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ABSTRACT

It is shown that the only possible static charged dust distribution with cylindrical symmetry admitting a translation along the axis of symmetry is obtained when the density of charge is numerically equal to that of matter.

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A functional relationship between g_{00} and ϕ, the elestrostatic field was first introduced by Weyl [1] for the axially symmetric case. In a more recent work De and Raychaudhury [2] found that for a charged dust in equilibrium a functional relationship between g_{00} and ϕ follows directly from the field equations and equality of charge density and matter density is a consequence, if it is assumed that the ratio of the density of charge to the density of matter is constant irrespective of any symmetry condition. We investigated this result for an axially symmetric static charged dust distribution with the condition that space-time admits a translation along the axis of symmetry and observed that the only possible static charged dust distribution for this case is that the charged density numerically equal to the matter density.

The Einstein - Maxwell equations are

$$R^i_j - \frac{1}{2}\delta^i{}_j R = -8\pi T^i{}_j \tag{1}$$

$$F^{ij}{}_{;j} = 4\pi J^i = 4\pi\sigma V^i \tag{2}$$

$$F_{[ij;k]} = 0 \tag{3}$$

with

$$T^i_j = \rho V^i V_j + \frac{1}{4\pi}\left(F^{ik} F_{kj} + \frac{1}{4}\delta^i_j F^{\ell m} F_{\ell m}\right) \tag{4}$$

where ρ and σ are densities of matter and charge respectively and V^i is the 4-velocity vector satisfying condition

$$V^i V_i = 1 \tag{5}$$

The electromagnetic field tensor is given by

$$F_{ij} = A_{i,j} - A_{j,i} \tag{6}$$

Since for cylindrically symmetric electrostatic field only F_{14} exists, we have from the field equation (1) $R^3_3 + R^4_4 = 0$. Therefore the line element may be taken in Weyl's canonical form: [3]

$$ds^2 = -\bar{e}^{2(\nu-\lambda)}(dr^2 + dz^2) - r^2\, \bar{e}^{2\lambda}\, d\phi^2 + \bar{e}^{2\lambda}\, dt^2 \tag{7}$$

where ν and λ are functions of r only.

Now we consider $A_4 = \phi(r)$ as the electrostatic potential, the remaining A_i being zero, and $T^4_4 = \rho$ is the only surviving component of $T^i_{\ j}$; with these assumptions the field equations give

$$\bar{e}^{2(\nu-\lambda)}\left(\frac{\nu_1}{r} - \lambda_1^2\right) = -\,\bar{e}^{2\nu}\mu\,{\phi_1}^2 \tag{8}$$

$$\bar{e}^{2(\nu-\lambda)}\,(\nu_{11} + \lambda_1^2) = \bar{e}^{2\nu}\,{\phi_1}^2 \tag{9}$$

$$\bar{e}^{2(\nu-\lambda)}\left(\nu_{11} + \lambda_1^2 - 2\,\lambda_{11} - 2\,\frac{\lambda_1}{r}\right) = -\,8\pi\rho - \bar{e}^{2\nu}\,{\phi_1}^2 \tag{10}$$

where subscripts 1 and 11 represent first and second derivatives with respect to r.

From equation (8) and (9) we get

$$\nu_{11} + \nu_1/r = 0$$

which on integration gives $\nu = A + B \log r$. To avoid singularity at r=0, we take B=0, so that ν is equal to a constant which may be taken to be zero. Hence from equation (8) we have

$$e^{2\lambda} \lambda_1{}^2 = \phi_1{}^2 \tag{11}$$

Again one obtains by taking the covariant divergence of equation (1) and utilizing equations (2) - (4)

$$v^i{}_{;j}\, v^j = - \frac{\sigma}{\rho} F^i{}_j\, v^j \tag{12}$$

The static condition gives

$$v^i = \frac{\delta^i{}_4}{\sqrt{g_{44}}} \tag{13}$$

Then we have from (9), (13) and (7) with $i = 4$

$$e^{\lambda} \lambda_1 = - \frac{\sigma}{\rho} \phi_1 \tag{14}$$

From equation (11) and (14) one gets

$$\frac{\sigma}{\rho} = \pm 1$$

Thus the condition of cylindrical symmetry alone determines the distribution of charged dust.

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