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GALAXY FORMATION AND VACUUM QUANTUM FLUCTUATION OF THE GRAVITATIONAL FIELD

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ABSTRACT: A model of vacuum quantum fluctuations of the gravitational field leads to an exponentially grow of the contrast density in Friedman perturbed cosmology. We conjecture about this effect having a dominant role in galaxy formation

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It has been widely conjectured that to explain the observed inhomogeneities of the Universe we live in, one should use the small perturbation analysis of spatially homogeneous cosmological models. One starts by assuming a Friedmann Universe and then perturbe Einstein's set of equations of General Relativity (GR) under almost arbitrary conditions. The pertinent variable we have to look at is the density constrast $\delta\rho/\rho$. A typical dependence of $\delta\rho/\rho$ with the cosmical time, in a pressure free background is $t^{2/3}$ [1]. So, there is an instability but, as it has been remarked by many authors [1, 2, 6] density fluctuations grow too slowly to be the real cause of galaxy formation. There has been many proposals of how to overcome this difficulty (see [3, 6, 9, 10] for a review) without much succes.

The purpose of this letter is to present a conjecture which would change drastically this situation. We start by remarking that Einstein's equations of GR do not give a full description of gravity when vacuum quantum fluctuations of the gravitational field are not null. This has been pointed out many times in the literature [4, 5].

In order to describe these fluctuations we will follow a modified version of a suggestion made by Ginzburg et al [4] some years ago. The essential point is to regard Einstein's equations of GR as exact microscopic equations for the metric tensor $g_{\mu\nu}$. This geometry undergo small fluctuations around some mean value $< g_{\mu\nu} >$. We can write $g_{\mu\nu} = < g_{\mu\nu} > + \delta g_{\mu\nu}$ where $\delta g_{\mu\nu}$ is a small perturbation term.

The equations of evolution of the mean metric $< g_{\mu\nu} >$, obtained by substitution of the above relation into Einstein's equations are

$$R^{\mu}{}_{\nu} - \frac{1}{2} R \, \delta^{\mu}{}_{\nu} = \phi^{\mu}{}_{\nu} \qquad (1)$$

In this equation the lhs is to be constructed with the mean metric and the rhs depends on the fluctuation. Then the hypothesis is made that $\phi^{\mu}{}_{\nu}$ may be expanded in a series in the mean curvature. We will follow this procedure here but instead of working with the set (1) of equations we will deal with the so-called quasi-Maxwellian approach of GR [7, 8]. The equation for the mean metric reads:

$$C^{\alpha\beta\mu\nu}{}_{||\nu} = I^{\alpha\beta\mu} + Q^{\alpha\beta\mu} \qquad (2)$$

where the Weyl tensor $C^{\alpha\beta\mu\nu}$ is constructed with $< g_{\mu\nu} >$, the current $I^{\alpha\beta\mu}$ depends on the matter content of the Universe and $Q^{\alpha\beta\mu}$ depends on the fluctuating term $\delta g_{\mu\nu}$. Using a Ginzburg-type expansion we will write

$$Q^{\alpha\beta\mu} = \sum_{\kappa} c_{(\kappa)} \, P^{\alpha\beta\mu}_{(\kappa)} \qquad (3)$$

where $P^{\alpha\beta\mu}_{(\kappa)}$ are polynomials of the Weyl tensor. Actually, we will decompose $C^{\alpha\beta\mu\nu}$ into its electric and magnetic parts, $E^{\alpha\beta}$ and $H^{\alpha\beta}$ respectivelly, as defined by

$$E_{\alpha\beta} = - C_{\alpha\mu\beta\nu} \, V^{\mu} \, V^{\nu} \qquad (4a)$$

$$H_{\alpha\beta} = C^{*}_{\alpha\mu\beta\nu} \, V^{\mu} \, V^{\nu} \equiv \frac{1}{2} \eta_{\alpha\mu}{}^{\rho\sigma} \, C_{\rho\sigma\beta\nu} \, V^{\mu} \, V^{\nu} \qquad (4b)$$

So, Weyl tensor can be decomposed, for an arbitrary observer with velocity V^{α} , as:

$$C_{\alpha\beta}{}^{\mu\nu} = 2\, V_{[\alpha}\, E_{\beta]}{}^{[\mu}\, V^{\nu]} + \delta^{[\mu}_{[\alpha}\, E^{\alpha]}_{\beta]} -$$

$$- \eta_{\alpha\beta\lambda\sigma}\, V^{\lambda}\, H^{\sigma[\mu}\, V^{\nu]} - \eta^{\mu\nu\rho\sigma}\, V_{\rho}\, H_{\sigma[\alpha}\, V_{\beta]} \qquad (5)$$

where $\eta^{\alpha\beta\mu\nu} = - \frac{1}{\sqrt{-g}}\, \varepsilon^{\alpha\beta\mu\nu}$. $\varepsilon^{\alpha\beta\mu\nu}$ is the completely anti--symmetric Levi-Civita symbol. [ ] means anti-symmetrization.

Now let us turn to a perturbation of Friedmann's Universe. In Einstein's theory (no vacuum fluctuation) the equation of perturbation reads

$$C^{\alpha\beta\mu\nu}{}_{||\nu} = \delta I^{\alpha\beta\mu} \qquad (6)$$

where $\delta I^{\alpha\beta\mu}$ is to be constructed with matter fluctuating term. It is wothwhile to remark that, as the Weyl tensor of the background is null we will denote the perturbation as $C^{\alpha\beta\mu\nu}$, $E^{\alpha\beta}$ instead of $\delta C^{\alpha\beta\mu\nu}$, $\delta E^{\alpha\beta}$, etc. Now, by taking the correct equation (2) the real perturbation set is

$$C^{\alpha\beta\mu\nu}{}_{||\nu} = \delta I^{\alpha\beta\mu} + Q^{\alpha\beta\mu} \qquad (7)$$

The most general linear expression on Weyl tensor reads

$$Q^{\alpha\beta\mu} = M\, V^{[\beta}\, E^{\alpha]\mu} + N\, V^{[\beta}\, H^{\alpha]\mu} + P\, \eta^{\alpha\beta\lambda\sigma}\, E_{\lambda}{}^{\mu}\, V_{\sigma} +$$

$$+ Q\, \eta^{\alpha\beta\lambda\sigma}\, H_{\lambda}{}^{\mu}\, V_{\sigma} \qquad (8)$$

As we are interested only on the effects of perturbations of the density contrast we can set $N = P = Q = 0$. The reason for this is simply that these terms affects only the time evolution of $H^{\alpha\beta}$ and as is well-known $\dot{H}^{\alpha\beta}$ is not directly coupled with fluctuation of the density.

To deal with a specific model we give to the background geometry the following features

i) pressure free

ii) the fundamental length $ds^2 = dt^2 - a^2(t)\, d\sigma^2$
with $a = a_o\, t^{2/3}$

iii) $\rho = \rho_o\, a^{-3}$

We then project the set (7) of equations in the 3-dimensional rest space of the observer V^{α} . The pertinent equations for our analysis are given by

$$E_{\ell}{}^{\kappa}{}_{||\kappa} = \frac{1}{3}\,(\delta\rho)_{|\ell} \tag{9}$$

$$\dot{E}^{\kappa\ell} - \frac{1}{2}\, H_m{}^{(\kappa}{}_{||j}\,\eta^{\ell)\alpha m j}\, V_\alpha + (\Theta - M)\, E^{\kappa\ell} = - \frac{\rho}{4}\, \delta\sigma^{\kappa\ell} \tag{10}$$

(latin indices goes from 1 to 3).

We have set the perturbation of the fluid velocity $\delta V^{\alpha} = 0$ just to simplify our exposition. It has no effect on our conclussions.

The case in which M is negative has been discussed elsewhere [11] . It will have a damping effect. We turn to the case in which M is positive. A typical solution of set (9, 10) can be written:

$$E^{\kappa}{}_{\ell} = \chi^{\kappa}{}_{\ell}(x^{\kappa})\ t^{-4/3}\ e^{Mt} \qquad (11)$$

and for the time-dependence of $\delta\rho/\rho$:

$$\frac{\delta\rho}{\rho} \simeq t^{2/3}\ e^{Mt}$$

We see that the time dependence of the contrast factor grows exponentially (compare with Lifshitz: that is take the limit $M \rightarrow o$).

So, for the above model the perturbation machanism is efficient enough to have a fundamental role in explaining galaxy formation. Finally it is worthwhile to remark that the actual value of M should be a matter for future investigations.

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