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SUMMARY

The synthesis of heavy and neutron-rich elements (with the mass number A $\gtrsim$ 70) is reconsidered in the framework of a dynamical supernova model. The synthesis equations for the rapid neutron-capture (or, the r-)process and the hydrodynamical equations for the supernova explosion are solved simultaneously. Improved systematics of nuclear parameters (nuclear masses, β^--decay half-lives etc) are used, and the energy release due to β-decays as well as the energy loss due to neutrinos (and their feedback on the hydrodynamics) is taken into account. It is shown that the observed solar-system abundance curve can be well reproduced by assuming only one supernova event on a time-scale of the order of 1 sec.

Key words: supernova —— nucleosynthesis

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1. Introduction

One of the very puzzling questions in nuclear astrophysics is to find a reasonable model to explain the full abundance curve of nuclei beyond iron-peaks. Three processes are believed to be responsible for the formation of these elements: the p-process (or, proton-capture and photo-neutron processes), and the s-and r-processes (slow and rapid neutron-capture processes). While the principal mechanisms of these processes are fairly well understood (see, e.g. Burbidge et al. 1957), the actual astrophysical sites as well as details of the nuclear systematics involved are still in question.

In this paper we deal with the r-process.

For matter to undergo rapid neutron-captures, neutron number densities in excess of 10^{20} cm^{-3} and temperatures greater than 10^9 K are needed, and thus a supernova event seems to be the most promising astrophysical candidate. Previous works on this subject suffered from two main difficulties: Neither the nuclear data for very neutron-rich nuclei (e.g. nuclear masses, β^--decay half-lives, fission data), nor detailed supernova models were available. But great progresses have been made in both fields during the last few years, and therefore it is worthwhile to reinvestigate the r-process.

Here, we used the nuclear systematics described in detail by Kodama and Takahashi (1975), hereafter referred as Ref. I) but with an improved nuclear mass-formula by v. Groote et al. (1976, hereafter Ref. II). In section 2, we give a brief description of the synthesis equations and the nuclear input data used.

The supernova model is discussed in section 3. The initial conditions for the hydrodynamics were obtained from fits to the recent results of Wilson (1975) and Arnett (1975). The initial element distribution for the matter undergoing the r-process is taken from a statistical-equilibrium calculation (El Eid, 1975). We then simultaneously solved the hydrodynamical equations and the synthesis equations by including energy feedback due to nuclear reactions (e.g. β-decay heating) and energy loss due to neutrinos as well as shock heating.

The results of our calculations are given and discussed in section 4. The most striking result is that the observed solar-system r-process abundance curve can be reproduced by assuming only one supernova event. In addition, the calculated abundance curves are fairly insensitive to astrophysical input parameters (neutron to proton ratio, expansion velocity, peak temperature) if varied within reasonable limits. Thus we can conclude that supernova explosions of massive stars ($M \gtrsim 10\ M_{\odot}$) will lead to almost the same relative abundance curve for the r-process elements. Only the amount of heaviest nuclei with $A \gtrsim 200$ produced in our model is sensitive to the input parameters and might serve as a test for supernova models. The minor discrepancies (compared to observed abundance curve) still present in the results of this paper may be explained by uncertainties in the nuclear data used.

2. Synthesis Equations and Nuclear Input Data

The basic assumption of our nuclear network is that the matter under consideration will always achieve a local (n,γ)-equilibrium described by

$$\mathrm{Log}\frac{n(Z,A+1)}{n(Z,A)} = \mathrm{Log}\ \frac{\omega(Z,A+1)}{\omega(Z,A)} + \mathrm{Log}\ n_n - \frac{3}{2}\ \mathrm{Log}\ \frac{A}{A+1}\ T_9 + \frac{5.04}{T_9}\ S_n(Z,A+1) - 34.075 \quad , \qquad (1)$$

where $n(Z,A)$ is the number density of nucleus with charge Z and mass-number A, n_n the neutron number-density per cm^3, S_n the neutron separation energy in MeV, T_9 the temperature in 10^9 K, and ω is the nuclear partition function.

In order to evaluate nuclear masses (here to get S_n), in this paper we use the droplet-model mass-formula with an empirical shell-correction term (Ref. II but with a different set of parameters which gives the best fit to the experimental masses with an rms of 0.58 MeV but spoils the fits to the nuclear-radius and quadrupole-moment data). For ω , we use an expression found in Ref. I with slight changes for the magic-number and deformation effects to be consistent with the here used mass-formula.

We assume Eq. (1) to be valid until the total number of free neutrons has become roughly equal to the total number of nuclei ("freezing condition"). It has been argued by Schramm (1973) that the r-process should be a dynamical one and that it should be treated in the framework of a (n,γ)-(γ,n)-network rather than by using Eq. (1), mainly to get a smoothing of the even-odd effects in the final abundance curve. However, we found that up to our freezing condition the (n,γ)-equilibrium is a fairly good approximation and after the freezing the contribution of free neutrons to the total baryon number-density is so small that we can safely neglect effects of additional neutron-captures. On the other hand, β-delayed neutron emissions after freezing do an even better job in smoothing the even--odd effects.

The β^--decays under (n,γ)-equilibrium are considered by the time-dependent equations for the isotopic abundances $n_Z := \sum_A n(Z,A)$:

$$\frac{d}{dt} n_Z(t) = -\lambda_Z n_Z(t) + \lambda_{Z-1} n_{Z-1}^{(t)} + \text{(feedback due to fissions)}, \quad (2)$$

where λ_Z is the effective isotopic β^--decay constant defined by

$$\lambda_Z := \sum_A \lambda_\beta(Z,A) P_Z(A), \quad \lambda_\beta := l_n\ 2/T_\beta, \quad P_Z(A) := \frac{n(Z,A)}{\sum_A n(Z,A)} \quad (3)$$

The β^--decay half-lives T_β are calculated in the same way as in Ref. I by consulting the gross theory calculation (Takahashi et al. 1973) with the use of the Q-values from the here used mass-formula.

The last term in Eq. (2) reflects the feedback of fissioning heavy nuclei to the r-process synthesis, for which we use the formula given in Ref. I. However, we emphasize that due to the rather short hydronynamical time-scale this term gives a negligible contribution to the final abundances: the r-process terminates by the lack of free neutrons rather than by neutron-induced (or, spontaneous) fissions as formerly thought.

After the freezing, we have included β-delayed neutron emission, α-decays and fission processes to obtain the final mass-abundance curve $n_A = \sum_Z n(Z,A)$, using the method of Ref. I, which is corrected for the new mass-formula used.

3. Supernova Model

a) Initial Conditions

Although there has been a great progress during the last years in constructing theoretical supernova models, no generally accepted model is available at the moment. We therefore had to parametrize our ignorance and to choose initial conditions relevant to solving the hydrodynamical equations, which should not contradict our present day's knowledge. Generally speaking, a "reasonable" model should obey the following restrictions. It should smoothly fit to advanced stages of stellar evolution. The peak density in the ejected material should be sufficiently high enough to give rather high degree of neutronization (the total neutron to proton ratio $N/P = 4 - 8$) in order to allow for the synthesis of very heavy elements (up to $A \simeq 240$). The velocity field of the ejected matter should be able to explain the observed supernova light-curves and to give a total luminosity of order of $10^{51} - 10^{52}$ ergs.

Following these ideas, we started from Arnett's

(1975) 22 $M_\odot$ presupernova star (with 8 $M_\odot$ initially ^4He-core), which after silicon flash has a central Ni-core of roughly 1.4 $M_\odot$. We then scaled the density of the interior parts of this star ($M(r) \leq 1.5\ M_\odot$) corresponding to a free fall solution, in the way that at the boundary of the forming neutron-star of $R = 10^6$ cm and $M(R) = 1\ M_\odot$ we have a density $\rho = 3\times10^{11}$ g cm^{-3} (neutron-drip density in cold catalyzed matter). The resulting density-distribution is shown in Fig. 1 in comparison with the presupernova distribution of Arnett (1975). The temperature distribution was then obtained by assuming an adiabatic law with an effective adiabatic index of 1.2 - 1.33, the temperature at the boundary of the neutron star being a free parameter. In the next step, we superimposed on this density-temperature profile of the star a velocity field which was similar to Wilson's calculation (1975), e.g. to his model with the cross-section for the elastic neutrino-nucleus scattering being $1.7\times10^{-44}\bar{A}$ (where $\bar{A}$ is the average atomic weight of the nuclei). Although from the theoretical point of view this model might not be a very realistic one, it has the advantage of leading to the formation of a neutron star of $\sim 1\ M_\odot$ as well as to an explosion of the rest of the star, and deposits of enough energy to account for the observed supernova-luminosities. Our velocity profile in comparison to Wilson's results for 0,03 sec after the start of the explosion is given in Fig. 2. Within all the uncertainties involved in the models the agreement is fairly good.

Finally the initial composition has to be defined.

For the matter being outside the Ni-core, we adopted the composition given by Arnett (1975). For the matter of the central 1.4 $M_\odot$ -core we calculated the composition at the onset of the explosion by using a nuclear-statistical-equilibrium calculation by El Eid (1975).

To check the sensitivity of our model to the choice of the initial conditions, the maximum velocity of the expanding matter was also used as a free parameter.

b) Hydrodynamics

The dynamical evolution of the system under consideration is given by the following set of equations (in Lagrangian coordinates),

$$\frac{\partial v}{\partial t} = - \frac{1}{\rho}\frac{\partial p}{\partial r} - G\frac{M(r)}{r^2} \quad \text{(momentum conservation)}, \qquad (4)$$

$$\frac{\partial M}{\partial r} = 4\pi r^2 \rho \quad \text{(mass conservation)}, \qquad (5)$$

$$\frac{\partial \varepsilon}{\partial t} - \left(\frac{p+\varepsilon}{\rho}\right)\frac{\partial \rho}{\partial t} + \sum_i \frac{\partial \varepsilon}{\partial n_i}\frac{\partial n_i}{\partial t} = - q_\nu + q_{nr} \qquad (6)$$

(internal-energy conservation),

$$p = p(\rho,T,n_i),\ \varepsilon = \varepsilon(\rho,T,n_i) \quad \text{(equation of state)}, \qquad (7)$$

where v is the velocity, p the pressure, n_i the number density of each species i, ε the internal energy density, ρ the mass-density, M(r) the mass inside a shell of radius r, G the gravitational constant, q_ν the energy loss due to neutrinos and q_{nr} the energy release of nuclear reactions (which might be positive or negative), the latter two being per unit time per volume. Since the time-scale of evolution is very short ($\lesssim$ 1 sec) we have neglected radiative transfer. The equation of state (7) is given by a sum of contributions from the radiation field, the electron gas, the neutrons and the nuclei.

Equations (4) to (6) have to be simultaneously solved with a set of nucleosynthesis equations. In order to save some computation time we used several reasonable simplifications.

For temperatures $T_9 \gtrsim 5$ we calculated the composition of matter and the equation of state from nuclear statistical equilibrium at fixed neutron-proton ratio. Then Eq. (6) read (the index eq indicating equilibrium quantities)

$$\frac{\partial \varepsilon_{eq}}{\partial t} - \left(\frac{p_{eq}+\varepsilon_{eq}}{\rho}\right) \frac{\partial \rho}{\partial t} = - q_\nu \quad , \qquad (6')$$

where q_ν is given predominantly by pair-annihilation neutrinos. The assumption of fixed neutron-proton ratio was very well justified since on a time-scale less than 10^{-2} sec the temperature in all expanding regions of the star T_9 dropped below 5.

At temperatures $T_9 \lesssim 5$ and for sufficiently high number of free neutrons ($n_n \gtrsim n_{nucl}$) we solved Eqs. (4) to (6) simultaneously with Eqs. (1) and (2), the equation of state being calculated from the actual nuclear-abundances. We approximated g_{nr} by the energy release by β^--decays. Equation (6) then became

$$\frac{\partial \varepsilon}{\partial t} - \left(\frac{p+\varepsilon}{\rho}\right) \frac{\partial \rho}{\partial t} + \sum_i \frac{\partial \varepsilon}{\partial n_i} \frac{\partial n_i}{\partial t} = - q_\nu + q_\beta \quad . \qquad (6'')$$

In the equation of state, the radiation field was treated as a black-body radiation and the contribution of the electrons was handled by an analytic fit to the Fermi integrals.

Finally, for the matter after the freezing of the r-process and at the initial densities $\rho \lesssim 10^7$ g cm^{-3}, we assumed fixed nuclear-abundances and adiabatic expansion leading to

$$\frac{\partial \varepsilon}{\partial t} - \left(\frac{p+\varepsilon}{\rho}\right) \frac{\partial \rho}{\partial t} = 0 \quad , \qquad (6''')$$

with the equation of state being calculated as before.

The hydrodynamical equations as well as the synthesis equations were simultaneously solved by finite differencing. For the hydrodynamics we used a code similar to that of Colgate and White (1966) and Arnett (1966). It was assumed that up to $r = 10^6$ cm we have a static neutron-star of mass $M = 1\ M_\odot$ which enters into our calculations just as an interior boundary condition and as an external gravitational field. The part of the star reaching out to $r \simeq 3 \times 10^9$ cm was divided into 20 mass-shells of roughly equal mass giving totally another solar mass. The outer boundary of this region was almost in hydrostatic equilibrium. On the time-scale of the r-process (~1 sec), the density, temperature and position of the outermost shell changed only by a few percent. Therefore we did not include more shells into our calculations. The shock heating was taken care of by adding an artificial viscosity term σ to p in Eqs. (4) and (6)

$$\sigma = \begin{cases} 2\rho(\Delta v)^2 & \text{if } \rho(r,t+\Delta t) > \rho(r,t) \text{ and} \\ & \quad v(r+\Delta r,t+\Delta t) < v(r,t+\Delta t) \\ 0 & \text{otherwise.} \end{cases} \qquad (8)$$

The r-process was assumed to occur only in the inner 8 mass-shells containing 0.43 $M_{\odot}$ of matter (the mass of the initial Ni-core minus the mass of the neutron star), because only this part of the star can reach during the implosion a high enough degree of neutronization and disintegration to allow for enough free neutrons to start with the r-process.

4. Results and Discussions

The supernova model outlined in the preceding section left us with some parameters which however are not really arbitrary. One parameter was the initial temperature at the surface of the neutron star. Assuming adiabatic compression of the presupernova star we would expect this temperature T_{max} to be of order 10^{10} K. In most of the calculations, we used 1.2×10^{10} K in agreement with Wilson's model (1975). To check the sensitivity of our results with respect to this parameter, we have varied it from 8×10^{9} K to 4×10^{10} K but the changes in the abundance curves were not significant.

The second parameter is the maximum velocity in the outgoing shock-wave, V_{max}. We varied V_{max} from 4×10^{9} cm sec^{-1} to 6×10^{9} cm sec^{-1} (giving an excess of kinetic energy over potential energy of order 10^{51} to 10^{52} ergs). But again the resulting abundance curves were rather insensitive. On the average the abundances were only changed by less than a factor of 3 but the main characteristics remained the same.

The third parameter is the onset temperature for the r-process, T_{onset}, i.e. the temperature at which the composition of matter starts to depart from nuclear statistical equilibrium. Calculations using a nuclear network have shown (Thielemann and Hillebrandt, 1976) that this critical temperature for the density region of intrest in the present calculation is roughly 5 x 10^9 K. Therefore we used this value in most of our calculations. As it will be shown later the most significant difference between our calculated abundance curve and the observed one is a slight shift of the abundance peaks around A = 130 and A = 195 to smaller mass-numbers. We had the suspicion that this effect could be eliminated by choosing a higher onset-temperature to shift the (n,γ) equilibrium to lower mass-number in every isotopic chain (thus bringing the r-process path closer to β-stability line). But again due to self-adjustments in the solution of the nonlinear set of differential equations, this effect was not sufficient enough to get the abundance peaks in the right positions.

The last set of parameters in the present model is the ratio of neutrons to protons (N/P) in each shell which is assumed to undergo the r-process. In a consistent supernova model, this parameter should be calculated from the degree of neutronization during the implosion phase. But since we had no detailed information about this process we assumed N/P to be highest in the innermost shell and to fit smoothly to the value ~1 in the shells containing mostly Si and lighter elements. As expected our results were rather sensitive to the choice of the maximum (N/P), especially the amount of nuclei with A $\geq$ 200 produced by the r-process was strongly dependent on this ratio. The best fits to the observed abundance curve was obtained by assuming $(N/P)_{max} \simeq 7$ and we had to assume at least $(N/P)_{max} \simeq 5$ to get any nuclei with $A \gtrsim 200$.

To summarize the preceding discussion, we give a set of parameters which led up to now to the best abundance curve:

$$
\begin{aligned}
T_{max} &= 1.2 \times 10^{10} \quad , \\
V_{max} &= 5 \times 10^{9} \text{ cm sec}^{-1} \quad , \\
T_{onset} &= 5 \times 10^{9} \text{ K} \quad , \\
(N/P)_{max} &= 7 \quad ,
\end{aligned}
\tag{9}
$$

which are not in contradiction to our knowledge of supernova explosions.

The results obtained by using the set of parameters (9) are given in Figs. 3 to 6: Figures 3 and 4 show the time evolution of the velocity and the density of the exploding star. After the freezing of the r-process ($t \simeq 0.74$ sec), the outer parts ($r > 3 \times 10^{9}$cm) are still almost in hydrostatic equilibrium and therefore it is well justified that we did not add more shells. Figure 5 gives the path of some shells of matter in a $(T - \rho)$ diagram. It may be seen that up to the onset of the r-process the path of the matter is almost an adiabat of index 4/3. After the r-process starts, the path becomes non-adiabatic due to changes of nuclear abundances (e.g. by neutron captures) and due to β^{-}-decay heating, but in the mean it is again nearly adiabatic with an effective adiabatic index ~1.2. The sudden increase of density and temperature in the outer shells reflects the effect of the shock-front (see also Figs. 3 and 4). For comparison we have also included the path used by Schramm (1973). It may be seen that his assumed path is in rather good agreement with our hydrodynamical calculations for $\rho \gtrsim 10^6$ g cm^{-3}. But to the lower density the temperature is considerably higher in our model. As a consequence the time-scale for the freezing (~ 0.7 sec) is more than a factor 2 longer than in his calculations. We want to emphasize that this is the main reason why we could produce also very-heavy nuclei ($A \gtrsim 220$) in reasonable amounts, in spite of the fact that the β^--decay half-lives used in our work are much longer than those of Schramm.

The final abundance curve obtained from our model is shown in Fig. 6 in comparison with the observed solar-system r-process abundances. The overall agreement is fairly good, especially the relative heights and shapes of the peaks corresponding to the neutron magic-numbers 50,82 and 126 are reproduced. It is interesting to note that in our model the r-process synthesis terminated after ~1 sec by an exhaustion of free neutrons rather than by neutron-induced (or spontaneous) fissions as formerly thought: at the freezing, the maximum charge Z synthesized did not go beyond 100. The consistent inclusion of β-delayed neutron emissions sufficently smoothed out the even-odd effects in the frozen abundance curve and led to the final mass abundance curve comparable with the observed one. However, there are still some differences which we could not eliminate by changing the astrophysical parameters of the model and which therefore will most probably have their origin in uncertainties in the nuclear systematics used in our work. For example, our r-pro cess path turned out to be too far away from the β-stability line to the neutron-rich side, resulting in the abundance peaks shifted to smaller mass-numbers. This seems to be partly due to the fact that the mass formula used tends to over--estimate the neutron-separation energies of very neutron-rich nuclei with neutron magic numbers. More detailed studies of these nuclear effects as well as of the age of the Galaxy and of the superheavy-element production rates are under way and will be given elsewhere.

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FIGURE CAPTIONS

Fig. 1 - Density profile of the model star at the onset of the explosion. ρ is the density, m(r) the mass inside a shell of radius r in 10^{33}g. The dashed line corresponds to Arnett's (1975) 22 $M_{\odot}$ pre-supernova star.

Fig. 2 - Velocity distribution in the model star after the onset of the explosion. V is the velocity, r is the distance from the center of the star. The dashed curve is taken from Wilson (1975). The upper solid curve corresponds to $V_{max} = 5x10^9$ cm sec^{-1} at 0.033 sec after the explosion starts, the lower one to $V_{max} = 4x10^9$ cm sec^{-1} at t = 0.078 sec.

Fig. 3 - Velocity profile in the exploding shells ad different times. The time t is given in seconds. $V_{max}(t=0) = 5 \times 10^9$ cm sec^{-1}. Other notations as in Fig. 2.

Fig. 4 - Density profile in the exploding shells ad different times. Notations as in Fig. 1 and Fig. 3.

Fig. 5 - The path of r-process material in a temperature-density diagram. T is the temperature in K, ρ the mass density in g cm^{-3}. The path is given for three different mass-shells (shell number 1,3 and 5 in our model). Circles mark the onset of the

r-process, crosses the termination point. In shell 1 the freezing is at $T = 5.5 \times 10^9$ and $\rho = 2.3 \times 10^3$ g cm^{-3}. For comparison the adiabatic path of Schramm (1973) is given as a dashed line.

Fig. 6 - r-process mass-abundance. Dotts are the observed solar-system r-process abundances (Allen et al.1971). The solid curve is the theoretical result for the set of astrophysical input parameters (Eq. (9)), shortly after the termination. Alpha decays from nuclei beyond A = 210 lead to the accumulated abundances of $^{206-208}$Pb and ^{209}Bi, which together with those of ^{232}Th, ^{235}U and ^{238}U are calculated at a time of 5×10^9 years after the event and shown by crosses. For comparison, the present day's observed data of the latter three isotopes (Cameron, 1973) are added.

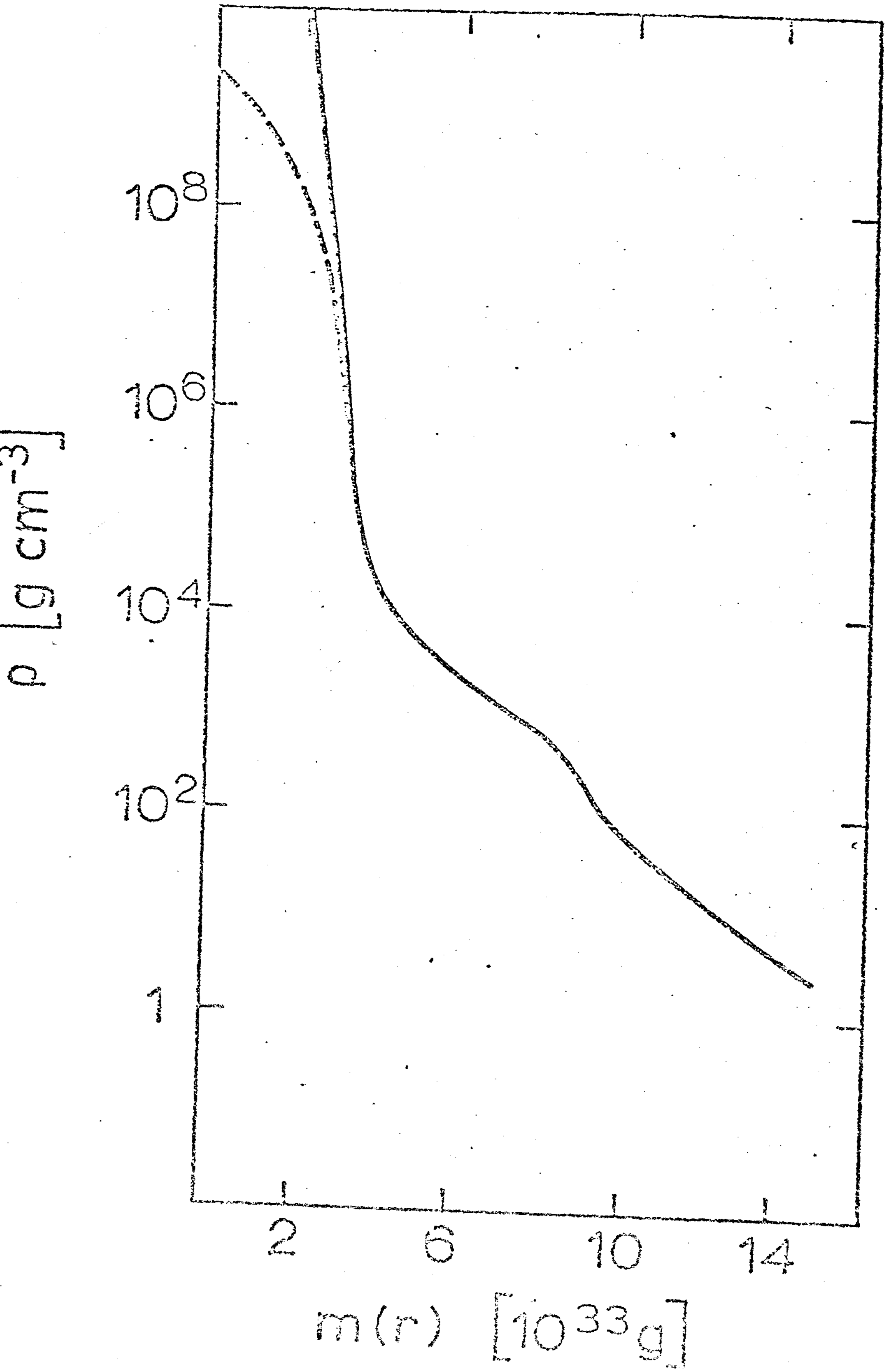
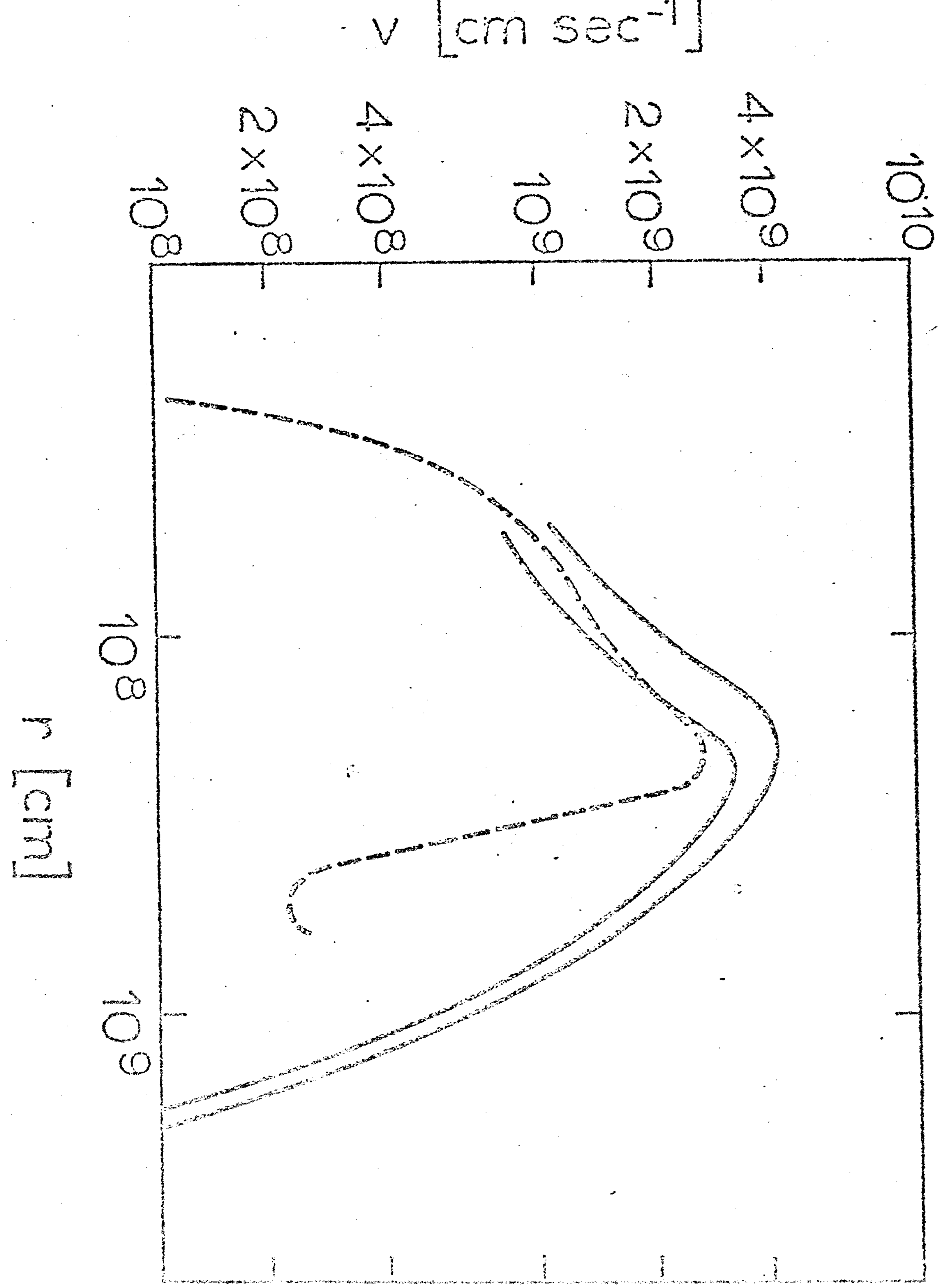
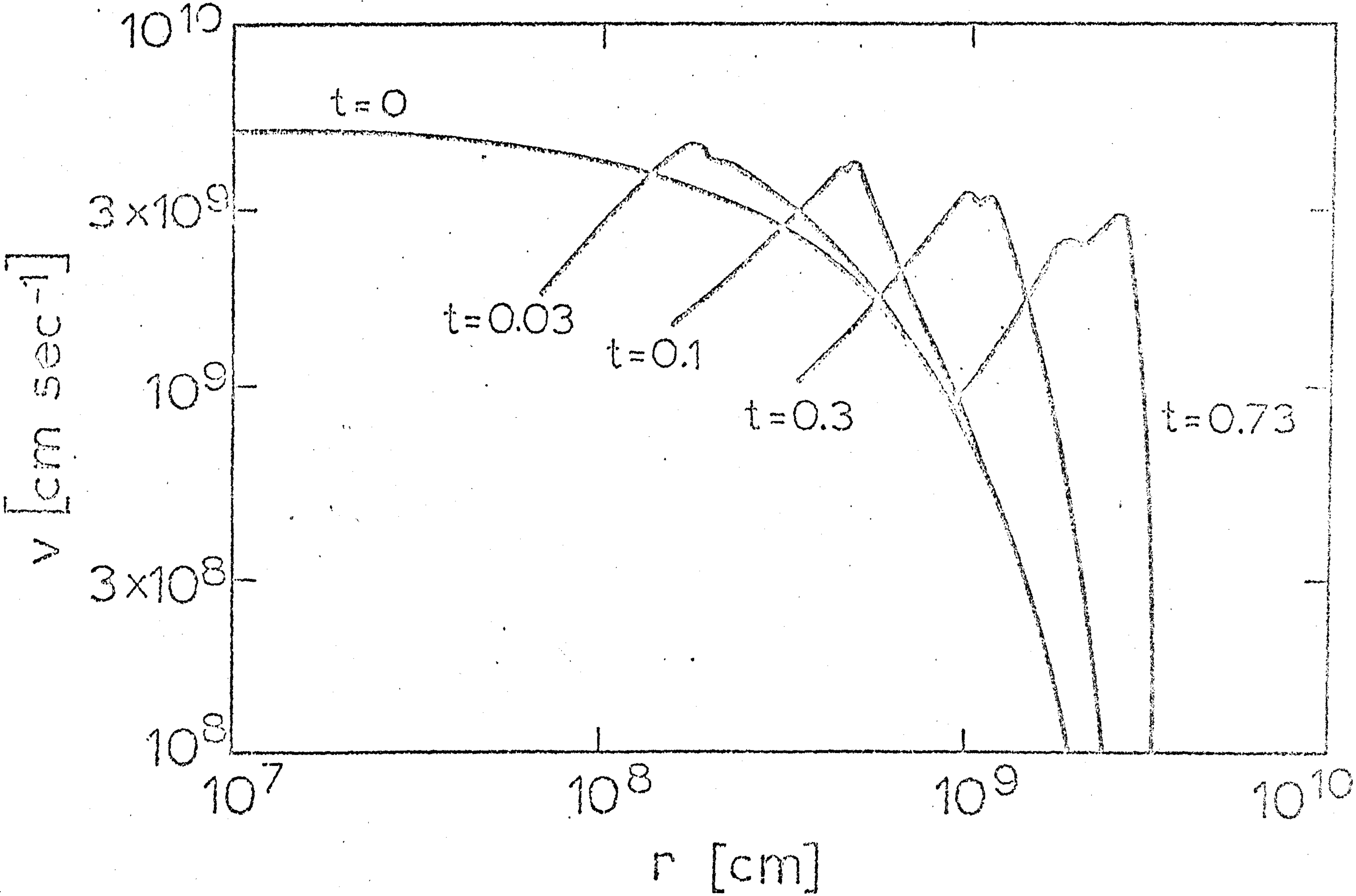

ρ [g cm^{-3}]
10^8
10^6
10^4
10^2
1
2
6
10
14
m(r) [10^{33}g]

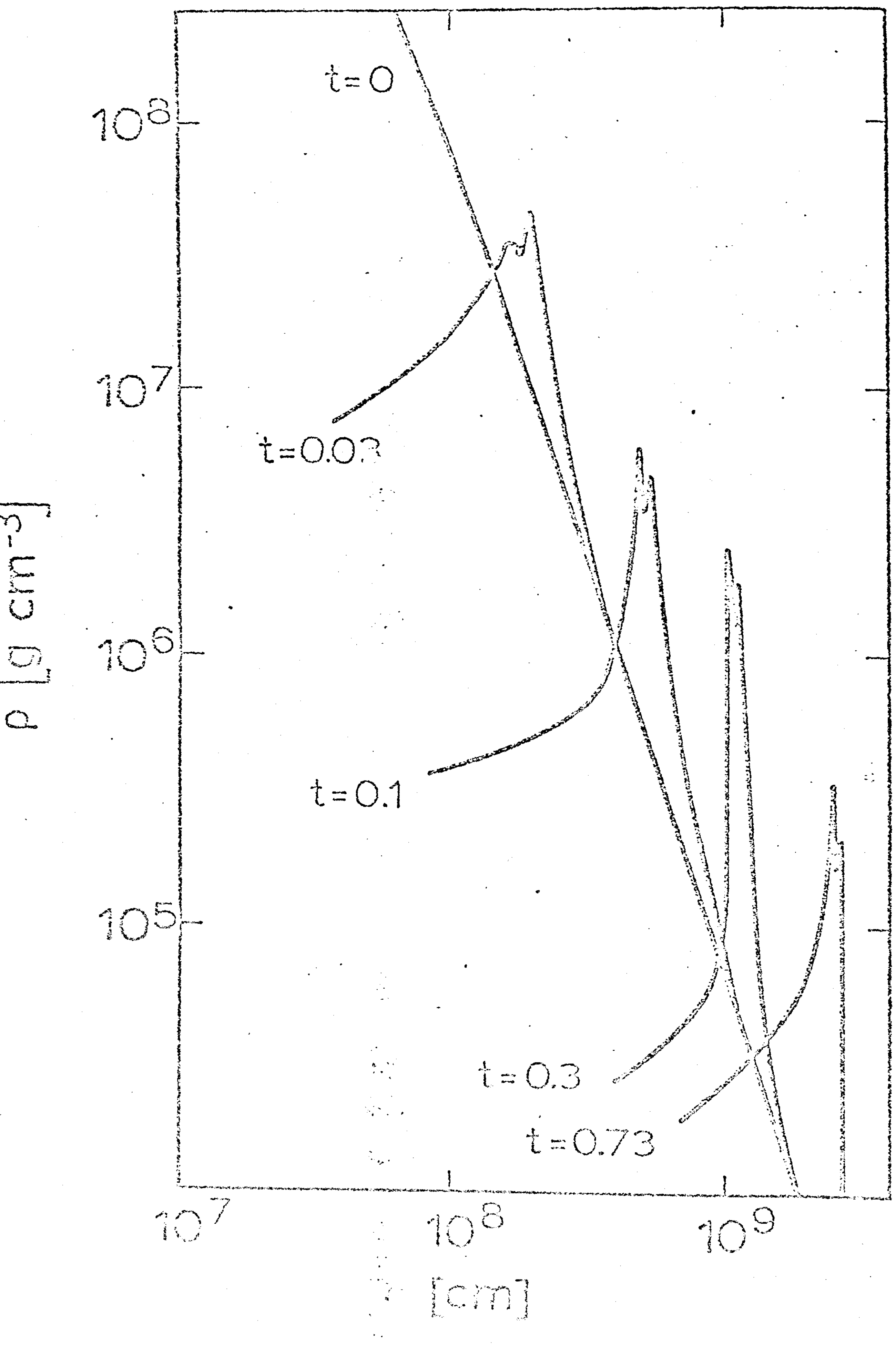
Fig. 2

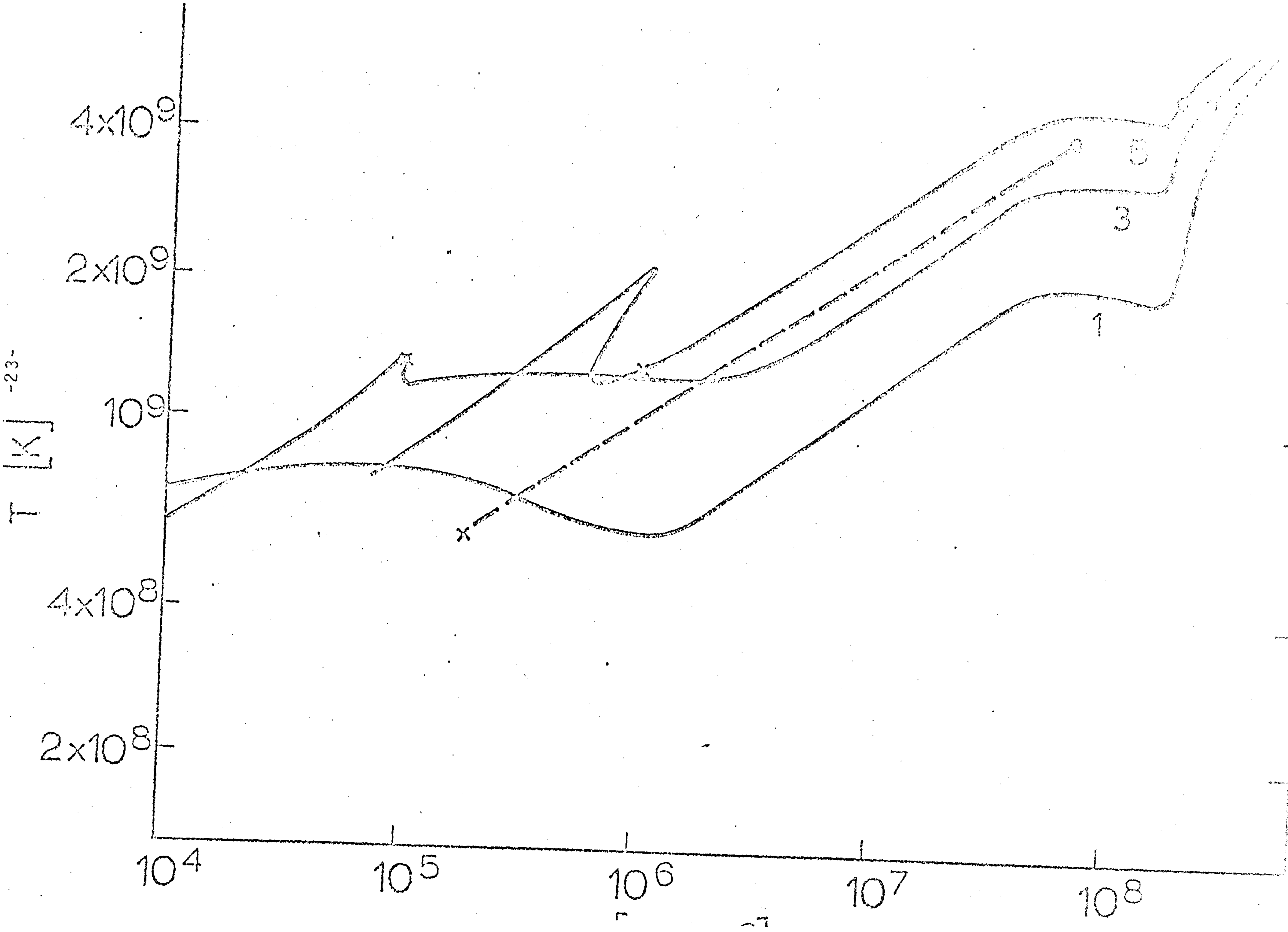

v [cm sec^{-1}]
10^{10}
4×10^9
2×10^9
10^9
4×10^8
2×10^8
10^8
10^8
10^9
r [cm]

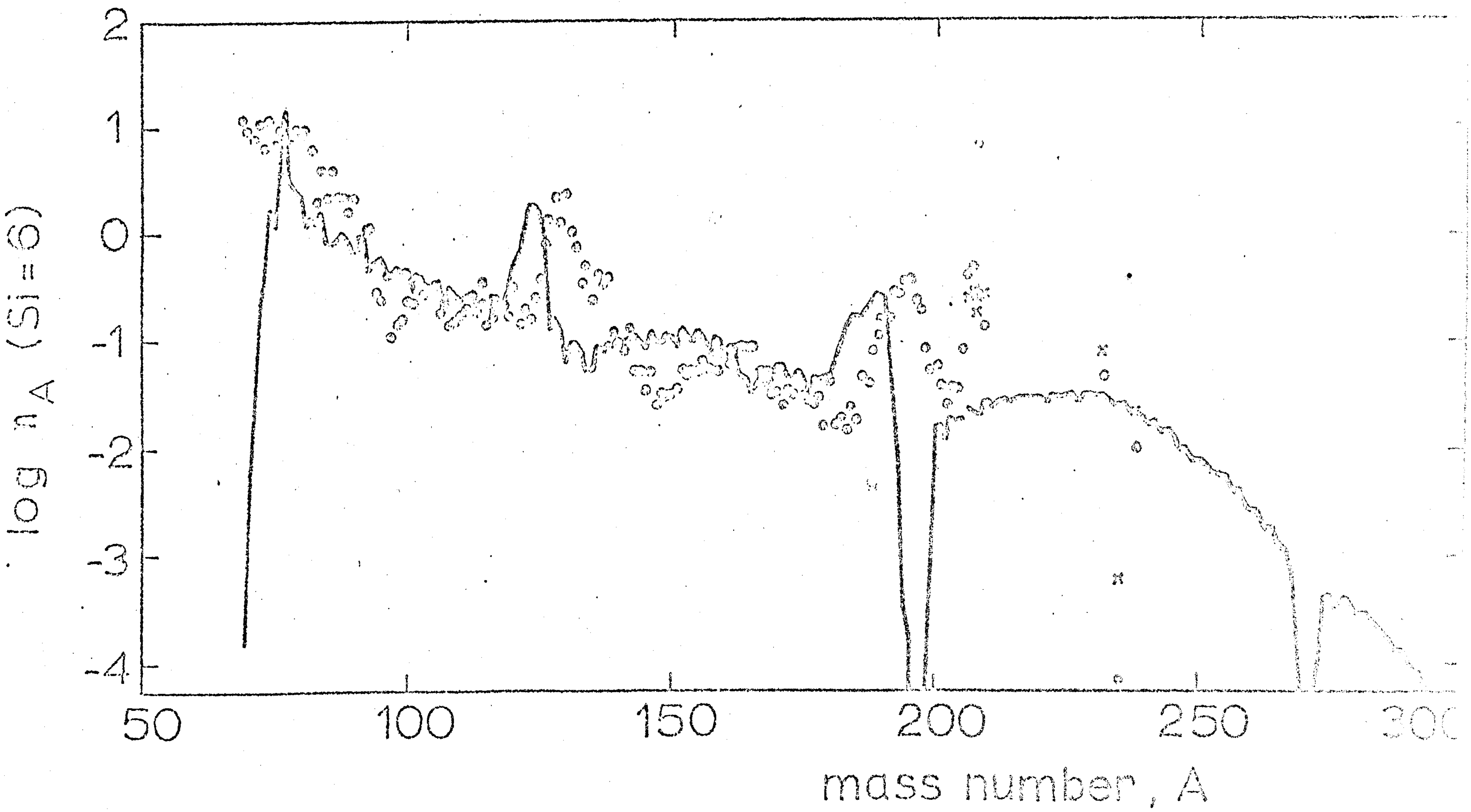


10^{10}
3×10^{9}
10^{9}
3×10^{8}
10^{8}
v [cm sec^{-1}]
t = 0
t = 0.03
t = 0.1
t = 0.3
t = 0.73
10^{7}
10^{8}
10^{9}
10^{10}
r [cm]

Fig. 8


ρ [g cm^{-3}]
10^8
10^7
10^6
10^5
t=0
t=0.03
t=0.1
t=0.3
t=0.73
10^7
10^8
10^9
[cm]


T [K]
4x10^9
2x10^9
10^9
4x10^8
2x10^8
10^4
10^5
10^6
10^7
10^8
1
3
5



log n_A (Si=6)
2
1
0
-1
-2
-3
-4
50
100
150
200
250
300
mass number, A