CBPF-NF-015/82

BIFURCATION IN THE COSMOS

by

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Abstract: We show that the presence of dissipative processes in the Cosmos provokes the indeterminacy of its future.

Relativistic cosmologists reserved the most of their activities in the investigation of the properties of the early cosmos. Few articles are concerned with the final configuration of the Universe. This is simply due to the fact that we observe only the past. No information of the future can be received and, consequently, any comment on later eras may appear either as a trivial extension of the observed present configuration of the Universe or may rise the suspicion that it is highly speculative.

In recent years, however, some scientists dare to be involved in the examination of standard or alternative final configuration of the Cosmos [Dyson,[1] Narlikar[2], Fabri and Melchiorri[3]]. Although different authors may present distinct views of the future, there is a common belief that, assuming the known laws of physics to be valid throughout the whole space-time, one expects to be able to deduce the future from our present Knowledge(+). In other words, the deterministic aspect of the cosmos seems guaranteed.

The purpose of the present note is to question on this and to show how the Universe may possibly be running into an unpredictable era. We are led to this idea not as a consequence of the violation of any of the common principles of classical relativistic cosmology but, as we will see, due basically to the non-linear character of Einstein's equations of motion for the metric of space-time.

(+) It is clearly understood that as far as our Knowledge of the Nature advances, the perspectives for the future will change correspondingly. This, of course, does not affect our argument.

It is almost astonishing that the highly non.linear system of Einstein's equations admits its reduction, in some special cases, to an autonomous planar system of equations. This, of course, is possible only in some restricted situations and certainly it is not applicable in general. However, the important point is that those cases in which such reduction is possible are of great interest and they contain precisely the most typical and traditional cosmological models like, for instance, Kasner, Gödel and Friedmann Universes. Denoting by ρ the density of energy and by θ the expansion factor associated to the galactic fluid, the complete system of Einstein's equations reduces to the set(*).

$$\begin{aligned} \dot{\rho} &= F(\rho,\theta) \\ \dot{\theta} &= L(\rho\ \theta) \end{aligned} \tag{1}$$

in which a dot means the derivative with respect to the global time t. The explicit form of the non-linear functions F and L depends on the model which describes the distribution of the energy throughout the space-time. We set for the stress-energy tensor the form

$$T^{\mu}{}_{\nu} = \rho\, V^{\mu} V_{\nu} - \tilde{p}(\delta^{\mu}{}_{\nu} - V^{\mu} V_{\nu}) \tag{2}$$

in which we assume the constitutive relations

$$\tilde{p} = p - \beta\theta^{2} \tag{3}$$

(*) We limit here our analysis to the case of homogeneous and isotropic Universes. We can without difficulty extend our discussion to anisotropic cases of the classes of Bianchi.

with $p = \lambda\rho (0 \leq \lambda \leq 1)$, β is a constant (which may depend at most on the energy ρ). We treat the source of the global metric as a quadratic Stokesian fluid. It seems worth to point out that we can interpret such $T^{\lambda}{}_{\nu}$ either as representing a viscous galactic fluid or some other cosmic field. Indeed, it has been shown (Novello[4]) that by means of the decomposition of the energy-momentum tensor of a neutrino field in its irreducible parts, with respect to an arbitrary observer, it can equivalently be interpreted as the stress-energy tensor of a quadratic Stokesian fluid of the type we are considering in our definitions (2,3). This is nothing but a consequence of the non-uniqueness of the description of a given distribution of energy by an observer. In this vein we will analyse the future of the Universe assuming a quadratic Stokesian regime(*).

Dissipative phenomena associated to irreversible processes have been in the realm of recent investigation on non equilibrium Thermodynamics. Some authors, like for instance Prigogine[8], have examined the non-linear response of certain chemical reactions, the dynamics of which are described by a set of equations similar to eq.(1), with the purpose to create an alternative interpretation of the evolutionary process.

The main lesson we can learn from these works is that the explicit deterministic feature of a system of equations describing an irreversible process may contain a germe of in-

(*) The contribution of the cosmic neutrino on the curvature of space time has been examined and reviewed by B. Kuchowicz[5]. The discussion of linear Stokesian fluid was made by Belinskii and Khalatnikov[6]. Quadratic functions has been discussed by Novello and Araujo[7].

determinacy. This appears very clearly if the system develops a bifurcation. In this case the instability of a singular point in the phase plane, coupled to the aleatory character of the fluctuations which the system can undergo(and which are always present) extinguish almost completely the possibility of predictions. In other words, the system arrived at the vicinity of a bifurcation point evolve in a non-deterministic way, which is a situation already implicitly contained in the equations used to describe the system. In Prigogine's words[8] : ..."le déterminisme des équations qui permettent de calculer la stabilité et l'instabilité des différents états, et le hasard des fluctuations qui décident vers quel état le système se dirigera effectivement, y sont inséparablement associés."

The exam of the Universe described by equations (1) with the source given by (2)(3) has a bifurcation point at the origin $(\rho,\theta) = (0,0)$, which corresponds to the final configuration of Friedmann-like cosmological open models. The proof of this is based on Bendixson's theorem[9] which states that the Poincaré index of a multiple equilibrium state is given by the formula

$$I_p = \frac{E-H}{2} + 1 \tag{4}$$

in which E and H represents, in the phase plane, the number of elliptical and hyperbolic sectors, respectively. Now, the system (1) depends, for a fixed value of λ, on the parameter β, that is we should write instead of (1):

$$\begin{aligned} \dot{\rho} &= F(\rho,\theta,\beta) \\ \dot{\theta} &= L(\rho,\theta,\beta) \end{aligned} \tag{5}$$

The examination of the value of the Poincaré index I_p for different β's at the multiple equilibrium state $(\rho,\theta) = (0,0)$, shows that $\beta_0 = \frac{1+\lambda}{3}$ is a bifurcation point; that is, a point in which I_p changes discontinuously.

The sudden modification of the topological properties of the integral curves of the system (5) in the plane (ρ,θ) represents an abrupt change of behavior of the physical system in the vicinity of the unstable point. The crucial consequence of this is the appearance of non-deterministic features on the metrical properties of the Cosmos, in the far future. In other words, the dissipative process generated by the quadratic viscous term, destroys the quiescent end of the open Friedmann cosmos and make impossible the prediction of its behavior beyond our present era.

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