NOTAS DE FÍSICA

VOLUME XIII

Nº 17

A NEW CLASS OF SUPERCONVERGENT SUM RULES FOR SCATTERING AMPLITUDES

by

D. S. Narayan, R. P. Saxena
and Prem P. Srivastava

CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Av. Wenceslau Braz, 71

RIO DE JANEIRO

1968

Notas de Física - Volume XIII- Nº 17

A NEW CLASS OF SUPERCONVERGENT SUM RULES FOR SCATTERING AMPLITUDES

D. S. Narayan *
Tata Institute of Fundamental Research
Bombay, India

R. P. Saxena *
University of Delhi, India

Prem P. Srivastava *
Centro Brasileiro de Pesquisas Físicas
Rio de Janeiro - Brazil

(Received March 6, 1968)

In a recent paper [1] the authors have proposed a class of superconvergent sum rules by postulating that the difference of two suitable invariant amplitudes, one representing the scattering of a vector particle against some target and the other representing the scattering of the "corresponding" axial vector particle against the same target satisfies a superconvergent dispersion relation. The vector particles under consideration

* This work was accomplished while the authors were at the International Center for Theoretical Physics - Trieste.

are ρ, K^*, $\bar{K}^*$, ω_8 and ω_1, (collectively represented as V_i where $i = 0, 1, \ldots, 8$ are the SU(3) indices) which are supposed to determinate the octet and singlet vector currents. One assumes as usual, that a suitable mixing between ω_8 and ω_1 gives rise to the observed isoscalars ω and φ. The corresponding axial vector particles are A_1, K_A, $\bar{K}_A$, D_8 and D_0 (denoted collectively by A_i) which are supposed to dominate the octet and singlet axial vector currents. We assume, in analogy to the previous case, that a mixing between D_8 and D_0 gives rise to the observed isoscalars D(1285) and E(1420). In (I) it was stated that the proposed superconvergent sum rules resemble in a sense the exact chiral symmetry sum rules at high energies. The assumed superconvergence of the difference of suitable invariant amplitudes, however, can be justified, as mentioned in the earlier paper, from the considerations based on (unitarity [2]) Regge pole theory [3].

In (I) we considered the sum rules obtainable by considering the scattering of $\rho(A_1) - \pi$ and $K^*(K_A) - \pi$ and predicted the widths of A_1 and K_A in terms of those of ρ and K^* respectively. In this letter we wish to consider the sum rules which can be obtained by considering the scattering of ω and φ (D and E) on pions and also those involving the scattering of vector and axial vector particles on K-mesons.

As in the previous paper, we decompose the full amplitude $T^{V(A)}$ representing the scattering of a vector (axial-vector) particle on any given target into the invariant amplitude [2] as

$$T^{V(A)} = \epsilon_1 \cdot P\ \epsilon_2 \cdot P\ A^{V(A)} + \frac{1}{2}\left(\epsilon_1\ P\ \epsilon_2 \cdot Q + \epsilon_1 \cdot Q\ \epsilon_2 \cdot P\right) B^{V(A)} +$$

$$+ \epsilon_1 \cdot Q\ \epsilon_2 \cdot Q\ C_1^{V(A)} + \epsilon_1 \cdot \epsilon_2\ C_2^{V(A)} \qquad (1)$$

From the postulated superconvergence we have the sum rule

$$\int \mathrm{Im}\left[B^{(V)}(s') - B_I^{(A)}(s')\right] ds' = 0 \qquad (2)$$

for any isospin state I in the s-channel.

We consider now the implications of the sum rule on the reactions $\pi^+ + \omega_8(\omega_1) \to \pi^+ + \omega_8(\omega_1)$ and $\pi^+ + D_8(D_1) \longrightarrow \pi^+ + D_8(D_1)$. Keeping the contributions of the important poles B(1210) and $\rho(760)$ in the first case and $\pi_V(1003)$ in the second case, and using the couplings similar to those defined in (I), we obtain the following sum rule for the coupling constants:

$$g^2_{D_8\pi_V\pi} = \left[g^{(L)2}_{B\omega_8\pi} + g^{(T)2}_{B\omega_8\pi} \left(\frac{m_B^2 - m_\pi^2}{m^2_{\omega_8}} \right) \right] k^2_{\omega_8} - \left(m_\rho^2 - m_\pi^2 \right) g^2_{\omega_8\rho\pi} \quad (3)$$

and a similar equation obtained by replacing $\omega_8(D_8)$ by $\omega_1(D_1)$. Here $k_{\omega_8(\omega_1)}$ is the centre-of-mass momentum of the decay E into $\omega_8(\omega_1)$ and π. The hypothetical coupling constants $g_{\omega_8\rho\pi}$ and $g_{\omega_1\rho\pi}$ are related to the physical coupling constants $g_{\omega\rho\pi}$ and $g_{\varphi\rho\pi}$ through the $\omega - \varphi$ mixing angle and satisfy

$$g^2_{\omega_8\rho\pi} + g^2_{\omega_1\rho\pi} = g^2_{\omega\rho\pi} + g^2_{\varphi\rho\pi} \ . \qquad (4)$$

Similar relations hold for the $g_{D(E)\pi_V\pi}$ and $g_{B\omega(\varphi)\pi}$ coupling constants. If we replace [4] to a good approximation $m^2_{\omega_8(\omega_1)}$ by an average value $\langle m^2_\omega \rangle$ and $k^2_{\omega_8(\omega_1)}$ by an average value $\langle k^2_\omega \rangle$

we can combine the two equations above to obtain

$$g^2_{D\pi_V\pi} + g^2_{E\pi_V\pi} = \left[\left(g^{(L)^2}_{B\omega\pi} + g^{(L)^2}_{B\varphi\pi}\right) + \left(g^{(T)^2}_{B\omega\pi} + g^{(T)^2}_{B\varphi\pi}\right)\frac{\left(m_B^2 - m_\pi^2\right)}{\langle m_\omega^2\rangle}\right]\langle k_\omega^2\rangle$$

$$-\left(m_\rho^2 - m_\pi^2\right)\left(g^2_{\omega\rho\pi} + g^2_{\varphi\rho\pi}\right) \qquad (5)$$

This sum rule can be tested [5] for the prediction of the $D \to \pi_V\pi$ decay width. We use the experimental width to determine $g_{E\pi_V\pi}$ and the sum rule $m_\rho^2\left(g^2_{\omega\rho\pi} + g^2_{\varphi\rho\pi}\right) = 4\, g^2_{\rho\pi\pi}$ derived in ref. 2. To determine the first term on the right-hand side we assume that the coupling is purely longitudinal and that the small branching ratio (say 1%) for $B \to \varphi\pi$ mode is purely due to the much smaller phase space available for it compared with the dominant mode $B \to \omega\pi$. The coupling constants $g_{B\omega\pi}$ and $g_{B\varphi\pi}$ are then comparable in magnitude. The partial width for $D \to \pi_V\pi$ decay is predicted to be ~ 18 MeV, while the total width of D has the experimental [6] value 32 ± 8 MeV.

We next consider the scattering of vector and axial vector particles on K-mesons and the consequent sum rules. Considering the reactions $\rho_o + K^+ \to \rho_o + K^+$ and $A_1^o + K^+ \to A_1^o + K^+$ and retaining the important pole contributions from K^+ and K_A^+ in the former case, while retaining the contribution from K^* in the latter case, we get the relation

$$4g_{\rho KK}^2 + k_{K_A \to K\rho}^2 \left(g_{K_A K\rho}^{(L)^2} - g_{K_A K\rho}^{(T)^2} \right)$$

$$- \left[\frac{(m_{A_1}^2 + m_{K*}^2 - m_K^2)^2}{4m_{K*}^2} - m_{A_1}^2 \right] \left(g_{A_1 K*K}^{(L)^2} - g_{A_1 K*K}^{(T)^2} \right) \qquad (6)$$

where $k_{K_A \longrightarrow K\rho}$ is the c.m. momentum in the decay $K_A \longrightarrow K + \rho$.

Similarly, considering the reactions $K^{*o} + K^- \rightarrow K^{*o} + K^-$ having contributions from the poles π^- and A_1^- and the process $K_A^o +$ $+ K^- \longrightarrow K_A^o + K^-$ getting contribution from the ρ^- pole, we obtain

$$4g_{K*K\pi}^2 - \left(\frac{m_{K_A}^2}{m_\rho^2} \right) k_{K_A \longrightarrow K\rho}^2 \left(g_{K_A K\rho}^{(L)^2} - g_{K_A K\rho}^{(T)^2} \right)$$

$$+ \left[\frac{(m_{A_1}^2 + m_{K*}^2 - m_K^2)^2}{4m_{A_1}^2} - m_{K*}^2 \right] \left(g_{A_1 K*K}^{(L)^2} - g_{A_1 K*K}^{(T)^2} \right) \qquad (7)$$

From eqs. (6) and (7) we can eliminate the unknown coupling A_1 K* K to obtain

$$4g_{\rho KK}^2 + 4\left(\frac{m_{A_1}}{m_{K*}} \right)^2 g_{K*K\pi}^2$$

$$= \left(\frac{m_{A_1}^2 \; m_{K_A}^2}{m_\rho^2 \; m_{K*}^2} - 1 \right) k_{K_A \to \rho\pi}^2 \left(g_{K_A K\rho}^{(L)^2} - g_{K_A K\rho}^{(T)^2} \right) \qquad (8)$$

We use the SU(3) relation $g_{\rho KK} = \frac{1}{2} g_{\rho\pi\pi}$ and determine $g_{K*K\pi}$ from the experimental width for the decay $K^* \to K\pi$. The calculated [7] partial width for the decay $K_A \longrightarrow \rho\pi$ turns out to be ~ 33 MeV. Taken along with the $K_A \longrightarrow K^*\pi$ width calculated (~ 67 MeV) in (I) the contribution to the total width from our sum rules, due to these two principal modes of decay of K_A is found to be ~ 100 MeV, to be compared with the experimental value of 80 ± 20 MeV for the total width. Moreover, the branching ratio is predicted to be

$$\frac{\Gamma_{K_A \longrightarrow K^*\pi}}{\Gamma_{K_A \longrightarrow K\rho}} \simeq 2$$

In conclusion we note that in spite of the simplicity of the sum rules, the assumption of saturation with important poles and the neglect of unknown transverse coupling constants, the predictions of the sum rules proposed in (I) and in the present paper seem to be fairly well verified by experiments.

* * *

ACKNOWLEDGEMENTS

The authors are grateful to Professors Abdus Salam and P. Budini and the IAEA for the hospitality extended to them at the International Centre for Theoretical Physics, Trieste.

REFERENCES:

1. D. S. Narayan, R. P. Saxena and P. P. Srivastava, ICTP, Trieste, preprint IC/67/47. Hereafter referred to as (I).

2. V. de Alfaro, S. Fubini, G. Rossetti and G. Furlan - Phys. Letters 21, 576 (1966). We follow the notation of this paper.

3. We can write the scattering amplitude in the s-channel as

$$T^{V(A)}_{I_s} = \sum_{I_t} R_{I_s I_t} \; T^{V(A)}_{I_t}$$

where R is the t-to s-channel crossing matrix. The trajectory corresponding to $I_t = 1$ is the ρ trajectory with $\alpha_\rho \sim 0.55$, to $I_t = 0$ we have the Pomeranchuk trajectory (α_P) and to $I_t = 2$ the trajectory with $\alpha_{I_t=2} < 0$, an assumption made in several recent papers with interesting consequences We assume, at high energies, the equalities of the residue functions $\beta_{VV\rho} = \beta_{AA\rho}$ and $\beta_{VVP} = \beta_{AAP}$. On this assumption, whatever the isospin in the s-channel may be (see eq. (1) below), $\left[C^{(V)}_{1,2} - C^{(A)}_{1,2}\right]$ is convergent, $\left[B^{(V)} - B^{(A)}\right]$ is superconvergent and $\left[A^{(V)} - A^{(A)}\right]$ is doubly superconvergent. The strong assumption associated with Pomeranchuk trajectory implies, probably, $\sigma^{VV}_{total} = \sigma^{AA}_{total}$. The present experiments, however, are insufficient to rule out this possibility.

4. Unitary symmetry predicts the octet mass $m_{\omega_8} \sim 927$ MeV and the $\omega - \varphi$ mixing theory gives a mass $m_{\omega_1} \sim 870$ MeV for the singlet. See for example M. Gell-Mann, "The Eightfold Way", Benjamin, N.Y., 1964.

5. With the coupling constants used, the decay widths are, for example,

$$\Gamma_{B \to \omega\pi} = \frac{1}{24\pi} \left(g^{(L)^2}_{B\omega\pi} + 2\, g^{(T)^2}_{B\omega\pi} \right) \frac{k^5_{C.M.}}{m^2_\omega}$$

$$\Gamma_{D \to \pi_V \pi} = \frac{1}{8\pi}\, g^2_{D\pi_V\pi}\, \frac{k^3_{C.M.}}{m^2_D}$$

6. A. H. Rosenfeld et al., Rev. Mod. Phys. 39, 1 (1967).

7. We neglect the transverse coupling, e.g., $g^{(T)}_{K_A K_\rho} = 0$ as was also done for $K_A \to K^*\pi$ in ref. (1).