ELECTRON-HYDROGEN ATOM COLLISION IN THE PRESENCE OF A CIRCULARLY POLARIZED LASER FIELD

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ABSTRACT

Electron-hydrogen collision in the presence of a circularly polarized laser field is studied within a formalism based in an appropriate space-translations transformation and the Green's function formalism. The Born-Oppenheimer approximation for the scattering amplitude is obtained and the dependence of the differential and total excitation cross section on the eletromagnetic field polarization is studied.

INTRODUCTION

The excitation of an atom by electron impact in the presence of a laser field is an important mechanism for the production of electrons in a gas at relatively high pressures[1].

This problem has been treated within several different approximations; usually the dipole approximation is assumed for the laser field[2-9].

Recently we suggested a theoretical treatment for the excitation of hydrogen by electrons in the presence of an intense linearly polarized eletromagnetic field (EMF) represented by the vector potential $\vec{A}(t) = A_0 \hat{z} \cos \omega t$ ($\hat{z}$ is the direction of the incomming electron)[2]. This approach is based in a space-translation transformation in order to transform the hamiltonian to an adequate interaction picture. Using the Green's function formalism and a first order perturbation theory to calculate the bound state of the atomic system, and expression for the scattering amplitude was obtained. Calculations for the total excitation cross section were done for 1s-2s excitation of hydrogen in a rubi laser field using a modified Born-Oppenheimer approximation for the scattering amplitude[2].

Experiments[10] and theoretical calculations[11-15] on two- and three-photon ionization of atoms have shown that circularly polarized light gives a larger cross section than linearly polarized lights.

The purpose of the present work is to obtain, using the same basic formalism of refs. (2) and (3), an expression for the scattering amplitude for elastic and inelastic excitation of

hydrogen by electrons in the presence of a circularly polarized laser field, and to compare the dependence of the differential and total excitation cross section on the EMF polarization.

THEORY

The Schrödinger equation for the system e-H in the presence of an EMF represented by the vector potential $\vec{A}$ is ($\hbar = c = 1$):

$$HF(\vec{r}_1,\vec{r}_2,t) = \left\{ \frac{1}{2m}\left[\vec{p}_1 + e\,\vec{A}(\vec{r}_1,t)\right]^2 + \frac{1}{2m}\left[\vec{p}_2 + e\,\vec{A}(\vec{r}_2,t)\right]^2 + V(\vec{r}_2) + U(\vec{r}_1,\vec{r}_2)\right\} F(\vec{r}_1,\vec{r}_2,t) = 1\,\frac{\partial F(\vec{r}_1,\vec{r}_2,t)}{\partial t} \quad (1)$$

where the subscript 1 refers to the incomming electron and 2 to the bound electron.

$$U(\vec{r}_1,\vec{r}_2) = V(\vec{r}_1) + W(\vec{r}_1,\vec{r}_2) \quad (2)$$

with $$V(\vec{r}) = -e^2/r \quad (3)$$

and $$W(\vec{r}_1,\vec{r}_2) = e^2/r_{12} \quad (4)$$

$\vec{p}_j = -i\,\vec{\nabla}_j$ is the usual linear momentum operator of the eletron j.

We assume the dipole approximation for the EMF and the vector potential $\vec{A}$ for a circularly polarized field can be written as:

$$\vec{A}(\vec{r},t) = \vec{A}(t) = A_o\,(\hat{x}\ \text{sen}\ \omega t - \hat{y}\ \cos\omega t) \quad (5)$$

where $\hat{x}$ and $\hat{y}$ are the unitary vectors on the x and y directions.

We introduce the unitary space-translation transformation, simetric with respect to the electrons coordinates[2.16]:

$$T = \exp\left\{ i\, \vec{\alpha}(t)(\vec{p}_1 + \vec{p}_2) + 2\, i\, \eta(t)\right\} \tag{6}$$

with

$$\vec{\alpha}(t) = -\frac{e}{m}\int^{t} \vec{A}(t')dt' = -\frac{e\,\vec{E}(t)}{m\,\omega^2} \tag{7}$$

and

$$\eta(t) = \frac{e^2}{2m}\int^{t} A^2(t')dt'$$

$\vec{E}(t)$ is the electric field of the EMF:

$$\vec{E}(t) = -A_o\omega\left[\hat{x}\cos\omega\, t + \hat{y}\,\text{sen}\,\omega\, t\right] =$$

$$= E_o\left[\hat{x}\cos\omega\, t + \hat{y}\,\text{sen}\,\omega\, t\right]$$

Under this transformation the time-dependent Schrödinger equation, eq. (2.1), takes the form:

$$\tilde{H}\ \psi(\vec{r}_1,\vec{r}_2,t) =$$

$$= \left\{ \tilde{H}_o + \frac{p_1^2}{2m} + U(\vec{r}_1+\vec{\alpha}(t),\vec{r}_2 + \vec{\alpha}(t))\right\}\psi(\vec{r}_1,\vec{r}_2,t) =$$

$$= i\,\frac{\partial\psi(\vec{r}_1,\vec{r}_2,t)}{\partial t} \tag{8}$$

$$\text{with}\quad \psi(\vec{r}_1,\vec{r}_2,t) = T\,F(\vec{r}_1,\vec{r}_2,t) \tag{9}$$

$$\text{and}\quad \tilde{H}_o\ \phi(\vec{r}_2,t) = \left\{\frac{p_2^2}{2m} + V(\vec{r}_2+\vec{\alpha}(t))\right\}\phi(\vec{r}_2,t) =$$

$$= i\,\frac{\partial\,\phi(\vec{r}_2,t)}{\partial t} \tag{10}$$

is the transformed Schrödinger equation for the hydrogen atom in the presence of the laser field. Treating $\vec{\alpha}(t)$ as a per -

turbation parameter of the Coulomb potential we obtain, in first order of $\vec{\alpha}(t)$, the bound wave functions for the electron 2[3]:

$$\phi^{(1)}(\vec{r}_2,t) = \phi^{(0)}(\vec{r}_2,t)\, e^{-i\rho_n(\cos\omega t + \operatorname{sen}\omega t)} \tag{11}$$

where $\phi^{(0)}(\vec{r}_2,t)$ is the unperturbated wave function of the hydrogen atom and ρ_n is determined using time dependent perturbation theory[3].

Following refs. (2) and (3) the expression for the scattering amplitude is given by:

$$f^{\vec{k}_0\vec{k}(\nu)}_{n\,n'\nu} = \frac{m\,\omega\,i}{(2\pi)^2}\int_0^{2\pi/\omega} dt' \langle \psi_{k(\nu)n'}(\vec{r}_1',\vec{r}_2',t')\,|U(\vec{r}_1'+\vec{\alpha}(t'),\vec{r}_2'+\vec{\alpha}(t'))$$

$$|\psi_{k_0 n}(\vec{r}_1',\vec{r}_2',t')\rangle \tag{12}$$

with

$$\psi_{\vec{k}(\nu)n'}(\vec{r}_1,\vec{r}_2,t) = e^{i\vec{k}(\nu)\cdot\vec{r}_1}\, e^{-i\frac{k^2(\nu)t}{2m}}\, \phi^{(1)}(\vec{r}_2,t) \tag{13}$$

and

$$\frac{k^2(\nu)}{2m} = \frac{k_0^2}{2m} + \varepsilon_n - \varepsilon_{n'} - \nu\omega \tag{14}$$

and $\vec{k}_0$ is the momentum of the incident electron.

$\nu<0$ ($\nu>0$) represents absorption (emission) of $|\nu|$ photons by the e-H system during the collision process.

$\vec{k}(\nu) = k(\nu)\,\hat{r}_1$ is the momentum of the scattered electron.

$f^{\vec{k}_0\vec{k}(\nu)}_{n\,n'\nu}$ is the scattering amplitude for a process where the scattered electron has its momentum $\vec{k}_0$ modified to $\vec{k}(\nu)$. The

atom suffers a transition from the state n to n', with simultaneous absorption ($\nu<0$) or emission ($\nu>0$) of $|\nu|$ photons by the whole system.

$\psi^{+}_{k_0 n}(\vec{r}_1,\vec{r}_2,t)$ is the solution of eq.(8) that satisfies the appropriate asymptotic condition[2,3].

Eq. (12) has the same analytical form as the expression obtained in ref. (2) for the scattering amplitude in the case of linearly polarized field and under the same approximations for the EMF and for the bound electron.

The Born-Oppenheimer approximation for the scattering amplitude is obtained when in eq. (12) we substitute the exact total wave function $\psi^{+}_{k_0 n}$ by the approximate function $\overline{\psi}^{+}_{k_0 n}$ defined as:

$$\overline{\psi}^{+}_{k_0 n}(\vec{r}_1,\vec{r}_2,t) = \psi^{+}_{k_0 n}(\vec{r}_1,\vec{r}_2,t) \pm \psi^{+}_{k_0 n}(\vec{r}_2,\vec{r}_1,t) \qquad (16)$$

where sign + (-) is for singlete (triplete) states of the system e-H.

This leads to an expression for the scattering amplitude with one term corresponding to the direct process and other corresponding to the exchange process. If we neglect the contribution of the one particle potential ($V(\vec{r}+\vec{\alpha}(t))$ in the exchange part of the scattering amplitude[2] we obtain a modified Born-Oppenheimer approximation, similar to that obtained by Ochkur[17].

Within this approximation the scattering amplitude is given by:

$$f^{\vec{k}_0\vec{k}(\nu)}_{n0\,\nu}\Big)^{c} = 2\,m\,e^2 i\cdot e^{i\nu\gamma}\,\frac{J_\nu\left(\sqrt{(\vec{q}(\nu).\hat{x})^2+(\vec{q}(\nu).\hat{y})^2}\,\alpha_0\right)}{q^2(\nu)}\,\delta_{n0} +$$

$$(\mathcal{F}^{\nu}_{n0} \pm \mathcal{G}^{\nu}_{n0})\; J_\nu\;(\sqrt{2}\;\rho_n) \qquad (17)$$

where inittialy was assumed the ground state (1S) $\vec{q}(\nu)$ is the transfered momentum defined by:

$$\vec{q}(\nu) = \vec{k}(\nu) - \vec{k}_0$$

$J_\nu(x)$ is the Bessel function of ν-th order and real argument. γ is defined by:

$$\cos 2\gamma = \frac{(\vec{q}(\nu).\hat{x})^2 - (\vec{q}(\nu).\hat{y})^2}{(\vec{q}(\nu).\hat{x})^2 + (\vec{q}(\nu).\hat{y})^2}$$

$\mathcal{F}^{\nu}_{n0}$ and $\mathcal{G}^{\nu}_{n0}$ are atomic form factors defined by:

$$\mathcal{F}^{\nu}_{n0} = \frac{m\,i}{2\,\pi}\int d^3r_1 d^3r_2\; e^{-i\vec{q}(\nu).\vec{r}_2}\;\phi^*_n(\vec{r}_1)\phi_0(\vec{r}_1)W(\vec{r}_1,\vec{r}_2) \qquad (18a)$$

$$\mathcal{G}^{\nu}_{n0} = \frac{m\,i}{2\pi}\int d^3r_1 d^3r_2\; e^{i(\vec{k}_o.\vec{r}_1-\vec{k}(\nu).\vec{r}_2)}\;\phi^*_n(\vec{r}_1)\phi_0(\vec{r}_2)W(\vec{r}_1,\vec{r}_2) \qquad (18b)$$

and

$$\alpha_0 = \frac{e\,A_0}{m\,\omega} \qquad (19)$$

The expression for the scattering amplitude in the case of linearly polarized field, obtained within the same approximations is[2]:

$$f^{L}_{n0\nu}\left(\vec{k}_o \vec{k}(\nu)\right) = 2\, m\, e^2 i\, e^{i\nu\pi}\, \frac{J_\nu(\vec{\alpha}_o \cdot \vec{q}(\nu))}{q^2(\nu)}\, \delta_{n0} +$$

$$(\mathcal{F}^{\nu}_{n0} \pm \mathcal{G}^{\nu}_{n0})\; J_\nu(\rho_n) \qquad (20)$$

(The first term of eq. (20) is not presented in ref. (2) since in that work it was considered only atomic excitation processes).

From eqs. (17) and (20) it is clear that for atomic excitation processes it is fundamental to take into account the distortions of the bound electron wave functions. For free-free transitions, where atomic excitation does not occur during the collision, the scattered electron absorbs $|\nu|$ ($|\nu| \neq 0$) photons. In this case the first term of eq. (17) (and (20)) is the one which describes the process.

The differential cross section for unpolarized target and unpolarized beam is given by:

$$\frac{d\sigma}{d\Omega} = \frac{k_o}{k(\nu)} \left\{ \frac{1}{4} \left| f_{sing} \right|^2 + \frac{3}{4} \left| f_{trip} \right|^2 \right\} \qquad (21)$$

where $f_{sing}(f_{trip})$ is the scattering amplitude for singlet (triplet) excitation obtained from eq. (17) or eq. (20) using signal + (-).

The total cross section σ is given by:

$$\sigma = \int \left(\frac{d\sigma}{d\Omega}\right) d\Omega$$

3. RESULTS AND DISCUSSIONS

The ratio between the differential cross section for circularly and linearly polarized field is:

(a) Free-free transitions, with absorption (or emission) of $|\nu|$ ($|\nu| \neq 0$) photons:

$$R_{FF}^{\nu} = \frac{(d\sigma/d\Omega)\text{circular}}{(d\sigma/d\Omega)\text{linear}} = \frac{J_\nu^2\left(\sqrt{(\vec{q}(\nu).\hat{x})^2+(\vec{q}(\nu).\hat{y})^2}\;\alpha_o\right)}{J_\nu^2\,(\vec{q}(\nu).\hat{z}\;\alpha_o)} \qquad (22)$$

For the linearly polarized case the field was chosen in the $\hat{z}$ direction. R_{FF}^{ν} depends on the direction of polarization and on $\vec{q}(\nu)$. Rotating the axis of polarization of the laser field change R_{FF}^{ν}.

(b) Excitation of the n hydrogen state from the ground state.

$$R_{EP}^{\nu} = \frac{(d\sigma/d\Omega)\text{circular}}{(d\sigma/d\Omega)\text{linear}} = \frac{J_\nu^2(\sqrt{2}\;\rho_n)}{J_\nu^2(\rho_n)} \qquad (23)$$

For 1s-2s excitation of hydrogen $\rho_2 = e^3E_o/2m\;\omega^3$.

Figure 1 shows R_{EP}^{ν} for 1s-2s excitation of hydrogen as a function of ρ_2. For rubi laser ($E_o \sim 5\times10^6$ V/cm and $\omega = 1.78$ eV) $\rho_2 = 1.72$ and $\alpha_o \simeq 10^{-3}$ a.u. In the limit of low electric field or high frequencies, $\rho_2 << 1$ and $R_{EP}^{\nu} \sim 2^{|\nu|}$. For laser fields of low frequencies its possible to obtain high values of ρ_2 and small values of α_o, within the validity of the perturbation theory used to obtain the bound state wave functions.

The ratio between the total excitation cross section

for circularly and linearly polarized field is given also by eq. (23) since ρ_n does not depends on the direction of $\vec{q}(\nu)$ or $\vec{\alpha}(t)$. However for free-free transitions this ratio has not a simple analytical form because the argument of the Bessel's functions depends on the direction of $\vec{q}(\nu)$ and $\vec{\alpha}(t)$.

4. CONCLUSIONS

The present results show that the excitation of the hydrogen ground state to its first excited states due to electronic collisions in the presence of an EMF with absorption (or emission) of $|\nu|$ photons is enhanced by circularly polarized light. Unfortunaly there are no experimental results concerning atomic excitation nevertheless experiments concerning atomic ionization indicates that linear polarization is less effective for these processes than circularly polarized light[11-15]. Calculations of hydrogen ionization in the presence of ultra-strong laser fields are in progress.

References:

1. Raizer, Yu. P., 1966 Soviet. Phys. USP, 8, 650-673.
2. Brandi, H.S., Koiller, B. and Lins de Barros, H.G.P., to be published in the Phys. Rev. A.
3. Brandi, H.S., Koiller, B., Lins de Barros, H.G.P., Miranda, L.C. and Castro, J.J., 1978, Phys. Rev. A17, 1900-1906.
4. Mohan, M. and Chand, P., 1978, Z. Phys. A 285, 115-120.
5. Mohan, M. and Chand, P., 1977, Phys. Lett. 63A, 245-247.
6. Mohan, M. and Chand, P., 1977, Phys. Lett. 63A, 257-258.
7. Choudhury, B.J., 1975, J. Phys. B: Atom. Molec. Phys. 8, 1420-1427.
8. Kovarskii, V.A. and Perel'man, N.F., 1971, Soviet. Phys. JETP 33, 274-276.
9. Gersten, J.I. and Mittleman, M.H. 1976, Phys. Rev. A13, 123-130.
10. Fox, R.A., Kogan, R.M. and Robinson, E.J. 1971, Phys.Rev. Lett. 26, 1416-1417.
11. Lambropoulos, P., 1972, Phys. Rev. Lett. 29, 453-455.
12. Lambropoulos, P., 1972, Phys. Rev. Lett. 28, 585-587.
13. Klarfeld, S. and Marquet, A., 1972, Phys. Rev. Lett. 29, 79-81.
14. Reiss, H.R., 1972, Phys. Rev. Lett. 29, 1129-1130.
15. Hermandez, J.P. and Gold, A., 1967, Phys. Rev. 156, 26-33.
16. Hernneberger, W.C. 1968, Phys. Rev. Lett. 21, 838-841.
17. Ochkur, V.I., 1966, Soviet. Phys. JETP 18, 503-508.

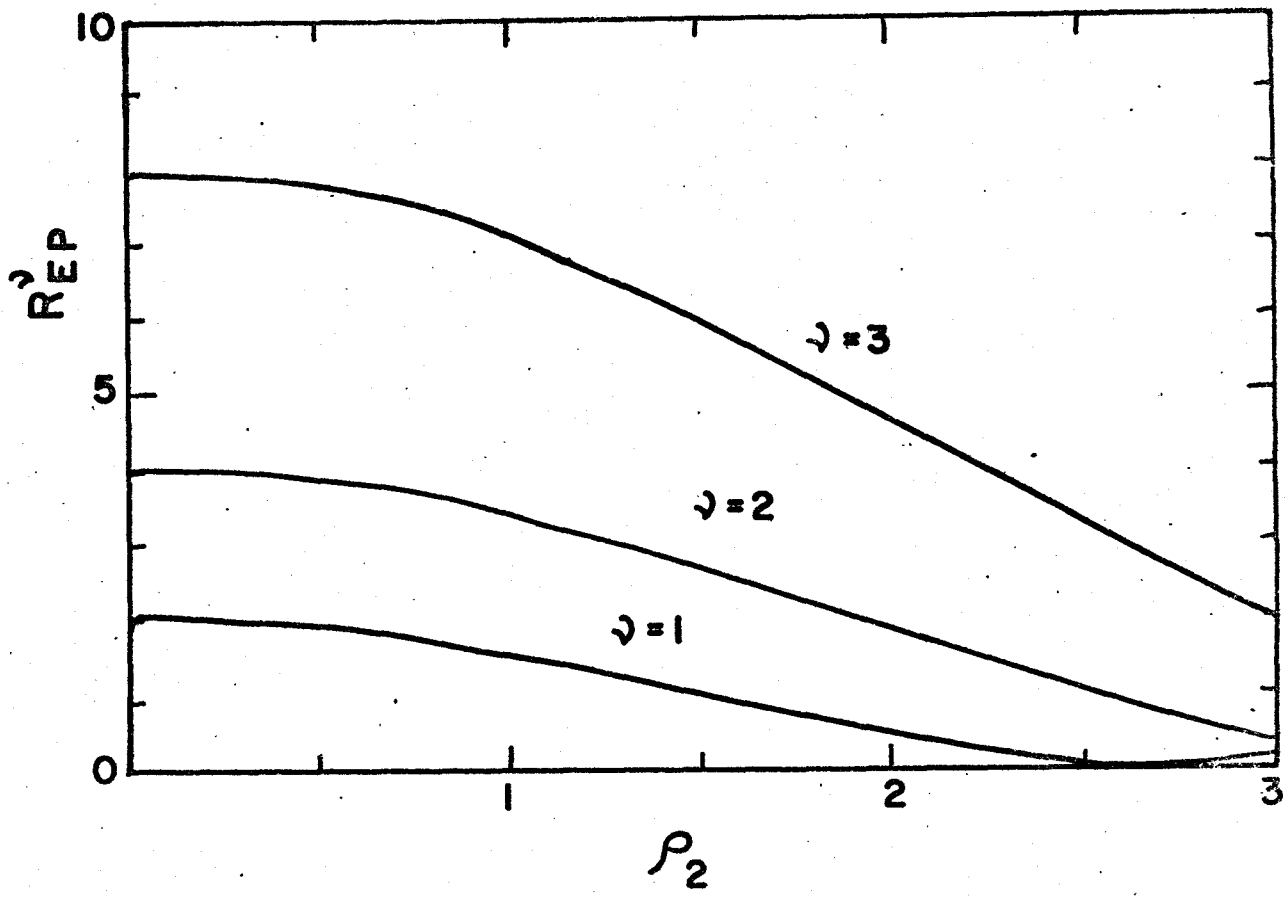

FIGURE 1

Figure Captions

Figure 1: Ratio between the differential excitation cross section for circularly and linearly polarized field, R^{ν}_{EP}, as a function of ρ_2 for some values of ν.