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ABSORTION OF GRAVITATIONAL WAVES BY AN EXCITED VACUUM SPACE - TIME

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ABSTRACT: We discuss modifications of Einstein's equations generated by vacuum fluctuations of the gravitational fields. A special case of excitation is shown to be responsible by absortion of gravitational waves.

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It has been known since a long time [1, 2, 3] that vacuum quantum fluctuations of the gravitational field induces some drastic modifications on the classical system of Einstein's equations of Genral Relativity (GR). Until recently, however, traditional relativists were very cautious in considering these alternative forms for the equation of the curvature of space time. The discovery that singularities (in a broad sense) are very common properties of solutions of Einstein's equations and the almost general feeling that a good model of gravity should be singularity free, have changed the situation. Independently of this matter of opinion, the true modifications imposed on Einstein's equations by vacuum quantum fluctuation have to be taken into account in a complete theory of gravity.

In the present paper we will work within the so called quasi-Maxwellian approach of GR. Thus, we will deal mainly with Weyl's conformal tensor $C^{\alpha\beta\mu\nu}$. Actually, we will use the decomposition of Weyl tensor into its electric and magnetic parts introduced by an arbitrary observer which has (co-ordinate frame) velocity V^{α}. We define the electric $E_{\alpha\beta}$ and the magnetic $H_{\alpha\beta}$ parts by expressions:

$$E_{\alpha\beta} = - C_{\alpha\mu\beta\nu} V^{\mu} V^{\nu} \tag{1}$$

$$H_{\alpha\beta} = C_{\alpha*\mu\beta\nu} V^{\mu} V^{\nu} \equiv \frac{1}{2} \eta_{\alpha\mu}{}^{\rho\sigma} C_{\rho\sigma\beta\nu} V^{\mu} V^{\nu} \tag{2}$$

So, we can write

$$C_{\alpha\beta}{}^{\mu\nu} = 2 V_{[\alpha} E_{\beta]}{}^{[\mu} V^{\nu]} + \delta^{[\mu}_{[\alpha} E^{\nu]}_{\beta]} -$$

$$- \eta_{\alpha\beta\lambda\sigma} V^{\lambda} H^{\sigma[\mu} V^{\nu]} - \eta^{\mu\nu\rho\sigma} V_{\rho} H_{\sigma[\alpha} V_{\beta]} \tag{3}$$

where $\eta^{\alpha\beta\mu\nu} = - \frac{1}{\sqrt{-g}} \varepsilon^{\alpha\beta\mu\nu}$. $\varepsilon^{\alpha\beta\mu\nu}$ is the completely anti--symmetric Levi-Civita symbol. [ ] means anti-symmetrization. Einstein's equations can be written

$$C^{\alpha\beta\mu\nu}{}_{||\nu} = I^{\alpha\beta\mu} \qquad (4)$$

for a conveniently choice of the boundary conditions [5, 6]. The current $I^{\alpha\beta\mu}$ has to be constructed with the energy momentum tensor $T^{\mu\nu}$ and its derivatives [4]. The very fact that $I^{\alpha\beta\mu}$ depends only on the matter content and not on space time curvature is a consequence of Einstein's choice of equations of GR.

Equation (4) has to be considered as an equation for the classical umperturbed curvature. By allowing the geometry to fluctuate around an arbitrary classical background a direct calculation shows [1] that the set of equations of the perturbed geometry is similar to equation (4) but with the crucial difference that the current term depends on geometry besides on the matter content. In general $I^{\alpha\beta\mu}$ will be a very envolved expression. However Ginzburg et al have given a very suggestive way to treat these terms in a simplified and worthwhile procedure. In his approach, which uses the conventional Einstein's equations $R^{\mu}{}_{\nu} - \frac{1}{2} R \, \delta^{\mu}{}_{\nu} = - T^{\mu}{}_{\nu}$ Ginzburg is able to treat the perturbed terms by developping them in polynomials of the umperturbed curvature.

This approach is attractive because it permits us to develop specific models independently of the particular features of the background geometry. However, it is worthwhile to call attention to the fact that the Ginzburg expansion can be justified only if the vacuum fluctuations do not destroy completely the primordial features of the umperturbed system. In other words, the fluctuations must, in some precise sense, be small quantities. In an anlogous way we assume we can develop the fluctuations on the Weyl tensor in terms of the unperturbed one. The general form of the fluctuating current will be given by the expansion

$$\tilde{I}^{\alpha\beta\mu} = q_1 \, V^{[\alpha} E^{\beta]\mu} + q_2 \, V^{[\alpha} H^{\beta]\mu} + q_3 \, E^{\lambda[\alpha} E^{\beta]}{}_{\lambda} V^{\mu} + q_4 \, E^{\lambda[\alpha} H^{\beta]}{}_{\lambda} V^{\mu} + q_5 \, H^{\lambda[\alpha} H^{\beta]}{}_{\lambda} V^{\mu} + q_6 \, V^{\alpha[} E^{\beta]\lambda} E_{\lambda}{}^{\mu} + q_7 \, V^{[\alpha} E^{\beta]\lambda} H_{\lambda}{}^{\mu} + q_8 \, V^{[\alpha} H^{\beta]\lambda} E_{\lambda}{}^{\mu} + q_9 \, V^{[\alpha} H^{\beta]\lambda} H_{\lambda}{}^{\mu} + \ldots \tag{5}$$

The constants q_k represents the contribution weight of each term and should be evaluated by taking into account the quantum properties of the geometry. They can depend on the perturbation free system in a very direct way. This can be seen by the fact that in general, only a restricted set of perturbations do not destroy completelly the features of the unperturbed system.

Let us restrict ourselves here only to the simplified case in which the perturbation excites only the q_1 mode, that is, all q_k's are null except $q_1 \equiv q$. So we can write

$$C^{\alpha\beta\mu\nu}{}_{||\nu} = q \, V^{[\alpha} E^{\beta]\mu} \tag{6}$$

The rhs has a striking ressemblance with Maxwell's current term inside a conducting media, i.e $J^{\alpha} = \sigma F^{\alpha\mu} V_{\mu}$.

This analogy will appear more clearly when we project the set (6) of equations in the 3 dimensional rest space of the observer V^{α} .

We will consider here the simplest case of perturbing a gravitational wave in a Minkowskii space time. Projecting equation (6) will give rise to the following set:

$$E^{i}{}_{j|i} = 0 \qquad (7a)$$

$$H^{i}{}_{j|i} = 0 \qquad (7b)$$

$$\dot{H}^{ij} + \frac{1}{2} E^{(i}{}_{m|l}\, \varepsilon^{j)ml} = 0 \qquad (7c)$$

$$\dot{E}^{ij} - \frac{1}{2} H^{(i}{}_{m|l}\, \varepsilon^{j)ml} = - q\, E^{ij} \qquad (7d)$$

in which we have chosen $V^{\alpha} = \delta^{\alpha}{}_{0}$, (latin indices varies from 1 to 3), a simple bar means derivative, a point means derivative in the V^{α}-direction, that is time derivative.

Multiplying equation (7d) by the factor $\frac{1}{2}\, \delta^{m}{}_{(i}\, \varepsilon_{j)}{}^{kl} \frac{\partial}{\partial x^{k}}$ and arranging terms we find, using equation (7c):

$$\ddot{E}^{kl} - \nabla^2 E^{kl} = -q\, \dot{E}^{kl}$$

where ∇^2 is the 3-dimensional Laplacian operator.

A typical solution of the wave equation (8) is given by

$$E^{ij}{}_{(p)} = L^{ij}\, e^{i(px - wt)}\, e^{- \frac{q}{2} t}$$

This shows explicity the analogy with the electromagnetic field in a conducting media, as we said above.

Here, it is the excitation of the space time which is responsible by the absorption phenomenum. The weight q of the linear term in the perturbation expansion measures the attenuation of the wave. It can thus be interpreted as the conductivity of the vacuum.

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