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ANGULAR DISTRIBUTION AND POLARIZATION OF Λ's IN $\pi - p$ COLLISIONS * +

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ABSTRACT. The angular distribution and polarization of Λ's in π-p collisions is studied assuming that all but the S-wave comes from the exchange of a K'-meson.

The data were analysed at two different energies 960 and 1300 MeV of the incident pion. A satisfactory result is obtained for the higher energy and favours a vector K'-meson. In the region of 960 MeV the model seems inadequate.

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I - INTRODUCTION

A theoretical explanation of the backward peaking of the Λ's produced in $\pi - p$ collisions has been proposed by Tiomno [1] on the assumption that the pion is absorbed by a meson K' (a resonant state of the $K\pi$-system) which is transformed into a K-meson (fig. 1)

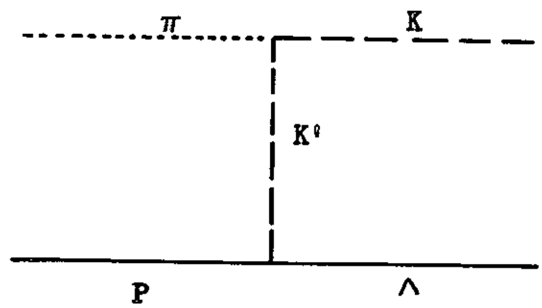

Fig. 1 - Feynman diagram for the exchange of a K'-meson

Such a K'-particle has been experimentally observed by M. Alston et al. [2] in the reaction $K^- + p \longrightarrow K^0 + \pi^- + p$, which shows a $K\pi$ resonance at 885 MeV.

The simple picture of a K'-meson exchange is however clearly incomplete, because:

a) It predicts no polarization for the Λ's in contrast with the large polarization found experimentally;

b) The mass of the K'-meson required to explain the angular distribution of Λ's at incident energies around 960 MeV is too small and imcompatible with the experimental mass.

It thus appears that the exchange of a K'meson is probably a main source of interaction but there are also some other important factors which must be taken into account.

In this paper we have investigated a modification of the original scheme by adding to it an arbitrary complex S-wave. It is indeed reasonable to expect that the S-wave is not entirely given by the graph in Fig. 1. On the other hand, if there are no strongly absorption channels in the other partial waves, then they should be mostly given by the K'-exchange term, except the $p_{\frac{1}{2}}$ wave, which has also a contribution from the term with one nucleon intermediate state. It should be emphazised that this model is primarily intended to verify the influence of the K'-meson exchange term and not as a full description of the process. It is in this spirit that the one nucleon term is not being included.

We have calculated the matrix elements for the process, assuming either scalar or vector K'-meson, at the energies of 960 and 1300 MeV. The S-wave parameters and coupling constants were adjusted to give the best fitting to the angular distributions. It was found that with the same coupling constant one could not fit the data at both energies. The inclusion of the term with one nucleon intermediate state is unlikely to improve substantially this situation.

The average polarization obtained at 960 and 1300 MeV was not in disagreement with the experimental values both for scalar or vector K'-meson, although the experiments seem to indicate a higher polarization (specially at 960 MeV), thus favouring the vector case, as indicated in Table I.

On can conclude that for the higher energies (~ 1300 MeV) and a vector K'-meson this model reproduces fairly well the main features of the process. However, for lower energies ($\sim$ 960 MeV), this model seems inadequate, and we believe that a different mechanism is involved, which modifies the magnitude and phase of at least one p-wave.

II - DYNAMICAL APPROACH

The Feynman amplitude for the reaction $\pi^- + p \longrightarrow \Lambda + K^0$ may be written in the form:

$$F_{sr} = \bar{u}_{\Lambda s}(p_\Lambda)\left[U + V(p_\pi + p_k)_\mu \gamma^\mu\right] u_{pr}(p_p) \qquad (1)$$

where $u_{pr}(p_p)$ and $u_{\Lambda s}(p_\Lambda)$ are Dirac spinors for the proton and Λ-particle, respectively, and U and V are functions of the invariants:

$$E^2 = (p_p + p_\pi)^2 = (p_\Lambda + p_k)^2, \quad K^2 = -(p_p - p_\Lambda)^2 = -(p_k - p_\pi)^2 \qquad (2)$$

The Feynman amplitude is related to the scattering matrix in the c.m. system by:

$$F_{sr} = 4\pi \frac{E}{(m_\Lambda m_p)^{\frac{1}{2}}} \chi_s^+ (f_1 + \sigma \cdot \epsilon' \, \sigma \cdot \epsilon \, f_2) \chi_r \qquad (3)$$

where the χ's are Pauli spinors and ϵ', ϵ are unit vectors in the directions of the momentum of the Λ and the proton, respectively. The amplitudes f_1 and f_2 have the following partial wave expansions:

$$f_1 = \sum (f_\ell^+ P'_{\ell+1} - f_\ell^- P'_{\ell-1}), \quad f_2 = \sum (f_\ell^- - f_\ell^+) P'_\ell \qquad (4)$$

where $f_\ell^\pm$ are the transition amplitudes in states of total angular momentum $j = \ell \pm \frac{1}{2}$ and orbital angular momentum ℓ.

From (1) and (3) one obtains:

$$4\pi\, f_1 = \frac{1}{2E}\left[(E_p + m_p)\,(E_\Lambda + m_\Lambda)\right]^{\frac{1}{2}}\left[U + (2E - m_p - m_\Lambda)\,V\right] \quad (5)$$

$$4\pi\, f_2 = \frac{1}{2E}\left[(E_p - m_p)\,(E_\Lambda - m_\Lambda)\right]^{\frac{1}{2}}\left[- U + (2E + m_p + m_\Lambda)V\right] \quad (6)$$

The differential cross section and the polarization of the Λ's, normal to the plane of production in the direction of $\vec{p}_\pi \times \vec{p}_k$ are given by:

$$\frac{d\sigma}{d\Omega} = \frac{|\vec{p}_k|}{|\vec{p}_\pi|}\left[\left|f_1\right|^2 + \left|f_2\right|^2 + 2\ \mathrm{Re}\ (f_1^* f_2)\cos\theta\right] \quad (7)$$

$$P = \frac{|\vec{p}_k|}{|\vec{p}_\pi|}\ \frac{2\ \mathrm{Im.}\,(f_1\, f_2^*)\sin\theta}{(d\sigma/d\Omega)} \quad (8)$$

where $\vec{p}_k$ and $\vec{p}_\pi$ are center of mass momenta and θ is the angle between them.

Let us now consider the contribution of the graph shown in Fig. 1 to the Feynman amplitudes. Since the observed resonance of the $K-\pi$ system at 885 MeV is very sharp, with full width of 16 MeV we shall neglect the spread of mass of the K'-particle. Thus, we shall use for K' the propagator of a stable particle, $1/(m_{k'}^2 + K^2)$, as in reference (1). The mesonic and baryonic vertices have the following structure for either a scalar or a vector K'-meson:

	Scalar K'-meson	Vector K'-meson	
Meson vertex	$2m_{k'}\, f_s\, \tau_i$	$f_v\, \tau_i(p_\pi + p_k)_\mu\, \eta^\mu$	(9)
Baryon vertex	$g_s\, \bar{u}_\Lambda\, \bar{u}_p$	$\bar{u}_\Lambda\left[g_{1v}\gamma_\mu + g_{2v}(p_p + p_\Lambda)_\mu\right]\eta^\mu u_p$	(10)

where τ_i is the isospin matrix, and η^μ is the spin function of the vector K'-meson, satisfying the conditions:

$$\eta^\mu p_{k'\mu} = 0, \quad \eta_s^{\mu +} \eta_{r\mu} = -\delta_{sr}, \quad \sum \eta_{s\mu} \eta^{+}_{s\nu} = -g_{\mu\nu} + p_{k'\mu}\, p_{k'\nu}/m_{k'}^2 \qquad (11)$$

The coupling parameters f's and g's are, in general, functions of K^2. In our calculations we take them as constants. We then obtain the following expressions for U and V:

Scalar case:

$$U = 2\sqrt{2}\, m_{k'}\, f_s\, g_s\, \frac{1}{m_{k'}^2 + K^2}, \quad V = 0 \qquad (12)$$

Vector case:

$$U = \sqrt{2}\, f_v \frac{1}{m_{k'}^2 + K^2} \Big\{ g_{1v}\, (m_\Lambda - m_p)\, (m_k^2 - m_\pi^2)/m_{k'}^2 + $$

$$g_{2v} \left[(p_k + p_\pi)\cdot(p_\Lambda + p_p) + (m_k^2 - m_\pi^2)(m_\Lambda^2 - m_p^2)/m_{k'}^2 \right] \Big\} \qquad (13)$$

$$V = \sqrt{2}\, f_v\, g_{1v} \frac{1}{m_{k'}^2 + k^2}$$

Bringing these expressions into (5) and (6) we obtain f_1 and f_2. As explained in the introduction, we shall now add an arbitrary S--wave. That means that we replace f_1 by $f_1 + \rho\, e^{i\alpha}$. Hence the cross section becomes:

$$\frac{d\sigma}{d\Omega} = \frac{|\vec{p}_k|}{|\vec{p}_\pi|} \left[f_1^2 + f_2^2 + 2\, f_1\, f_2 \cos\theta + \rho^2 + 2\rho \cos\alpha (f_1 + f_2 \cos\theta) \right] \qquad (14)$$

The relation between the momentum transfer and the scattering angle is:

$$K^2 = 2\, E_p\, E_\Lambda - m_p^2 - m_\Lambda^2 - 2\, p_p\, p_\Lambda \cos\theta$$

Thus the differential cross sections and polarization as a function of $Z = \cos\theta$ have the general forms:

$$\frac{d\sigma}{dZ} = \frac{|\vec{p}_K|}{|\vec{p}_\pi|} \left[A + \frac{B}{\lambda - Z} + \frac{C}{(\lambda - Z)^2} \right] \tag{16}$$

$$P = \frac{|\vec{p}_K|}{|\vec{p}_\pi|} \; \frac{2\, h_2\, \rho \sin\alpha\, \sin\theta}{(d\sigma/dZ)} \; \frac{1}{\lambda - Z} \tag{17}$$

where:

$$A = \rho^2 - 2\rho \cos\alpha\, h_2$$

$$B = 2\left[\rho \cos\alpha (h_1 + \lambda h_2) - h_1 h_2\right] \tag{18}$$

$$C = h_1^2 + h_2^2 + 2\lambda h_1 h_2$$

and

$$\lambda = (2\, E_p\, E_\Lambda - m_p^2 - m_\Lambda^2 + m_{K'}^2)/2\, p_p\, p_\Lambda \tag{19}$$

In these expressions h_1 and h_2 are functions of the energy only. For scalar K'-meson they are:

$$4\pi\, h_1 = 2\sqrt{2}\, m_{K'}\, f_s\, g_s\, \frac{1}{4E} \left[(E_p - m_p)\,(E_\Lambda - m_\Lambda)\right]^{-\frac{1}{2}}$$

$$4\pi\, h_2 = 2\sqrt{2}\, m_{K'}\, f_s\, g_s\, \frac{1}{4E} \left[(E_p + m_p)\,(E_\Lambda + m_\Lambda)\right]^{-\frac{1}{2}} \tag{20}$$

For vector K'-meson, and taking $g_{2v} = 0$ we have:

$$4\pi\, h_1 = \sqrt{2}\, f_v\, g_v\, \frac{1}{4E} \left[(E_p - m_p)(E_\Lambda - m_\Lambda)\right]^{-\frac{1}{2}} \left[(m_\Lambda - m_p)(m_K^2 - m_\pi^2)/m_{K'}^2 + 2E - m_p - m_\Lambda\right]$$

$$4\pi\ h_2 = \sqrt{2}\ f_V g_V \frac{1}{4E} \left[(E_p + m_p)(E_\Lambda + m_\Lambda)\right]^{-\frac{1}{2}} \left[-(m_\Lambda - m_p)(m_K^2 - m_\pi^2)/m_{K'}^2 + \right.$$
$$\left. + \ 2E + m_p + m_\Lambda\right] \qquad (21)$$

III - GENERAL RESULTS AND CONCLUSIONS

The values of A, B and C at the energies of 960 MeV and 1300 MeV where determined by a least mean square fitting of the experimental data for angular distribution from F. Eisler et al. [3] The results are shown in Figs. II and III. The areas were normalized as to give the total number of events.

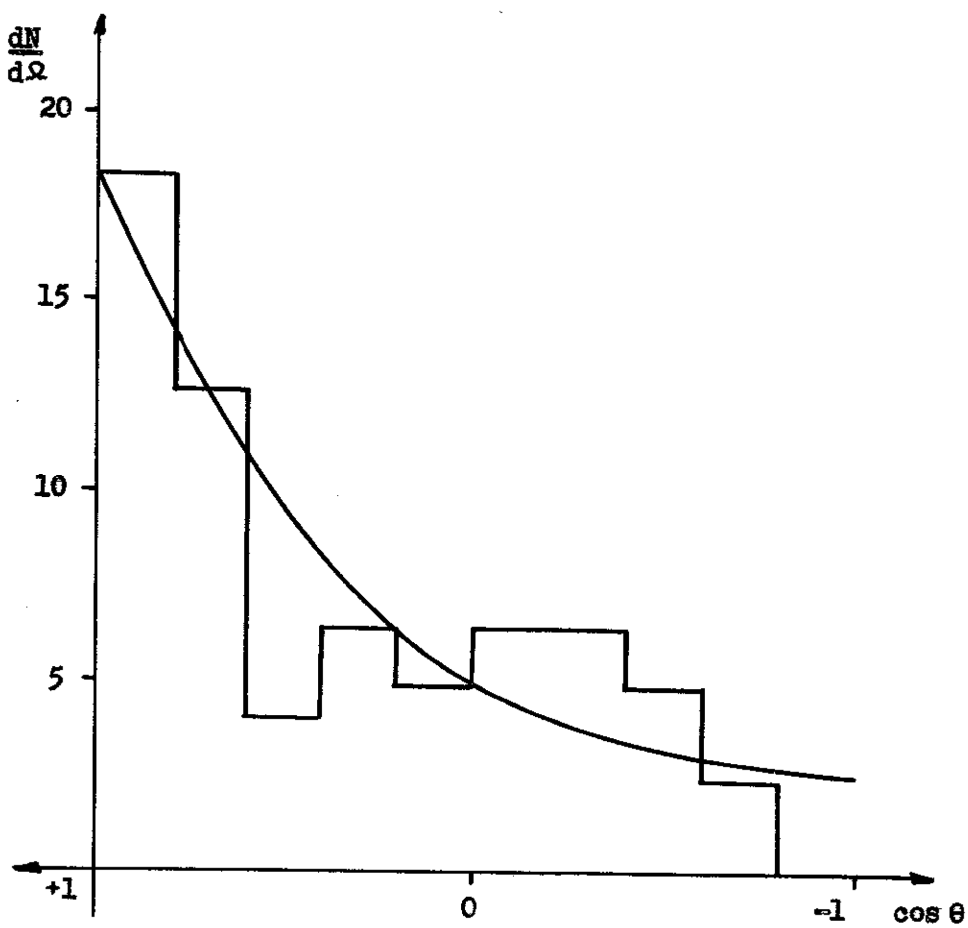

Fig. II - Angular distribution of Λ's for an incidente pion energy of 960 MeV.

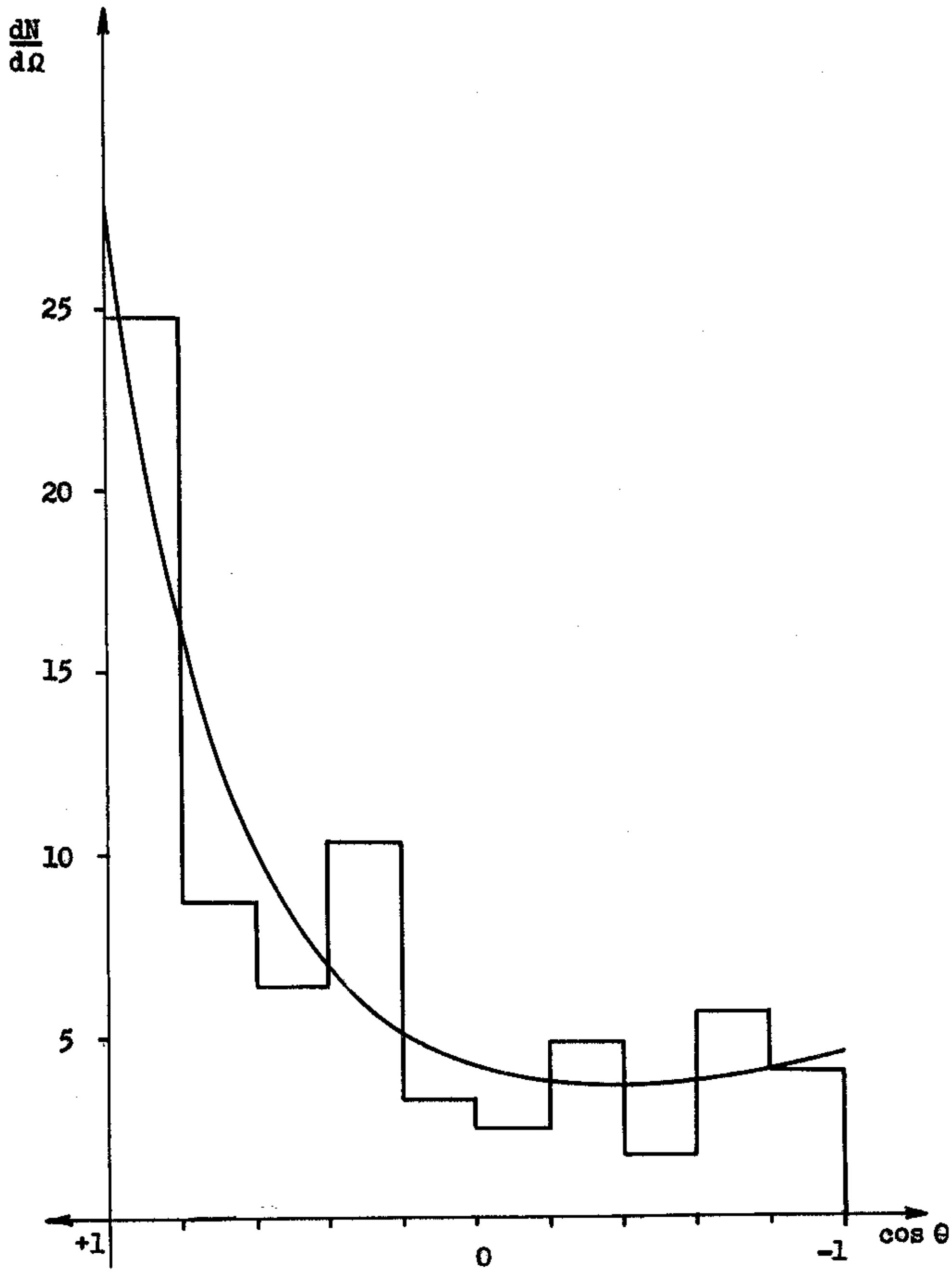

Fig. III - Angular distribution of Λ's for an incident pion energy of 1300 MeV.

The coupling constants and S-wave amplitudes at each energy are determined from the coefficients of the angular distributions by means of (18), (20) and (21). They are given in Table I. One remarks that the best value of $(fg/4\pi)^2$ required to fit the data at the two energies differ by a factor of 5 in the case of a vector K'-meson and a factor of 3.4 in the scalar case. The coupling constant in the K'Kπ-vertex may be directly calculated from the experimental value of the K'-meson lifetime. Then one can deduce the ΛPK'-coupling constant $g^2/4\pi$, also given in Table I. One remarks that for scalar K'-meson $g^2/4\pi$ is rather too large even at 1,300 MeV. It is unlikely that one could fit the data at both energies with a unique coupling constant, even allowing for statistical errors.

We point out that in the vector case one can dispose of another coupling constant, g_{2V}, which was set equal to zero in our calculations; however, even with g_{2V} arbitrary, there remains the same discrepancy in the magnitude of the coupling constants. The source of the discrepancy in the coupling constant is mainly traced to the fact that the total cross-section at 960 is larger than that at 1.300 MeV by a factor of 2.6.

We have calculated the polarization averaged over angles using the parameters listed in Table I. The results are in general lower than the experimental values [4]. However, at 1,300 MeV the polarization predicted in the case of vector K'-meson is 32%, in good agreement with the experimental value.

From these results one can draw the following conclusions: the model us inconsistent with K'-meson being scalar in view of the large value found for $(g^2/4\pi)$. The angular distribution and polarization around 1,300 MeV can be fairly well explained, assuming that the contribution from a vector K'-meson exchange term is dominant in the sense that even p-waves come mostly from that source without distorsion. Our analysis favours a vector K'-meson because it gives a satisfactory polarization, as well as a reasonable value for the coupling constant.

On the other hand, in the region of 960 MeV the model seems inadequate, failing particularly to produce a large polarization. It was already remarked by Phillips [5] that one could not obtain at low energies both a large polarization and strong assymetry in the angular distribution if only the S-wave were complex. The reason is that in order to obtain a large polarization the S-wave must be out of phase with respect to the p-waves. But then there will be little interference between them and, therefore, small assymetry.

We conjecture as a possible mechanism that in the region of 960 MeV there exists a channel strongly affecting one of the p-waves but with little influence in the higher energy region around 1,300 MeV*. Such a mechanism is similar to the one proposed by Kanasawa [6]. However if, as he suggests, there exists a resonance of the $K\Lambda$ system around 960 MeV in the p-wave, then the phase would go over $\pi/2$ and the effect of the resonance would still be felt at energies in the region of 1,300 MeV.

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* For instance, if the parity of the Σ-particle were opposite to that of the Λ and nucleons then the S-wave, ΣK-channel (with threshold at 904 MeV) would modify the $p_{1/2}$-wave in that region of energies.

TABLE I

K'-vector

T^{lab} (Bev)	Ref^o (Bev^{-1})	Im f^o (Bev^{-1})	$(fg/4\pi)^2$	$g^2/4\pi$	< P >
0.960	0.35	0.37	1.08	4.4	0.25
1.300	0.10	0.20	0.19	0.76	0.32

K'-scalar

T^{lab} (BeV)	Ref^o (Bev^{-1})	Im f^o (Bev^{-1})	$(fg/4\pi)^2$	$g^2/4\pi$	< P >
0.960	0.39	0.37	1.85	196	0.11
1.300	0.13	0.22	0.55	58	0.18

T_π^{lab} is the kinetic energy of the pion in the lab. system.

Ref^o is the real part of the total S-wave, that is, the sum of the S-wave coming from the K'-exchange term plus $\rho \cos \alpha$, and Im f^o= $= \rho \sin \alpha$.

IV - ACKNOWLEDGEMENT

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