the best possible field theoretic restrictions on the general connections between the representation theory of the permutation group (statistics) and the internal symmetry groups (the multiplicity structure of fields) in a Lagrangian setting. Some time after Heisenberg's SU(2) isospin paper, Schwinger rediscovered (without reference to Jordan) the SU(n) results using a similar method. Schwinger's contribution became widely known (especially through a book written by Biedenharn and van Dam [51]).

A complete de-mystification of the internal symmetry issue was not possible at the time of the Jordan's and Schwinger's contributions since the interrelations between causality and localization properties with a generalized concept of charges through the omnipotent vacuum polarization properties were not really understood. Even though in the 1930s most of the physical principles of QFT were in place, several important concepts which reveal deep connections between them were still missing; the complete de-mystification of interal symmetry in terms of the causal observable strucure had to wait more than 50 years [49].

Another important corner-stone was added to particle physics when Wheeler and Heisenberg introduced the scattering matrix. The S-matrix, as it was later referred to, was not only an invariant measure of interactions between particles, but became the dominant observable of particle physics in terms of which most experimental results could be analyzed. Its formal use in renormalized perturbation theory by Dyson contributed a significant amount of conceptual order to the renormalization recipes. Its detailed relation to quantum fields turned out to be one of particle physics profound problems.

A complete solution was attained only when the asymptotic convergence in the LSZ theory was finally backed up by mathematical proofs which showed that the timelike asymptotic convergence and the invariance of the asymptotic transition operator S is a consequence of spacelike causality and suitable spectrum conditions (existence of a mass gap). For the first time conceptual clarity in the particle-field relation substituted the intuitive but somewhat ambiguous particle-wave duality.

Scattering theory provided a Fock space structure to the representation space of a QFT even in the presence of interactions; if the application of operators to the vacuum state leads to vectors which have a one-particle Wigner component, then the locality structure of QFT guarantees the presence of (anti)symmetrized tensor product states of Wigner particles. This is more specific and precise than Jordan's "corpuscles" from quantized waves. There is a fine point here which tends to be sometimes overlooked even in rather sophisticated contemporary work. The multi-particle interpretation of state vectors is limited to Lorentz frames and there is no reason to believe that e.g. an LSZ asymptotic behavior holds in the time in the Rindler world of uniformly accelerated Unruh observers [52]. As a result the particle terminology in those cases does not seem to be appropriate; the arguments are based on linear (free) systems whose Rindler excitations are identified with particles[25] but one cannot expect an LSZ mutiparticle structure to hold in the frame of an uniformly accelerated Unruh observer (or in general curved spacetime situations).

The omnipresence of vacuum polarization renders any quantum mechanical bound-state picture obsolete; what gets fused are (superselected) generalized charges and not particles. Whereas the relation among particles is basically "democratic" ("nuclear democracy"), that between charges is hierarchical. The only hierarchy among particles arises from charges which they carry; it has nothing to do with the quantum mechanical distinction between elementary versus bound which is based on particle number conservation.

[25]Therefore it may be somewhat misleading to use the terminology of "particles" in connection with the Rindler-Unruh situation (as one find in the literature [53][54]) since there is no reason to believe that the Hilbert space analyzed in terms of thermal Unruh excitations has a Fock space structure (apart from the atypical case of non-interacting quantum matter).

In the Lagrangian approach a hierarchy between a "would be" fundamental field and its fused composites has been introduced via the quantization procedure; but even in such a context it is not clear whether this distinction has an intrinsic physical meaning; among the many possible scenarios it could happen that the objects carrying the fundamental charges are outside the Lagrangian description. This happens e.g. with the Sine-Gordon Lagrangian where the Lagrangian field creates a local net of operator algebras which in addition to its vacuum representation has additional physical representations. In this particular case the extended representation (the DR-field algebra representation) can be incorporated into the Thirring model Lagrangian, but there is no argument that the extension always permits a Lagrangian description.

At the time of Jordan's "neutrino theory of light" (see the first section) particles were simply identified with vectors obtained by applying the Fourier components of fields to the vacuum and for bound-states one used a covariantized quantum-mechanical description. Apart from the very peculiar Fermion/Boson relation in d=1+1 spacetime dimension, Jordan's theory was inconsistent with QFT.

However if one makes use of the fact that the omnipresent vacuum fluctuations couple all one-particle vectors with vectors obtained from the vacuum by applying localized operators which have a nonvanishing component with the same quantum numbers (charge, mass, spin), then Jordan's idea loses its "crazy" aspect (but unfortunately also some of its potential usefulness). In the electro-weak theory one can of course find composites of the neutrino field which fulfill these requirements and the application of scattering theory reveals that (apart from complicated normalization factors) they are indeed interpolating fields for photons i.e. their suitably defined large time limits are free photon fields. Jordan's problem was an academic one, there was never any need to have a quantum mechanical bound state description of photons in terms of neutrinos.

However the conceptual difficulties which come with such a bound-state picture return with full force when we say that, e.g., a meson is made-up of two quarks. In that case we really want to know in what sense an old-fashioned bound state picture could make (approximate) sense in the presence of vacuum polarization. Presently we are using the magic protection of the word "quark confinement" in order to legitimize an "effective" return of the quantum mechanical bound state picture which we hope to understand in a future theory of confinement. Perhaps the meaning of "bound state" or "made up" in quantum chromodynamics does not go beyond the message drawn from Jordan's neutrino example, namely it refers to the fusion of (confined) charges and should not be understood in the sense of binding of (confined) particles.

Scattering theory and the S-matrix played a crucial motivating role for passing from pointlike fields to the setting of LQP. The Haag-Ruelle scattering theory [46][47] showed that the LSZ scattering theory [48] can be derived from the principles of QFT (assuming the presence of mass gaps) and the S-matrix stays the same if one passes from one interpolating Heisenberg field to a composite field which acts cyclicly on the vacuum. In particular the local equivalence class of a given field (the Borchers class [55]) is associated with the same S-matrix. This insensitivity of the S-matrix against local changes was also seen within the perturbative Lagrangian formalism, but the proofs tend to be highly technical. The LQP formulation in terms of nets of operator algebras was the intrinsic way to express this since relative local fields generate the same net. The generalization of scattering theory to general operators (taken from e.g. double cone localized algebras in the net) presented no new problems [47].

Heisenberg [56] was the first to see that the S-matrix is a global object which poses no short ultraviolet problem since the particles participating in the scattering process are "on-shell" i.e. avoid the dangerous regions of short distance fluctuations. Hence if it is possible to avoid fields i.e to find a calculational scheme which stays on-shell, one could have a finite theory of interacting relativistic particles. The only properties which Heisenberg required of the unitary S-operator besides Poincaré invariance was a factorization property in case the wave packets of the particles involved become spatially separated. This is a special case of what we nowadays call the cluster property; it is automatically taken care of if the S-matrix is that of an underlying QFT. In this way Heisenberg formulated the first attempt for a pure S-matrix theory.

It is quite interesting to notice that Poincaré invariant S-matrices which cluster can be constructed in a systematic way by a method which was not available to Heisenberg. One again starts from tensor products of Wigner one-particle spaces but this time one uses this setting in order to formulate a quantum mechanical relativistic problem with a fixed number of particles without vacuum polarization.

The quantum-mechanical localization in the sense of the Born probability interpretation is called the Newton-Wigner localization; it is neither covariant nor causal[26]. However it acquires these two properties asymptotically for large timelike distances and this is the reason why it is used in time dependent scattering theory; the Haag-Ruelle vectors at finite times depend on the Lorentz frame but the S-matrix which is defined in terms of their large time limits is frame-independent.

Relativistic quantum mechanics starts from a tensor product of two Wigner particle representation and defines an interaction by going into the center of mass frame and modifying the center of mass Hamiltonian (=invariant mass) by adding an interaction potential which commutes with the Poincaré generators of the full two-particle system. The mass operator in the 3-particle system has to be determined by a cluster argument; one first transforms to the relative center of mass system of the various two-particle subsystems where one can use the original two-body interaction. Transforming back into the 3-particle center-of-mass frame one obtains a situation in which the third particle is a "spectator" which physically corresponds to a situation where it is spatially separated by an infinite distance from the 2-particle subsystem. Adding up the various two-particle pair interaction operators of these spectator situations, one finally obtains the minimal generators of the 3-particle Poincaré group in the presence of interactions. Non-minimal 3-particle interactions are obtained by adding to the 3-particle mass operator a term which corresponds to a direct 3-body interaction term (which vanishes if any one of the particles becomes a spectator). In the presence of 4 particles even the minimal implementation of clustering yields a 3-body interaction which is uniquely determined in terms of the original 2-body interaction.

It is evident from this iterative construction of n-particle interactions that the implementation of cluster properties requires the use of transformations between different Lorentz systems i.e. in contrast to field theoretic cluster properties, the cluster properties of a relativistic particle theory with "direct interactions" [57] is frame dependent. However this dependence drops out in the asymptotic limit i.e. for the Moller operators and the resulting S-matrix[27]. Therefore theories obtained in this way are Poincaré invarinat clustering S-matrix theories par excellence in the sense of Heisenberg's proposal.

The two-fold use of the Wigner theory as a point of departure for QFT as well as for relativistic QM is intimately related to the existence of two different kinds of localizations: the modular localization and the Newton-Wigner localization [60]. It is interesting that the two different localization concepts have aroused passionate discussions in philosophical circles as evidenced e.g. from a title like "Reeh-Schlieder defeats Newton-Wigner" in [59]. As it should be clear from our presentation, particle physics needs both, the first for causal (non-superluminal) propagation over finite distances and the second for scattering theory (where only asymptotic covariance and causality is required but where the dissipation of wave packets is important).

There is another historical reason why I have dedicated much space to recall theories based on particles rather than fields. This has to do with Dirac's alternative particle oriented method as compared to Jordan's quantization of wave-fields. There simply exists no alternative to QFT if one wants to maintain the vacuum properties. Hole theory as a theory of particles interacting with the electromagnetic field is not consistent; the relation to particles in the presence of vacuum polarization can only go through large time LSZ asymptotes i.e. there can be no particles at finite times. This is the reason why the Dirac-Fock-Podolski multi-time formulation cannot serve as a starting point for an alternative formulation of nth order QED perturbation theory[28], although with a bit of artistry and hindsight one can obtain correct low order results based on hole theory [61]. The scheme of Heisenberg was too general and physically bloodless; the subsequent progress on how to extract unambiguous results from the ill-defined perturbation theory of QED via renormalization directed attention away from S-matrix back to QFT.

The important distinction of Heisenberg S-matrices from those resulting from QFT is the crossing property which is related to causality and the ensuing vacuum fluctuation properties (which allows no natural formulation in a scheme of direct particle interactions).

Despite the great success of renormalization theory with respect to its experimental verification, not

[26] This fact caused Wigner's disapointment and made him believe that QFT cannot satisfactorily describe particles (private communication by Haag).

[27] This framework of relativistic direct particle interactions is being successfully used in meson-nucleon physics at energies where relativity matters but particle creation is limited to a few channels [58]. Since strictly speaking it is a pure S-matrix scheme, quantities such as formfactors of currents have to be added by hand (taking hints from QFT) i.e. its use is strictly phenomenological.

[28] This explains why one does not find an n^{th} order renormalization calculation in the setting of hole theory.

everybody was happy. There was on the one hand Dirac who could not reconcile the extreme technical and artistic formulation of the renormalization recipes (since he made important contributions to certain concepts such as the electron selfenergy we can however safely assume that he understood what was being done) with his sense of esthetics, mathematical rigor and elegance. We do not know what Jordan's reaction may have been, but reading the text of his Kharkov talk one gets the impression that he may have been a bit disappointed by the very conservative nature of renormalized perturbation theory since he believed that only very revolutionary conceptual changes and additions could resolve the problems described in his talk. The loss of faith in post QED perturbative field theory came from a growing number of physicists in the meson-nucleon physics community who realized that the perturbative methods are inadequate for strong interactions.

This led finally to a return of S-matrix theory, but this time it was enriched with the crossing property, an important additional requirement which was abstracted from QFT [62]. Looking at subsets of Feynman diagrams, it is not difficult to see that the connected parts of on-shell formfactors or scattering processes[29] for different numbers of (anti)particles in the outgoing bra and incoming ket states are related by a process of crossing. In this process one takes away one incoming particle from the ket state and adds it to the outgoing bra configuration with conjugate charge (antiparticle) and opposite 4-momentum. Here a 4-momentum on the lower real part of the complex mass shell is defined by analytic continuation from physical values, i.e. crossing cannot be a symmetry in the sense of Wigner. The crossing property for the connected part of formfactors of localized operators is very suggestive in the LSZ setting; the necessary analyticity properties follow in many cases from Lehmann-Dyson type of spectral representations [63]. There is however an important fine point in that in an on-shell setting one requires the analytic path connecting the original with the crossed configuration to be a path which stays within the complex mass shell. This makes crossing as it is needed in an on-shell constructive approach (see later) a quite subtle property [64].

This S-matrix program was closely linked with the adaptation of Kramers-Kronig dispersion theory to QFT. Originally it was thought of as having the same physical content as the quantum field theoretical construction of the S-matrix, but later it was formulated as a setting which is in opposition to the locality concepts of QFT. The opposition finally took the extreme form of a cleansing rage against QFT and culminated in the doctrine that the "S-matrix bootstrap program" has a unique solution and constitutes a theory of everything (a TOE except gravity).

This strange episode probably would have faded away on its own since the gap between the wild claims and expectations of its protagonists and the physical reality became insurmountably large, but the strong return of QFT in the form of (nonabelian) gauge theory shortened this process. The only relic which remained from this bootstrap program is the Veneziano dual model which later was reformulated and extended into modern string theory (again with the pretence of a TOE, this time including gravity). The latter does however not represent a solution of the original S-matrix program with the field theoretic crossing property i.e. it is not the result of an extension of existing particle physics but rather represents an invention without known connection to the experimentally secured physical principles underlying Jordan's quantization of wave fields. This will become clearer in the next section where we will show that the S-matrix with the crossing property plays a pivotal role in the implementation of classification and construction of wedge algebras.

An important hint in favor of this new operator algebra-based approach comes from the unicity theorem of the inverse scattering problem in LQP [65]. It states that an S-matrix with crossing admits at most one QFT in the LQP setting i.e. it cannot happen that a given S-matrix with crossing admits several inequivalent QFTs. Besides the Noether currents (or rather their algebraic counterparts [66]) it does not distinguish particular pointlike objects affiliated with the net of operator algebras. This complies with the fact that in passing from classical to quantum field theory, fields (except Noether currents) lose their individual nature since they are not directly measurable; their main role is to provide interpolating objects for the in/out going particles.

Contrary to what was believed at the time of the bootstrap approach, the S-matrix does contain local information and it is hidden behind crossing. A conceptually more appropriate way of stating this new insight is to emphasize that in LQP the S-matrix acquires in addition to its scattering interpretation an

[29] The S-matrix is a special formfactor, namely the identity operator sandwiched between outgoing bra and incoming ket states (the opposite convention can also be used).

important new role as a relative[30] modular invariant of wedge algebras which are the building blocks of LQP.

The careful shift of emphasis from Jordan's quantized wave fields to Haag's algebraic nets (which we use in this essay in order to convince the reader that Jordan's radical expectations have meanwhile found their appropriate conceptual and mathematical basis) should not be misunderstood as diminishing the importance and usefulness of pointlike fields. There is little doubt that physically relevant models of LQP have point- or string- like covariant generators. What is however delicate and problematic is the use of such singular coordinatizations in (perturbative) Lagrangian calculations. It is evident that a scheme whose only conceptual and mathematical tool consists of pointlike fields has no alternative to the implementation of local interaction than by locally coupling free fields i.e. by causal perturbation theory. But it is also evident that this brings in all those technical aspects of renormalization in which Dirac rightfully could not see neither beauty nor rigor.

The LQP setting offers other possibilities since e.g. a wedge algebra may be generated by extended objects which are not identical to smeared fields (smeared with wedge-supported test functions). The sharper localized operators may originate from algebraic intersections and the pointlike objects may only come into existence at the end, after one already constructed nontrivial double cone algebras (with pointlike localization-cores). Intersections can of course be trivial; in that case we would say that the LQP with the wedge structure from which we started does not exist. In the causal perturbation theory based on coupling pointlike free fields the candidates for nonsensical theories are those which as a result of their bad ultraviolet behavior cannot be renormalized in terms of a finite set of parameters. In that case the perturbative delineation of models would follow the logic of the power counting law for the interaction density[31]. The achievement of renormalization theory was that the ultraviolet divergence problems in the presence of interactions (which started to become noticed shortly after the discovery of quantization of wave fields and which led to despair for almost two decades) were finally solved. To be more precise, the divergence problem had been partially solved since it was unclear whether the power counting law which required the operator dimension of the interaction density (which is the sum of the dimensions of the fields participating in the interaction) to be bounded by the spacetime dimension is an intrinsic property following from a perturbative implementation of the principles of QFT or whether it is a technical property of this particular implementation of perturbation theory. This is of course an academic question as long as we cannot think of a different approach, but as will be outlined in the next section, such an alternative approach already exists for a special nontrivial family of d=1+1 QFTs.

The following illustration may serve as a warning against interpreting the renormalizability criterion in terms of power counting too naively as a well-defined procedure to screen between existing and non-existing QFTs. Suppose we are interested in interacting massive vector mesons. The easiest way to describe free massive vector mesons is by a transverse vector potential A_μ which obeys the Klein-Gordon equation. But since contrary to classical expectations its operator dimension is $d_A = 2$ (any other covariantization of the Wigner representation would only increase the operator dimension), and since each interaction is at least trilinear in free fields, the interaction density of any model involving physical vector potentials has inevitably operator dimension ≥ 5; which would make it nonrenormalizable i.e. the dimension of the interacting field would keep increasing with the perturbative order and these higher polynomials would cause an ever-increasing number of independent parameters.

At this point one would be reminded that massive vector mesons should be treated as gauge theories with the mass being generated by a Higgs mechanism. But let us ignore this advice and use instead a cohomological extension of the Wigner one-particle theory of a massive vector meson. By this we mean an enlarged unphysical (i.e. with indefinite metric ghosts) representation of the Poincaré group with a nilpotent δ-operation such that the original Wigner space is the cohomology of the extended situation. This is possible and the associated Fock space formalism is a particularly simple linear realization (bilinear

[30]The antilinear involutive Tomita S-operator of the interacting wedge algebra differs from that of the corresponding incoming algebra by the scattering matrix (see next section).

[31]In models which contain Fermions and Bosons one can sometimes tune the parameters in such a way that the highest divergences compensate (supersymmetry). But the wave function renormalization (integral over Kallen-Lehmann spectral function) can never be finite i.e. there are no completely ultraviolet finite interacting theories in d=1+3. For conformally invariant theories the rigorous proof is actually quite simple [78]. The so-called finite N=4 supersymmetric gauge theories may lead to a fake (gauge dependent) finite wave function renormalization in an indefinite metric formulation, but the problem of infinities returns if one constructs the gauge invariant composites.

in the Lagrangian) of the BRST formalism which is only possible in the massive case [67]. The unphysical field has formally operator dimension d=1, and therefore its coupling becomes renormalizable in the new power counting. The cohomological nature of the extension gives us the hope that as a benefit of deformation stability of cohomology we will be able to descend to a physical sub-theory. This is indeed the case but only after realizing that perturbative consistency of the renormalized theory requires the introduction of new physical degrees of freedom whose simplest realization is a scalar field. This is of course the Higgs field except that in the present case it has a vanishing vacuum expectation; the theory was massive to begin with, and there is no mass-generating role which would require a Higgs condensate. The physical expectation values are identical to those obtained from the gauge theoretical construction together with the Higgs mechanism.

The so obtained physical theory (after the cohomological descent) has no memory about the cohomological trick; the procedure is reminiscent of the use of a catalyst in chemistry in that the cohomological extension was used in an intermediate step and again removed through the descent [67]. This trick has stabilized the operator dimension d=2 of the physical vector meson modulo logarithmic corrections and (in contrast to the naive approach) did not require to introduce an infinite set of new parameters

The interest of this illustration is not the result itself, but rather the different way in which it is obtained (some of the following ideas can be traced back to extensive work of Scharf and collaborators [68]). It shows, as was always expected by some physicists, that the approach via gauge theory and Higgs mechanism of mass generation has no intrinsic physical meaning (but rather represents a useful technical tool for doing computations). In contrast to a classical field theories with vector fields for which the gauge principle is needed to characterize Maxwellian interactions, the quantum field theoretical renormalizability of massive vector mesons fixes the interaction all on its own uniquely in terms of one coupling strength. The semiclassical reading of that unique vector meson theory reveals of course that it looks exactly like the quantization of classical gauge theory with the Higgs mechanism. Since quantum principles (even if they are presently insufficiently understood) are more basic than classical principles, one should perhaps turn around the historical approach in favor of the opposite reading

$$\textit{renormalizability} \overset{\textit{semiclassically}}{\longrightarrow} \textit{classical gauge principle}$$

since a theory which is already unique in the perturbative setting of renormalizability does not need any further selection principle. Pragmatically speaking we have only reduced the somewhat mysterious classical gauge principle (for which we have beautiful differential geometric presentations, but geometry is no substitute for causal quantum physics) to the mystery of a renormalizability principle (of which we still lack an understanding within the beautiful setting of LQP). But whereas we still have a future chance to unravel the latter, one does not expect that anything can be added to the former. The strategy indicated works strictly speaking for massive vector mesons, one expects that in the massless limit the scalar particles will decuple (possibly coupled with the appearance of new charges).

This illustration also suggests that perhaps the nonrenormalizability statement concerning the coupling higher spin s>1 fields should be taken with a grain of salt. To be sure, if we start with a physical free field with higher spin, the short distance scale dimension is certainly beyond the canonical value one, which means that the perturbative iteration will increase the short distance divergences by inverse powers which increase together with the number of new parameters with the perturbative order. But there may exist a cohomology-like magic which leads to an unphysical but renormalizable situation such that after a descent to a physical subsystem the operator dimension of the higher spin Heisenberg field is stabilized around its initial physical free field value modulo logarithmic corrections. According to the above considerations this is precisely what happens for massive vector mesons, but since the systematics of such tricks is not known, the status of higher spin representations in interactions is unclear. This brings us back to Jordan's 1929 critical assessment of the situation and the persisting need for a radically different approach as compared to the standard quantization formalism, even after the discovery of renormalization.

There have been attempts to improve the short distance behavior in the conventional point-like field setting by enforcing a partial cancellation (see previous footnote) of divergencies between Fermions and Bosons via the so-called supersymmetry. But the tuning between Fermions and Bosons which achieves this supersymmetry has some consequences which show that this supersymmetry, unlike any other symmetry, is a very peculiar kind of symmetry. Whereas the coupling of systems with normal symmetries (internal, Lorentz) to a heat bath causes a sponteneous breaking, the supersymmetry "collapses" [69]. This means that there is no enlargement of the Hilbert space by combining different values of the breaking parameter (e.g. the preferred rotational direction in the case of a ferromagnet) such that the symmetry can be recovered (at the expense of the cluster property) in the larger reducible representation space as is the case for spontaneous breaking. Another more serious reason is that the modular method applied to observable algebras (which are neutral, i.e. on which inner symmetries act trivially and spacetime symmetry is described by the Poincaré group and not its covering) reveals all inner symmetries but, there is no indication for supersymmetry. Since it goes against the observable algebra–charged field algebra dichotomy by mixing observable fields with fields carrying the unimodular charge, a follower of the LQP logic may say what God has separated men should not force together. In fact in the framework of perturbative study of renormalization group flows in a multicoupling setting, supersymmetry appears as an accidental point at which certain short distance compensations take place. This special situation looks somewhat accidental (which corroborates with the previous failure of supersymmetry to show up in the modular setting). But of course these arguments do not convince somebody who wants to see a supersymmetric partner behind each particle (at least for asymptotically large energies).

The LQP approach certainly satisfies Jordan's requirements which he formulated in his Kharkov talk and as a result of its mathematical precision and conceptual beauty it may even have pleased Dirac. But the crucial question is whether it is capable of leading to a classification and construction of models which can explain important results in particle physics. There is reason for optimism, but the research is very much in its infancy. In the last section I will try to present some constructive ideas and a few encouraging results.

4 LQP and ultraviolet-finiteness

As soon as one decides to work with pointlike fields, it is very hard to think of a construction scheme which is different from the standard way of coupling free fields and using the Wick-ordered interaction density for the iterative computation of the renormalized time-ordered products via their vacuum expectation values. The only notable exception is chiral conformal theory which deals with fields which are localized on a lightray. Although such models are technically speaking not identical with free fields, they are (like free fields) uniquely determined in terms of their commutation structure (which in turn is fixed by braid group representations and possibly additional combinatorial data); with other words there are no deformable interaction parameters (coupling strengths). Although the complete descriptions in terms of n-point correlations (or explicit formulas in terms of auxiliary free fields) have only been constructed very special cases, the known facts makes them very interesting objects of field theoretic research. The methods of constructions of pointlike fields are representation-theoretic (representations of "Lie-field" algebras of the current-algebra or W-algebra type) and leave no room for ultraviolet divergencies since the scale dimension is part of the spacetime group theory. The operator algebraic approach and the pointlike formalism turn out to be equivalent [70]. Chiral theories arise as "conformal blocks" in a decomposition theory for 2-dimensional local (bosonic or fermionic) conformal field theories. This origin of this decomposition is basically group theoretical.

The relevant space-time symmetry group is the universal covering of the conformal group and the relevant "living space" is not Minkowski spacetime but rather the universal covering of compactified Minkowski spacetime which carries its own global causality structure [71].

A more convenient description in terms of operator distribution valued sections on the (compactified) Minkowski spacetime (as the base of a vector bundle) can be obtained by decomposing the global covering field according to the center of the conformal covering [72]. The resulting central components (conformal block fields) are the desired sections. For the special case that the central decomposition is trivial (no complex phase factors[32] under central transformations) the field is a free field which lives on the compactified Minkowski space (or on its two-fold covering in case of an odd number of spacetime dimensions) and therefore fulfills the quantum version of Huygens principle i.e. the (anti)commutator has its support on the lightcone. This happens e.g. for free photons. Jordan was aware of the validity

[32] The decomposition theory shows that the phases of the central phase factors are linear combinations of the (anomalous) scale dimensions which occur in the theory [72].

of this principle in his quantized wave field setting [73], but not of the fact that a conformal interactions gives rise to an algebraic modification in the Huygens region [74].

Conformal invariant theories offer presently the best chances for explicit classifications and constructions since the mechanism for interactions is fully accounted for by the algebraic structure in the Huygens (timelike) region (which in turn determines the spectrum of scale dimensions) i.e. the Jordan-Haag credo of an intrinsic access to local quantum physics without classical crutches is fully realized. There is no classical counterpart of the timelike "reverberation" structure for even spacetime dimension; the covering group representation aspect is an extremely rich generalization of the spin phenomenon i.e. one has the analogy

$$spin\ (spacelike) \backsim anomalous\ dimension\ (timelike)$$

The best studied case is that of two-dimensional conformal theories where the conformal group tensor factorizes into two Moebius groups with the center of the covering following suit. This leads to the very peculiar tensor decomposition of the two dimensional conformal QFT into two chiral theories. The restrictiveness of the resulting algebraic structure together with the acquired mathematical knowledge about Kac-Moody- and diffeomorphism- algebras permitted Belavin Polyakov and Zamolodchikov [75] to discover the first nontrivial family of chiral models which they called "minimal" models since their representation structure was completely fixed in terms of the commutation relations of only one field, which was the energy-momentum tensor. Whereas previous model illustrations of the block decomposition theory consisted of exponentials of free massless Boson fields (which appear naturally in the solution of the massless Thirring model) and one could not have been sure that the apparent generality of the decomposition can be realized by models, the discovery of the minimal models showed the enormous scope of its content and the richness of the ensuing anomalous dimension spectrum.

The above mentioned algebraic structure in the Huygens region for d=1+1 conformal theories splits into a separate algebraic structure of the left and right moving chiral components, i.e. it becomes reprocessed into the two lightray commutation structure. Since lightlike can also be seen as a limit of spacelike, the chiral commutation structure can be interpreted in terms of braid group statistics (of fields). This viewpoint turned out to be very useful for classification and constructions of chiral theories [76][77]. In fact the ultimate goal in chiral QFT is to obtain a complete classification and construction of the interval-indexed chiral nets or their spacetime pointlike field generators based on braid group statistics (and possibly additional combinatorial data). The presence of the powerful Tomita-Takesaki modular theory of operator algebras which has been successfully adapted to LQP [79] nourishes hopes, that this goal may be reached in the not too distant future.

The chiral theories are much more than a training ground for nonperturbative methods in QFT. Since physically speaking they arise as zero mass limits of massive theories, one expects that they still retain physical informations about highly inclusive scattering processes. But in view of the fact that standard scattering theory is not applicable (the LSZ limits in interacting conformal theories vanish [78]) and a formulation which aims directly at probabilities for inclusive processes has not yet been worked out to the extend that it can be directly applied [46], this still remains a problem of future research. It seems to be undeserved luck that precisely chiral theories, about which one has the presently most detailed knowledge, can be used for higher dimensional LQP. This is because it can be shown that chiral theories are the building blocks in the holographic lightfront projection of any local QFT (independent of the spacetime dimension) [81].

The idea leading to this mathematically and conceptually very precise formulation of holography in the setting of LQP is the following. One starts with the Unruh situation of a (Rindler-) wedge restricted QFT which automatically inherits the Hawking thermal aspects[33] The linear extension of the (upper) causal horizon is a lightfront which inherits a 7-parametric subgroup of the 10-parametric Poincaré group. The lightfront with its inherited unusual causal structure is very different from globally hyperbolic spacetimes, but precisely this makes it a powerful instrument for the exploration of QFT associated to the ambient Minkowski spacetime. With the exception of d=1+1 (where one needs data on both lightrays) the classical data on the lightfront determine the global ambient theory. Even more, the data on the wedge (upper) horizon determine the data in the associated wedge. The smallest regions on the lightfront which still cast ambient causal shadows are semiinfinite strips with a finite transverse extension on the lightfront.

[33]The vacuum state resricted to the wedge algebra becomes a thermal KMS state at the Hawking temperature.

Compact regions on the lightfront do not cast any ambient shadows; in fact they do not even cast shadows within the lightfront (the manifestation of its extreme deviation from hyperbolic propagation).

The marvelous aspect of LQP consists in enabling us to transfer these structures to the local quantum realm; the algebra localized on the horizon is identical to the global wedge algebra, but the net substructures of both algebras are very different (apart from the equality of the before mentioned semiinfinite strip algebras with their ambient causal shadows). For interaction-free theories these claims can be verified by standard methods i.e. by restricting the pointlike fields to the lightfront. For interacting fields, which in d=1+3 necessarily lead to Kallen-Lehmann spectral functions with affiliated infinite wave function renormalization factors, the lightfront theory cannot be obtained by restriction. In this case one has to use pure operator-algebraic modular methods. It turns out that the lightfront net has no transverse vacuum fluctuations, all vacuum polarization is compressed into the longitudinal lightray direction. This complies perfectly with the 7-parametric symmetry group aspect; the holographic projection of the two "translations" contained in the 3-parametric Wigner little group of the lightray in the lightfront precisely accounts for the transverse Galilei invariance of a fluctuationless transverse quantum mechanics.

The lightfront QFT is the only known case in which QM for a subsystem returns in the midst of QFT without any nonrelativistic approximation. In lightray direction one finds a Moebius invariant chiral theory. The rotational subgroup of the Moebius group does not result from the holographic projection of the Poincaré group (i.e. is not a subgroup of the mentioned 7-parametric group), rather it results from "symmetry enhancement" [82] on the lightfront[34]; the phenomenon of symmetry enhancement and its connection to universality classes is well known from the critical limit (massless limit, scaling limit). The difference with the lightfront holography and its associated universality aspect is that the ambient theory and its holographic projection live in the same Hilbert space; in fact the two theories are coming from identical global algebras, their difference is only in the spacetime net indexing of subalgebras which causes a relative nonlocality. Therefore the new symmetries which are represented by diffeomorphisms of the holographic projection have already been present in the ambient theory. They were not perceived as symmetries simply because their action on the ambient theory is "fuzzy" i.e. not presentable in terms of geometric transformations (but only as support-maintaining transformations of test function spaces if one uses the setting of pointlike fields). Whereas the holographic process from the ambient theory to its lightfront image is unique, the holographic inversion is not; it remains an interesting problem to classify the class of ambient theories having the same image or adding additional informations to the holographic projection which makes the inversion unique. Presently lightfront holography is not in a state where it could be useful for the construction of ambient QFTs.

The holographic approach becomes more favorable if one restricts ones interest to problems which do not require to solve the holographic inversion, but permit to be studied in the holographic projection. The most prominent such problem is the assignment of an entropy to the phenomenon of thermalization caused by causal localization. A special case is Hawking's black hole thermalization which through the imposition of the classic thermodynamic basic laws of heat bath thermality leads to Bekenstein's area law for the assigned entropy. The fundamental problem behind this pivotal observation is to find a direct derivation of a quantum entropy in the setting of localization-caused thermal aspects which applies not only to black holes but also to black hole analogs[35].

An excellent testing ground is the restriction of the global vacuum to the wedge algebra or that of its causal horizon i.e. the Unruh-Rindler thermal situation. The relevant KMS operator is the Lorentz boost which is not of trace class and hence permits no definition of entropy. This is a general property of all localized algebras; the restricted vacuum leads to (generally non-geometric) KMS states which cannot be tracial since sharply localized algebras are of hyperfinite von Neumann type III_1 which simply do not have tracial states, hence a notion of entropy cannot be defined. The physical origin of this phenomenon are the uncontrollably big vacuum fluctuations which accompany the creation of sharply defined causal localization boundaries. The remedy is to make a fuzzy surface which leaves the vacuum polarization to

[34] Since in chiral theories the diffeomorphisms of the circle turn out to be of modular origin [80], one expects the higher diffeomorphisms to be also symmetries of the holographic projection. It turns out that these transformations act in a fuzzy way on the ambient theory.

[35] The termal aspects of localization do not depend on the presence of spacetime curvature but rather on an extension of the idea of "surface gravity" which sets different scales for true gravity theories from those of their acoustical, hydrodynamic or optical analogs.

reorganize themselves in a "halo" surrounding the original localization region. This is done with the help of the so-called split inclusion property, which, different from a cutoff, maintains the original local theory while only reorganizing some local degrees of freedom in the finitely extended halo (whose extension is a control parameter for the size of vacuum fluctuations).

The vacuum state restricted to the split inclusion becomes a thermal density matrix (with the Hawking temperature) in an appropriately defined tensor factorization of the total Hilbert space. This sequence of density matrices for decreasing halo size converges (as expected) against the dilation operator (the holographic image of the wedge-associated boost) which is a non-trace class operator which sets the thermal KMS properties of the Unruh-Rindler effect; thus if one would be able to associate a split-localization entropy with the finite halo situation, one can be sure that this is naturally associated with the Hawking-Unruh temperature aspect. The transverse symmetry of the horizon (whose linear extension is the lightfront) forces the concept of an area (the dimension of the transverse edge of the wedge) density of split-localized entropy.

This shows that the prerequisites for a Bekenstein-like quantum area law for localization entropy are met in a surprisingly generic manner. But for the control of a limiting halo-independent area density two more properties remain to be established:

(1) The increase of the area density with decreasing halo size ε is universal (model calculations indicate that it goes like $ln\varepsilon$) so that it possible to have a finite relative area density between systems with different quantum matter content.

(2) The validity of thermodynamic basic laws for causal localization-caused thermal behavior which parallels those of the standard heat-bath thermal setting.

These problems are presently being investigated.

The technical advantage of the holographic approach for particle physics lies in the simplicity of chiral QFT, but the unsolved problems of the holographic inversion prevent presently the return to the particle setting of the ambient original theory (interacting conformal theories do not permit a Wigner particle interpretation [78] and an associated scattering theory; for this reason they are not of *direct* physical relevance to particle physics).

Fortunately there are constructive ideas which stay within the S-matrix particle framework. Their basis is the observation that the scattering matrix in LQP admits an interpretation in terms of the modular data of wedge algebras, which is characteristic for causal localization (and the ensuing vacuum polarization and TCP symmetry) property. This can be seen by combining two known facts. On the one hand the $\Theta \equiv TCP$ invariance of the $S-$matrix [55] can be written in the form

$$\Theta = \Theta^{in} S$$

where Θ^{in} is the TCP operator of the free field theory for the incoming fields. The second fact is that the Tomita S_T-operator for the wedge algebra $\mathcal{A}(W_0)$ relative to the vacuum state vector Ω which is the antilinear closable operator defined by

$$S_T A\Omega = A^*\Omega,\ A \in \mathcal{A}(W)$$

has a polar decomposition (choosing the reference wedge W_0 as $x > |t|$)

$$\begin{aligned} S_T &= J\Delta^{\frac{1}{2}} \\ J = \Theta U(Rot_x(\pi)),&\ \Delta^{i\tau} = U(\Lambda_{x-t}(2\pi\tau)) \end{aligned}$$

where the "radial" part $\Delta^{\frac{1}{2}}$of the polar decomposition is the positive operator $e^{-\pi K}$ with K =generator of x-t boost and J the antilinear involutive "angular" which is apart from a π-rotation around the x-axis identical to Θ. Since the S-matrix commutes with the connected part of the Poincaré group, the relation between J and S is the same as that with Θ and S namely[36]

$$J = J^{in} S,\ J^2 = 1$$

This is a special case of the Tomita Takesaki modular theory for operator algebras whose aim is to classify and characterize pairs of operator algebras and cyclic and separating state vectors $(\mathcal{A}, \Omega)$ in

[36] As in most derivations involving the S-matrix we assume asymptotic completeness $H = H^{in}$.

terms of their modular operators namely an antiunitary involution J (generalized "TCP") and a one-parametric unitary modular group Δ^{it} (a generalized boost "Hamiltonian" $\Delta^{it} = e^{2\pi itK}$). The Tomita operator S_T has characteristic properties which make it a unique object of mathematical physics[37], it is an anti-unitary operator which is involutive on its domain $S_T^2 \subset 1$ and "transparent" i.e. its range equals its domain. It is the only object which is capable to encode geometric properties into domain properties; modular theory is the key for understanding the connections between the operator aspects of LQP and their geometric manifestations (see more remarks below). In an essay about Jordan and his legacy we cannot do more than make an evocation of this rich mathematical theory in terms of some of its physics adapted formulas; the non-expert reader may consult [79] in order to learn about its deep content.

Modular theory of operator algebras is helpful and often indispensable for an intrinsic analysis of QFT which avoids the (always) singular pointlike field coordinatization forced upon us by field quantization while preserving its physical content. The first step of such an analysis consists in extracting informations about inner- and spacetime- symmetries from the algebraic net structure. Here modular theory provides a unifying viewpoint; not in the sense of a group theoretical marriage (which, as shown by O'Rafarteigh, is impossible under reasonable assumptions) but rather by providing a common origin: algebraic inclusions. The spacetime automorphism groups of Minkowski spacetime (Poincaré group) and Dirac-Weyl compactified Minkowski spacetime (the conformal group) are generated from the modular group of wedge algebras relative to the vacuum state (in fact one only needs a finite number of modular generators). There are indications that if one generalizes the modular theory of chiral nets (indexed by intervals on the lightray) to multi-intervals and permits also non-vacuum states one can modular generate the full diffeomorphism group of the circle [80].

Since the discovery of (symmetry) group theory by Galois, one has the option of analysing inclusions[38] in order to study symmetries. There are three types of inclusions of operator algebras: (1) V. Jones- or DHR-inclusions, (2) modular inclusions and (3) split inclusions.

The DHR inclusions originate from the DHR endomorphisms i.e. from globally inequivalent representations which restricted to the individual algebras forming the net yields unitary equivalent subalgebras which have the same spacetime indexing. On the mathematical side these inclusions belong to class of Jones inclusions which are characterized by the existence of conditional expectations from the larger to the smaller algebras. Modular theory provides a geometric characterization; a necessary and sufficient condition for Jones inclusions is that the modular group of the smaller algebra is the restriction of that of the bigger one (the Takesaki theorem). Physicists usually met conditional expectations in the commutative Euclidean Kadanoff-Wilson scheme of renormalization group decimation where modular groups are trivial and conditional expectations from the original- to the thinned out- algebra of infinite dimensional function spaces always exist. Jones inclusions can be encoded into group symmetries and their "para-group" generalizations; in the case of 4-dimensional observable nets indexed by subsets of d=3+1-dimensional Minkowski spacetime one obtains groups which always can be viewed as subgroups of SU(n) for large enough n; the DHR theory confirms precisely Jordan's results which led him to interpret inner symmetries as "dual" to permutation group statistics.

The modular inclusions result from going beyond Jones inclusions, but doing this in a controllable "minimalistic" way by retaining a halfsided action of the restricted modular group on the smaller algebra. The resulting group is isomorphic to the translation-dilation group on a line. By adding further algebraic properties (modular intersections) obtained from the concrete physical situation, one can extend the noncompact groups to the Moebius group etc. These methods are used in the aforementioned algebraic lightfront holography. The perhaps most surprising results obtained by modular methods, which highlight the inexorable connection between relative position of subalgebras and geometry, are those in [83] where it is shown that a finite set of subalgebras (3 in d=1+2, 6 in d=1+3) carefully tuned in a precise relative position with respect to each other (as operator algebras in a common Hilbert space) can define a full QFT; i.e. these data do not only determine the spacetime symmetry group but also the full net structure (by transforming with the original algebras with the symmetry group and forming algebraic intersecting)[39].

[37] It does not appear outside modular theory and therefore cannot be found in past and present mathematical physics textbooks.

[38] Galois studied the inclusion of a number field in its extension obtained by adjoining the roots of a polynomial equation with coefficients from the number field.

[39] The algebras used in this construction are identical (the unique hyperfinite type III_1 von Neumann factor), i.e. like in case of a finite collection of points the information is residing solely in the relations.

It seems that any serious-minded approach about connections between quantum physics and geometry (e.g. quantum gravity) should take these observations into account. According to the considerations about the thermal manifestations of localization at the beginning of this section, thermal aspects should be an important part of the quantum-geometry connection.

The third type of inclusion namely the split inclusion was already alluded to in connection with recovering quantum mechanical (inside/outside quantization box tensor factorization) properties in LQP. Although the individual local algebras in the algebraic net are well-defined mathematical objects, their mathematical type III nature (which is inexorably related to the infinitely large vacuum fluctuations at the boundary of the sharp localization region) does not permit a tensor factorization into the algebra and its commutant. The split property allows to recover a tensor factorization by permitting the vacuum fluctuations to spread into a fuzzy halo of arbitrarily thin diameter; unlike a momentum space cutoff which converts the local theory into an unknown object, the splitting is a local process which is well defined in the original theory and does not require to throw away degrees of freedom. It includes a local algebra into a larger local algebra with localization region equal to the original region plus halo and secures the existence of type I quantum mechanical algebras with a fuzzy intermediate localization. In contrast to the previous two types of inclusions it is not unique which corresponds physically to the many possible interpolating shapes for the vacuum polarization clouds in the fuzzy halo region. Both the modular groups of the two algebras with respect to the vacuum are generally not spacetime diffeomorphisms, but rather fuzzy transformations which in the setting of pointlike fields act on the test functions in such a way that their support in the respective regions is kept fix.

As in the case of the Wigner one-particle spaces in the previous section, we may define the Tomita operator in terms of the associated Lorentz boost, the TCP operator of the asymptotic particles and the S-matrix and use it as before to define a real subspace as the +1 eigenspace; its complexified span is the dense subspace of modular wedge localized vectors. But different from the one-particle case there exists no functor which encodes the spatial modular theory into its algebraic counterpart. In looking for a substitute, one should remember that the crossing property was essential for the uniqueness proof of the inverse scattering problem. Hence we should add this property to the causality and spectral requirements which define a theory within the LQP setting and worry about its conceptual position within LQP at a later state. There are two cases in which this would not be necessary: for formfactors in interaction-free theories (algebras generated by smeared free fields) and the interacting d=1+1 factorizing models. So the first step should consist in a clarification what do we mean by interactions if we do not have the Lagrangian "crutches" at our disposal. This can be done with the help of "polarization-free generators" (PFG [84]) i.e. operators F affiliated with localized operator algebras $\mathcal{A}(\mathcal{O})$ which applied to the vacuum state vector generate a one-particle state vector with no admixture of vacuum polarization (particle/antiparticle pairs)

$$F\Omega = one - particle\ \ vector$$

Such PFGs exist in case of algebras generated by free fields for any region O; according to modular theory they are also available for arbitrary QFTs if one chooses for O a wedge region. Hence one defines interacting theories as those for which no sub-wedge PFGs exist, a definition which obviously does not use Lagrangian crutches. Let us now characterize factorizing d=1+1 models as models for which the S-matrix is purely elastic and where the n-particle elastic S-matrix is moreover determined by the elastic two-particle S-matrix. The cluster properties then fix the form of the n-particle scattering as

$$S^{(n)}(\theta_{1,\dots..}\theta_n) = \prod_{i<j} S^{(2)}(\theta_i, \theta_j)$$

$$p = m(ch\theta, sh\theta),\ S^{(2)}(\theta_i, \theta_j) = S(\theta_i - \theta_j)$$

The momentum space rapidity parametrization is very convenient to formulate crossing and the presence of bound states[40] for the meromorphic two particle S-matrices; in fact meromorphy, unitary and crossing symmetry leads to a classification of scattering matrices $S^{(2)}(\theta)$, with other words the bootstrap program in this limited factorization context is completely solvable and there is no unique solution (no theory of

[40] The boundstate picture for an elastic S-matrix is a quantum mechanical one, however the S-matrix particle hierarchy passes into the charge hierarchy of the previous section as soon as one studies the operator-particle relation for operators localized in sub-wedge regions.

everything) but there are infinitely many S-matrices with the crossing property. Following Zamolodchikov one can associate a formal algebra with such S-matrices which fulfills the Z-F algebra which in the simplest case is of the form

$$
\begin{aligned}
Z^*(\theta)Z^*(\theta') &= S(\theta-\theta')Z^*(\theta')Z^*(\theta) \\
Z(\theta)Z^*(\theta') &= S(\theta'-\theta)Z^*(\theta')Z(\theta)+\delta(\theta-\theta')
\end{aligned}
$$

A closer examination reveals that such operators are the positive and negative frequency parts of PFGs for the wedge algebras [84]. For the proof of the modular properties, especially the KMS property of the generating PFG operators, the crossing property of $S(\theta)$ turned out to be crucial, a fact which was to be expected on the basis of the similarity between both of these cyclicity relations. A detailed account with all mathematical rigor for the case without bound states was recently presented in [85]. In the presence of bound states the formulation in terms of algebraic commutation relations between PFG generators is more involved since their presence changes the structure of the Fock space which would lead to the presence of projection operators and additional terms in the Z-F commutation relation. In that case a definition of the action of the Z-F operators on the multiparticle Fock space vectors is more convenient. The KMS property amounts to a compensation of the explicit bound state contributions with terms coming from contour shifts from S-matrix poles with new explicit antiparticle contributions coming from the contour shifts across the crossed poles. Further restrictions beyond the crossing symmetry in the presence of bound state pole situation do not seem to arise from the KMS property, but a treatment on the same level of rigor as for the case without bound states is still missing.

Knowing the wedge algebra, the remaining steps of the algebraic approach consist in computing better localized algebras (smaller localization regions) by forming algebraic intersection of wedge algebras in general position (by Poincaré transforming the wedge algebra in the reference position W_0). This is of course a step demanding a new type of mathematical skill, but the important point here is that nowhere in this constructive road map one has to face a short distance problem. The process of construction stops at the double cone algebras of arbitrarily small size whose localization-core is a point. Whenever the resulting net of operator algebras has pointlike generating fields (there are no known counter arguments) one may of course coordinatize the net in terms of such fields, but in doing this one looses the unicity and intrinsic nature of the description while possibly gaining simplicity and familiarity of the formalism. Intersections may of course turn out to be trivial i.e. just consist of multiples of the identity operator. In this case we would say that there is no local theory with the required data which would be the analog of saying that a theory in the standard setting is nonrenormalizable or irreparably ultraviolet divergent. The method of constructing the net of operator algebras from a reference wedge algebra is completely general. The main difference to the standard approach is that in the LQP setting one is not forced to work with objects which already have the best possible localization from the beginning throughout the calculation but rather one thinks of generating operators which are not better localized than the algebras they are supposed to generate i.e. their localization can never coalesce to a point.

In the case of d=1+1 factorizable models one can actually compute the intersections thanks to the simplicity of the PFG generators. In the case of absence of bound states one obtains a LSZ like[41] infinite series in Wick ordered PFG operators with meromorphic coefficient functions with Paley-Wiener-Schwarz fall-off properties which reflect the size of the localization region. These coefficient functions are the connected parts of the formfactors of the double cone localized operators i.e. the localized operators are only known as sesquilinear forms on multiparticle states, the existence of operators behind those infinite series remains an open problem as it did in the analogous LSZ case (where pointlike Heisenberg fields were represented as infinite series in terms of incoming fields). The structure of the formfactor equations from the intersecting property is identical to those relating the different formfactors in the approach based on Smirnov's recipe. A remarkable property of the formfactors and the S-matrix is that they are analytic in certain coupling strength parameters g which appear in some of the solutions around g=0[42]. There are however arguments that the power series for off-shell quantities as vacuum correlation

[41] I am referring to the representations of Heisenberg fields as infinite series in the incoming field with retarded coefficient functions whose Fourier transforms have known analyticity properties.

[42] In certain models for which a Lagrangian description is known, these parameters coalesce with the usual coupling strengths.

functions in the coupling strength can never be convergent (at best summable in some weaker sense). This raises the question whether the difference in perturbative convergence properties between on-shell and off-shell objects could be of a general nature.

Since the contemporary literature creates sometimes the wrong impression that in some sense the physical scope of QFT is exhausted by the Lagrangian quantization method (functional integral representations in terms of Euclidean actions), it may be helpful to point out that there are interesting models which do not support such a view. This is not only demonstrated by the quantum Wigner strings but also by many examples of d=1+1 factorizing theories (e.g. the Koberle-Swieca Z(n) models [87]) which cannot be "baptized" by a Lagrangian name.

Unfortunately the starting point of this approach, namely the wedge generating property of the PFG operators, is, apart from one exception[43], limited to the d=1+1 factorizing models (one needs "temperate" PFGs which are only consistent with purely elastic scattering [86]). According to [86], the existence of temperate PFGs in $d > 1 + 1$ result in the triviality of scattering (hence the adjective "free"). It is clear that in order to find a substitute for the non-existing temperate PFG in the general case one has to obtain a better understanding of the mysterious crossing property and its possible relation to modular aspects. For a step in this direction I refer to a forthcoming paper [89] in which the crossing symmetry for the connected formfactors of a special object (a "Master" field) admits an operator KMS modular interpretation in terms of an auxiliary nonlocal field theory on the rapidity space.

The historically minded reader will have noticed that the present ideas about an on-shell constructive program are closely related to the S-matrix bootstrap approach of the 60s, a fact which was already alluded to in the previous section. The main difference is that in the new setting the S-matrix is viewed as a special formfactor (of the identity operator between in and out states) and the program is extended to formfactors of localized operators with vacuum polarization which are, via the crossing property, able to link virtual particle creation with real creation. The aim of this new "on-shell approach" is to implement the classification and construction of nets of operator algebras within the LQP setting without falling into the trap of ultraviolet divergencies which is an inexorable aspect of quantization schemes and their emphasis on singular field coordinatizations. The success of the d=1+1 bootstrap-formfactor program for factorizing models, and the natural explanation of its recipes in terms of the modular aspects of operator algebras shows that the new ultraviolet-finite setting is not empty.

The faith in this program for the general case (beyond the factorizing models) comes primarily from the uniqueness of the inverse scattering problem in regard to the field coordinatization-independent LQP net formulation [65]. In a situation of uniqueness one expects (for those cases were a theory in the LQP sense really exists) to find at least in a perturbative method for classification and construction. Such a new on-shell perturbation theory should not only reproduce the perturbative expressions for standard renormalizable theories in the off-shell Lagrangian formulation upon restriction to on-shell formfactors, but also permit to treat interacting massive vector mesons without ghosts and with the perturbative presence of the scalar Higgs particle (with vanishing Higgs condensate one-point function) arising an inexorable companion of the interacting massive vectormeson by consistency rather than by a free choice in model building. In fact we should be able to see the true frontiers resulting from the physical principles instead of those set by short distance problems and power counting of Lagrangian quantization. The aforementioned idea of linking the crossing property to the KMS property of an auxiliary on-shell QFT whose thermal correlation functions describe the connected formfactors of a masterfield should be seen as a first attempt at understanding the constructive side of the uniqueness theorem[44].

One reason (in addition to the strong return of QFT in the form of gauge theories) why the old S-matrix approach was given up at the end of the 60s was the lack of an operator formalism for the implementation of crossing; ideas based on analyticity properties of formfactors using conjectured non-perturbative properties (Mandelstam representations, and Regge poles) did not lead to a framework in which one could systematically compute amplitudes, just as the Kramers-Kronig dispersion theoretical

[43]The exceptional case consists of "free anyons" in d=1+2. In the Wigner representation theory they correspond to $s \neq$ halfinteger (see previous section). The resulting braid group statistics is incompatible with the existence of subwedge PFGs but admits the existence of temperate PFGs for wedges [43]. In other words the local generation of anyon particles as opposed to Bosons and Fermions requires the presence of vacuum polarization clouds, even if there is no genuine interaction (vanishing scattering cross section).

[44]Any method by which one can compute formfactors without going through correlation functions of pointlike fields in intermediate steps is automatically ultraviolet-finite.

framework of the 50s was found unsuitable for systematic and trustworthy calculations (but at least very well suited for experimental checks of consequences of causality). Modular theory suggests that on-shell analyticity properties are deeply related to domain properties of modular operators, and therefore it seems to be more important for constructive attempts to unravel these operator properties rather then using analyticity properties.

Physicists unlike mathematicians are not easily deterred by hard problems. If they come across an important problem (as the Heisenberg-Chew S-matrix attempts of a formulation of particle physics without ultraviolet divergencies) they do not give up and leave it to the future because they are unable to solve that problem by presently known methods; they rather tend to invent another similar problem which is more susceptible to solution. In the case at hand, Veneziano discovered that by using properties of Γ-functions one can implement a property (called "duality") which formally looks like the field theoretic crossing by replacing the scattering continuum in the invariant energy of the elastic part of the S-matrix with a an infinite discrete particle "tower". The reasons given at that time for tolerating such an ad hoc looking structural violation of field-theoretic scattering theory were purely phenomenological and are long forgotten in the modern community of string theorist.

Veneziano's dual model construction based on properties of Γ-function would probably also have been forgotten if it was not for two important observations about two intriguing aspects. The first property was that the model allows an alternative description in terms of an auxiliary d=1+1 conformal field theory which in some sense gave more respectability than Γ-function engineering. The crossing property of the infinite particle tower (which we will refer to as Veneziano "duality" in order not to confuse it with the crossing property of a field theoretic S-matrix) was deferred to the duality property in chiral conformal 4-point functions which is basically the commutation structure (with braid group commutation relations in the general case) which relates the various permuted positions of the conformal operators[45]. More important was the later realization that models of this kind permit an interpretation in terms of the quantization of a classical string described by a Nambu-Goto Lagrangian. The required Poincaré invariance of the momentum space string led to covariant realizations in 26, and in the presence of supersymmetry in 10 spacetime dimensions; what appeared initially as a vice was (in the string theoretic credo) converted into a virtue by the invention of compactifying spatial dimensions in the spirit of Kaluza and Klein. The Nambu-Goto string quantization requires to picture the range of Veneziano's auxiliary chiral fields as a target space of a first quantized map defined by a classical reading of the chiral fields. This in turn introduces a rich differential geometric structure which the auxiliary chiral reading of the old dual model did not have. Therefore it is not surprising that the Green-Schwarz-Witten formulation of string theory [90] and its subsequent extensions has had a significant impact on mathematics; however there are no indications from experimental high energy particle physics that nature realizes such structures.

Since it is not the duty of a theoretician to worry about the status of the experimental situation but rather to support or criticize ideas by conceptual and mathematical analysis, the relevant question about this kind of string theory in the present context is how it relates to contemporary QFT. This boils down to the question whether the above duality property which replaced the field theoretic crossing has destroyed causality and localizability and if this turns out to be the case whether string theory offers a generalization of these structures. Ever since the time of Jordan's quantization of wave fields causality and localization structures are the most indispensable properties in quantum physics[46] which are directly linked to the physical interpretability of a theory. Without having a localization structure it is not possible to interpret structural properties in momentum space; particle momenta are primarily data which parametrize relations between asymptotic spacetime events [46], covariance alone is of no help. This is no problem for those natural quantum strings (e.g. Wigner massless strings) in the setting of LQP. These strings exist because the observable net in certain models allows "charged" representations and the superselected charges have internal degrees of freedom which cannot be dumped into pointlike field multiplets. Unlike Nambu-Goto strings these quantum strings have an internal structure which makes them "stringy" in the interpretable sense of semiinfinite string localization; the fact that they do not permit a "quantization reading" makes them more noncommutative than pointlike objects. There is no

[45]The tower structure arises from operator product expansions between pairs of operators in the 4-point function.

[46]Recent attempts to formulate so-called "Noncommutative Field Theories" have shown that although it is mathematically not difficult to abandon causality and localization, the construction of a generalized substitute which still permits a physical interpretation is an extremely difficult unsolved problem.

intrinsic physical meaning to quantum closed strings; they would be the analogs of particle-antiparticles pair objects and have the same charge as the vacuum. Since there is no Lagrangian formulation, the implementation of interactions cannot follow the standard pattern of causal perturbation theory (Wick-ordered couplings etc.).

In the standard setting, the quantum fields which commute for spacelike distances with a given causal field (local equivalence class or "Borchers class" of the given local field) follow a classical pattern, i.e. are local composites (normal products which reduce to Wick products in case of free fields). This relic of a classical picture is lost for those zero mass Wigner objects which have an internal structure. In that case the class of relatively local objects in the sense of spacelike distances is much larger than the classically suggested local composites. The mathematically simplest illustration of this new phenomenon is provided by generalized free fields which have a continuous (not denumerable) class of relative local objects [91].

LQP has a very elegant way of dealing with this new phenomenon of extension from the classically suggested picture of composites to that required by the algebraic notion of relative spacelike commutativity. The algebraic extension is called "Haag dualization". In the standard setting the operator algebras which are localized in double cones generated by pointlike fields in the vacuum representation are automatically "Haag dual" i.e. the commutant of the spacelike disjoint algebra is exactly equal to the original operator algebra. Haag dualization is a well-defined "maximalization" of local algebras which at the beginning are not dual in this sense; the Hilbert space remains unchanged in this operation. The Bisognano-Wichmann property implies Haag duality for wedges, and the double cone algebras obtained from intersecting Haag dual wedge algebras inherit this property. A new perturbative formalism which replaces the causal perturbation theory of the standard situation does not yet exist, but with the powerful guidance provided by the algebraic setting it should be possible to construct one. Such a new perturbative setting could be useful for implementing interactions for pure quantum objects which do admit a (Lagrangian) quantization interpretation as e.g. the aforementioned Wigner strings. Although they are still subject to those causality and localization principles which Jordan discovered together with his quantization of wave fields, they constitute generalized realizations for which his 1929 Kharkov credo of "abandonment of classical crutches" is much more than an esthetical improvement. For renormalizable interactions in terms of pointlike fields the hypothetical on-shell procedure should reproduce the renormalized mass-shell formfactors in a finite manner.

In the closed string theory based in the Nambu-Goto Lagrangian quantization setting, the interactions are implemented by geometrical pictures of combining and splitting tubes; however not only do these strings suffer from the severe spacetime dimensional restrictions, but one has lost the relations with causality and localizability, so that the physical interpretation becomes obscure. In fact even if one imposes a multi-string tensor product structure ("free string field theory"), the causality/localizabilty issue looks still contradictory: in the d=24+1 lightfront setting the string becomes "transparent" [92] (i.e. the algebraic commutation structure only shows the center and not the string itself), whereas in the d=25+1 covariant setting localization aspects seem to have been completely lost [93]. These calculations also reveal a fundamental difference in philosophy behind LQP quantum strings and (Nambu-Goto) strings of string theory. Whereas LQP views causality of observables and the resulting localization of charge-carrying fields (point- or semiinfinite string-like) as the physically indispensable properties[47] and the covariance aspects as consequences (the connection being provided by modular theory), string theorist place more emphasis on target space covariance and seem to be less concerned about localization properties.

As far as the achievements of the quantized Nambu-Goto strings are concerned, namely the incorporation of spin $s = 2$ objects ("quantum gravity") and the ultraviolet finiteness, there is no good reason to think that pure quantum strings (as the massless strings appearing in the Wigner zero mass representation theory) are doing worse on these issues. In fact the idea that the ultraviolet-finite formulation of the formfactor approach for factorizing models may possess a general higher-dimensional counterpart was the present Leitmotiv for what was considered as Jordan's legacy. Any approach which either succeeds to calculate directly in terms of algebras without the use of pointlike fields, or only uses only formfactors of fields and avoids correlation functions will be ultraviolet-finite and the uniqueness theorem of the inverse scattering problem [65] suggests that such a formulation exists.

[47] Momentum space properties acquire their physical interpretation through localization of spacetime events; e.g. particle momenta parametrize asymptotic localization events [46].

The development of concepts starting from Jordan's quantized wave fields passing through renormalization theory to the more intrinsic setting of LQP has been a historic dialectic process in which new ideas were confronted with established principles in an extremely profound and conscientious manner. Since different from mathematics, theoretical physics at certain times has to pass through very speculative phases, this dialectic confrontation (which either eliminates the speculative idea as unfounded, or ends with a historical synthesis which incorporates the old setting as a limiting case into a new framework) is the most valuable theoretical tool which separates genuine revolutionary enlargements of knowledge about nature from one way trips into the "blue yonder". The list of abandoned proposals is much larger than that of successful theories.

The present LQP setting, which is deeply rooted in the history of QFT by elevating the Jordan-Pauli causality [27][73] combined with the abandonment of "classical crutches", would view the string proposal based on the quantization of the Nambu-Goto Lagrangian (which leads to high spacetime dimensions and obscure localization aspects) as a step into the physical "blue yonder".

String theorists argue that their model of closed strings approaches quantum field theory in the low energy limit But for making such arguments involving scale-sliding on the formal level of actions more credible, one would need some form of localization on the string side. A more convincing argument would consist in showing that the dual S-matrix obtained from the string prescription approaches a S-matrix with the field theoretic crossing property, but the lack of conceptual understanding about the intrinsic meaning of the string recipes make this a hopeless task. The replacement of time-dependent LSZ scattering theory as a consequence of QFT locality by a string cooking recipe without any hope of regaining the old conceptual elegance is another worrisome aspect.

An even greater stumbling block for a conceptually conscientious quantum physicist is the way string theorists implement interactions between their first quantized strings by joining and splitting tubes. This picture does not only return to pre LSZ times, but it even falls back behind Heisenberg's dictum that the position of an electron is not an attribute of the electron but of the event produced by the interaction with the measurement apparatus. By not separating algebraic properties of observables from those which originate from states, one is returning to a conceptual level of the pre-Heisenberg Bohr-Sommerfeld quantum theory in which such a separation was not possible.

String computations are expected to produce a dual S-matrix in the form of an infinite series in ascending genii of Riemann surfaces. Instead of causality and localization properties the construction of the dual string S-matrix uses geometrical information about Riemann surfaces, Teichmueller spaces etc. But there is not the slightest indication that these geometric properties are intrinsic. Let us explain carefully what is meant by a property being intrinsic. In the standard formulation of QFT one computes a system of correlation functions of pointlike fields. A reconstruction theorem asserts [55] that all physical properties which went into the construction of these correlations can also be re-extracted from these correlations; i.e. if I pass this calculated system of correlations to my learned particle theory colleague, he will be able to reconstruct a Hilbert space with a system of causal and localizable operator algebras and verify all physical (local gauge-invariant) properties without my telling him in what way I computed my correlation functions; they are intrinsic properties of my calculated set of correlation functions. Examples for non-intrinsic properties are those properties which enter on the classical side if one started the computation with "classical crutches" as Lagrangians, actions in functional integrals etc.. Quantization is a form of art outside the range of a reconstruction theorem.

It is clear that reconstructability and intrinsicness are important conceptual achievements of QFT and the uniqueness of the inverse scattering problem (in case of asymptotically completeness) attributes an equally important role to a field theoretic crossing symmetric S-matrix. In string theory no such reconstruction theorem is known and none of those geometric properties which form the basis of string theory have been shown to be intrinsic; in fact their proximity to quantization properties casts serious doubts about their intrinsic nature.

Whereas temporary shortcomings are sometimes unavoidable at the start of new ideas, it is much harder to feel comfortable if after more than 20 years the unclear situation prevails and the historical awareness gets lost. It is not surprising but deeply lamentable, that the sociological dominance of string theory threatens to destroy the conceptual fabric of one of the most successful scientific endeavors which started with Jordan's quantization of wave fields almost 80 years ago. It is particularly worrisome that profound knowledge, which one expects to be important in the future development of particle physics, gets lost at an alarming rate[48].

Within the historical context it would have been more natural to study first those pure quantum strings (as e.g. the Wigner helicity tower representations) and their interactions since they have deep roots in the causality and localization principles. But generalizations along a given quantization formalism are often much simpler than extensions which are subject to physical principles for which a suitable formalism still needs to be developed. In the apparent clash between the time-honored causality and localization ideas which found their first expression in Jordan's work on quantization of wave fields (and for which the same author was already looking for a more intrinsic formulation free of classical "crutches") and the quantization-based Nambu-Goto strings and their geometric generalizations, I have opted for the path which saves the principles and sacrifices the quantization formalism. Therefore it should not surprise that I view the encouragements of experimentalizers by string theorists to detect the little curled up spatial poltergeist dimensions which account for the difference between the numbers 10 and 4 of the supersymmetrized quantized Nambu-Goto string) with a certain amount of suspicion and disbelief. Expensive experiments based on flimsy theory are the most visible signs of crisis in particle physics. One can only hope that this kind of experimental enterprises will not just end with a negative result and without any other theoretically unexpected observation, because this will damage more than the faith in the value of the string-oriented post electro-weak particle physics.

If string theory would be a just a fashionable mathematically challenging area which attracts young people and competes with other ideas, there would be no reason to be worried. Even if one does not accept the arguments in favor of a quantization interpretation[49] of the range of conformal fields as a string target space, the mathematical results of string theorists on subtle problems in conformal field theories (e.g. the intricate structure behind the Liouville field theory) are valuable extensions of knowledge about nonperturbative QFT. The worrisome negative aspects are not primarily coming from the scientific content, but rather from the sociological climate and the hegemonial ideological stance of its meanwhile totally globalized and omnipotent community. This is even visible in the terminology. Whereas the aim of natural sciences for centuries was the de-mystification of nature (essentially since the time of Lavoisier, when the phlogiston theory was abandoned), string theorist do not miss any opportunity to emphasize that their big Latin M could also be understood as "Mystery" and one does not get the impression that they are interested in changing this situation. At the time of the great quantum theoretical discoveries when the number of physicists was much smaller than presently, there was no danger of the scientific content being dominated by particular interests of particular schools outside the quest for knowledge. The various physics institutes existed in a multilateral equilibrium and a sociological amplification of an idea leading to a monoculture of a theory without experimental support would have been unimaginable ever since alchemy was abandoned. The large number of physicists and the globalization through instant communication have made this system very unstable. Whereas fashions in earlier times influenced preferences in the selection of candidates mainly on a local level, the globalized trend of the string ideology is placing a severe strain on the liberty of research.

We are accustomed to think of the exact sciences as an area which unfolds in a more detached and objective way than other human endeavors. But there is no rule which states that the exact sciences are immune against those human shortcomings and defects which led to catastrophic political ideologies. Physics was successfully defending its independence against infringements of political ideologies; attempts in this direction ended as a lip service in forewords of textbooks from the USSR, but were never able to influence the scientific content. There is however no defense against ideological strands which originates inside physics. Their power of control over minds and money does not depend on whether they were able to contribute anything of enduring value.

The start of string theory dominance occurred in the 70s when the dual model theory of strong interactions was converted to a theory of quantum gravity (the Paris "Bartholomew night-like massacre of the old string theory"). One would have expected that a theory after having had some success at laboratory energies in strong interactions would loose its physical credibility if one is asked to abandon this application and instead use that theory as a theory of gravity at energies corresponding to the

[48]Most string theorists in the younger generation know what a Calabi-Yao manifold is, but only a few have a rudimentary knowledge of LSZ scattering theory.

[49]In order to talk about a geometric target range, it would be necessary to be able to view the chiral conformal theory as the result of quantization of a classical field structure.

Planck length which are 15 orders of magnitudes higher. But it is also clear that in case one succeeds to convince a majority that this is a reasonable step, one has created a new area of research which different from the original strong interaction theory is only controlled by mathematics and outside obligations of experimental verifications. This new situation has led to the present state in which particle physicists solving string theoretic problems seem to be doing quite well, but particle physics is in a deep crisis (the particle physics analog of the economic Enron-Worldcom effect). Physics and what physicists are doing is presently not necessarily identical.

It may be seen as a historical irony that Germany as the country where most of it began almost 80 years ago, is also the place where this trend of a string oriented particle physics monoculture has most progressed. Research proposals in particle theory which does not relate in some way with strings have no chance of being approved, even if their scientific content is of the highest quality. The necessity of getting the string imprimatur is an insult to free research. The presence of physicists in leading positions of influence who have since their graduate students times not done anything else but string theory and who built their career on their social success in that area (there was yet no scientific success), is a heavy liability for the future of particle physics. Social success tends to destroy the intellectual modesty which one needs to start something new and original. It also diminishes the intellectual curiosity and ability to appreciate different ideas in particle physics and causes an amnesia regarding historical roots and continuity. Since there is not a single pure research institution dedicated to particle physics in Germany whose future is not compromised in this direction, this may very well be the end of the project which I have described in this essay as Jordan's legacy; this is the reason why in the present context one cannot be silent about this very serious problem.

There have been attempts by philosophers and historians to understand important discoveries in the context of their sociological settings. The 1925 discovery of quantum mechanics with its rejection of the causality in the sense of classical determinism has been related to the post World War I gloom and doom of a nation which only a short time before was thinking of itself as one of the intellectual centers of the world; i.e. playing with acausality as a way to overcome the gloom and stay in the intellectual avant-garde through a revolutionary act [95][1]. People who subscribe to such ideas may perhaps ask the question whether string theory and in particular those more recent proposals which come with big Latin letters as M-theory should be considered as the sociological manifestation of the Zeitgeist of the Hegemon in particle physics.

We have used some writings of Pascual Jordan as one of the protagonists of quantum field theory as a historical support of a new program which de-emphasizes pointlike fields in order to obtain an intrinsic singular-coordinate free formulation which bypasses the ultraviolet aspect of the standard formulation. One may receive additional support in this endeavor from his glorious friendly opponent Paul Dirac. In most of recent centennial articles his strong believe in mathematical beauty and conceptual harmony is emphasized and as an illustration his beautiful geometric presentation of the Dirac equation and construction of magnetic monopoles are cited. But the same Dirac also rejected renormalization theory, a fact which is mostly ignored as an unpleasant shortcoming of an otherwise brilliant theoretical physicist. It is my firm conviction that a man who contributed so much to the beginning of renormalization theory certainly did not reject its impressive results out of a lack of understanding, but rather because he considered it as conceptually unfinished and mathematically grossly imperfect. Indeed if one looks at standard approaches as canonical quantization or quantization via functional integrals, one realizes that this is what deserves to be called "as if" physics. By this I mean that the physical correlation functions (i.e. the results after renormalization) are Einstein-causal but definitely not canonical nor representable in terms of a functional integral. The "as if (everything would be consistent)" attitude (i.e. the ignoring of the mismatch between the original assumptions and the final mathematical structure /which one arrives at after mathematical doubtful intermediate steps) generated a false impression of a universality of the functional framework. The majority of particle physicists ignore this imbalance and some even may react irritated against anybody who points his finger at this serious imperfection. The "as if" aspect is often unavoidable with new discoveries; it is important to first secure the results and later worry about the mathematical legitimization. There is however nothing more contraproductive in theoretical physics than a solidified "as if scheme" which has been used for decades by generations of physicists since the interest to discover the mathematical correct way (which may require new conceptual investments far beyond the "as if scheme") decreases with time.

My explanation for Dirac's rejection of renormalization is that although he did not doubt the validity perturbative results, the "as if" attitude in their derivation and the mathematically delicate setting in terms of singular field coordinatizations was not in line with his sense of beauty. In fact he may even have rejected the mathematically improved setting of causal perturbation theory[50] because that more rigorous approach draws a technical frontier between renormalizable/nonrenormalizable without giving a hint about its relation relative to the underlying physical principles (there exists no dictum that one must implement interactions by coupling free fields, although it is difficult to think of anything else if one starts with Jordan's quantized wave fields). Unfortunately the "as if" attitude in physics which was totally absent in Jordan's times (and is still absent in LQP) pervades most formalisms used in particle theory, it is in particular characteristic of string theory whose intrinsic physical content is unknown.

The LQP setting enriched with modular theory, advocated here for constructive purposes, may be revolutionary enough to have satisfied Jordan's demands, and its conceptually concise setting may even have lived up to Dirac's expectations concerning beauty and rigor, but the main question: *is the success of this method in the case of d=1+1 factorizing model a strange accident or is there a general message behind,* despite many encouraging arguments, has to be left open. However the many new profound questions which arise in the present context show that despite its more than 75 years of existence after its discovery in 1927, QFT is very much alive and its deeper conceptual and mathematical layers still need to be understood in agreement with Jordan's expectations at the 1929 Kharkov conference. When string theorist these days point at old (and in their view understood) QFT as opposed to their new string theory, they tacitly identify QFT with its very limited Lagrangian implementation.

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