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IDENTIFICATION OF PARTON DENSITIES WITH PARTON FRAGMENTATION FUNCTIONS, DUALITY AND AVERAGE MULTIPLICITIES OF LEPTON-HADRON PROCESSES

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ABSTRACT

Suggesting that the hadronic and the current densities of quark-partons are identical and utilizing duality principles predictions for the average multiplicities of lepton-hadron processes in the deep inelastic scaling region are derived.

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I - INTRODUCTION

Recently the suggestion was made that the parton distribution functions and the parton fragmentation functions introduced by Feynman[1] are closely related[2]. The suggestion made in ref. 2 was based on obvious similarities between both sets of functions.

The actual physical content of such a close (or hopefully nearly complete) identification of these functions is the following:

It is the basic assumption of the Parton Model (P.M) that the distribution of the final state hadrons has much overlap in phase space with the final parton state obtained from scattering and decay of partons[3]. So by observing hadrons in final state in hadronic reactions we get information on the phase space behaviour of the partons. The same ind of information[4] can be got from inclusive lepton-hadron processes of the type $\ell h \to \ell' X$. What is measured in these latter reactions in the pertinent kinematical region are the parton distribution functions, which in the case of the Quark-Parton-Model (Q.P.M.) we write as $d_i^h(x)$. Here h is a specific hadron, i enumerates the six quark states and x is the longitudinal momentum fraction of the parton. (To be exact, in hadronic collissions the parton density in the rapidity intervall $\log P_{cm} < y < \log P_{cm}$ is measured, and $d_i^h(x)$ represents this density from $y = 0$ to $y = \log P$[1]).

On the other hand no information on the current fragmentation region can be gained in reactions of the type mentioned above[3],[4],[2]. The best way to learn about this region is the process $e^+e^- \to hX$[3],[5].

Now in the framework of the P.M. it is very suggestive, although one can not definitely prove it, that the current fragmentation region has a structure very similar to that of the hadronic distribution $d_i^h(x)$. This is because both can be thought of as being produced by the same mechanism of interacting and decaying partons.

Now the distribution of current fragments is described by the parton fragmentation functions[1],[2],[3],[5] $D_i^h(z)$, where z is the fraction of longitudinal momentum the outgoing hadron h takes off the fragmenting parton i.

So it is very stimulating to assume in the P.M. that the hadronic distribution $d_i^h(x)$ and the current distribution $D_i^h(z)$ are actually nearly the same. This means that between the parton and the hole fragmentation region there is a plateau[6] which in its physical properties is nearly identical with the hadronic central plateau[1] (neutral and universal) and the heights of the two plateaus are not very different. It further includes that the hadronic target fragmentation region corresponds closely to the parton fragmentation region[6]. This can be understood from the fact mentioned above, that final hadron and final state parton distribution have a large phase space overlap. Further the hole region must be universal, which is not unnatural since the hole (= antiparton) belongs to the diffraction 'sea'. In making this point it is essential that partons are supposed to interact only when they are nearby in phase space (So only soft events in the sense of ref. 8 are taken into account). Within our framework we will also come to feactures of long range correlations (see p. 11).

The ansatz of ref.2

$$D_i^h(z) = \psi(z)d_i^h(z) \qquad (1)$$

relates the current distribution and the hadronic distribution via a universal function $\psi(z)$[2]. If the plateaus of both regions are assumed to have equal height (this can also be checked by measuring multiplicities) and if exactly the same physics is supposed to go on in the current and in the hadronic region we are led to

$$\psi(z) = 1 \ ; \qquad (2)$$

or

$$D_i^h(z) = d_i^h(z) \ ; \qquad (3)$$

This includes e.g. that in the process $e^+e^- \rightarrow hX$ one should observe two jets with a pionization region in between. Also in the pertinent kinematical region of $\ell h \rightarrow \ell' h' X$ there should now be pionization. In a global sense eq. 3 menas that in the current fragmentation region no essential new physics will be discovered.

It will be most interesting to investigate this ansatz in the framework of lightcone analysis and crossing symmetry(2).

But before doing so one has to develop as much experimental suggestions to be tested as possible(2), because eq. 3 contains several rather strong assumptions, which are at most plausible from the P.M., but otherwise unproven.

In this paper we will discuss the consequences of eq. 3 combined with the duality constraints of ref. 7 for the average multiplicities of left moving hadrons in several lepton-hadron semi-inclusive processes.

In advance we want to make a brief remark concerning the convergence of certain integrals arising in the course of our investigations.

If the Regge analysis of deep inelastic data, which seems substantiated by experiment for small x will also apply down to wee x(1)

$(x \to 0)$, then the integrals $\int_0^1 d_i^h(x)dx$ must diverge at the lower end of the integration, because as a consequence of $\alpha_p(0) = 1$ the integrand behaves as $\frac{1}{x}$ in this region.

Such integrals will appear in what follows, so it is worthwhile to clarify that eventually there is no divergence. This comes about because at $x = 0$ the parton cascade(1) will be stopped in a characteristic way, which will imply a drastic change in the $\frac{1}{x}$ behaviour, such that the number of wee partons will be finite. (This by the means that simple Markoffian chains are no good model for the cascade.) Then Pomeron dominance is only given at an intermediate range of small x.

If the core should be asymmetric under isospin as argued in ref.15 then the number of partons must be finite because otherwise the Gottfried integral does not exist (see ref. 9). So in the absence of data one can perhaps speculate that at very small x an integrable behaviour results.

On the other hand we will rewrite part of our results in a way that infinities, should they occur, will be cancelled.

II - BARYONS IN FINAL STATE

We consider the process $J(q) + N(p) \to hX$, where $J(q)$ is the virtual electromagnetic or weak current, which carries the momentum q, and p is the momentum of the nucleon target. The detected hadron carries four-momentum h. The current fragmentation region is defined by:

$$q^2 \to -\infty; \quad M\nu = pq \to \infty; \qquad x = -\frac{q^2}{2M\nu} \text{ finite}$$
$$hp,\, hq \to -\infty; \quad z = \frac{hp}{M\nu} = \frac{2hq}{q^2} \text{ finite}; \; u = \frac{hq}{M\nu} \text{ finite} \tag{4}$$

The final hadron average multiplicity $<n_h>$ is then defined by the relation:

$$\int \frac{d^4\sigma}{dq^2\, d\nu du dh_T^2}(\ell N \to \ell' hX)\, du dh_T^2 = \tag{5}$$

$$= <n_h> \frac{d^2\sigma}{dq^2 d\nu}(\ell N \to \ell' X)\ ;$$

We work in this paper with the distributions $N^h_{\ell N}(x,z)$ defined in ref. 1, from which $<n_h>$ is obtained by z-integration (for details see ref.3).

From the work of ref. 9 the following integrals over quark densities can be calculated (eq. 35 of ref. 9 leaving out the 'unusual' Pomeron contributions).

$$\begin{aligned} \int u(x)dx &= \frac{5}{4} + \frac{3}{4} \int f_1^{ep}(x)dx; \\ \int \bar{u}(x)dx &= -\frac{3}{4} + \frac{3}{4} \int f_1^{ep}(x)dx; \\ \int d(x)dx &= \frac{1}{4} + \frac{3}{4} \int f_1^{ep}(x)dx; \\ \int \bar{d}(x)dx &= -\frac{3}{4} + \frac{3}{4} \int f_1^{ep}(x)dx; \\ \int \{s(x) + \bar{s}(x)\}dx &= -\frac{3}{2} + \frac{3}{2} \int f_1^{ep}(x)dx; \end{aligned} \tag{6}$$

Integration is taken over the whole kinematical region $0 < x < 1$ and we have used Feynman's notation[1] $d_u^p = u$, $d\frac{p}{u} = \bar{u}$,...etc., $f_1^{ep}(x) =$ $= \frac{4}{9}\{u(x) + u(x)\} + \frac{1}{9}\{d(x) + \bar{d}(x)\} + \frac{1}{9}\{s(x) + \bar{s}(x)\}$.

The relations are valid for an SU(3) singlet Pomeron (see also ref. 7) from duality principles.

For the special case of a non-SU(3) singlet Pomeron treated in ref. 9 (introduced to remedy the violation of the Gottfried sum rule) these equations are changed to

$$\begin{aligned}
\int u(x)dx &= \frac{9}{8} + \frac{3}{4} \int f_1^{ep}(x)dx ; \\
\int \bar{u}(x)dx &= -\frac{7}{8} + \frac{3}{4} \int f_1^{ep}(x)dx; \\
\int d(x)dx &= \frac{3}{8} + \frac{3}{4} \int f_1^{ep}(x)dx; \\
\int \bar{d}(x)dx &= -\frac{5}{8} + \frac{3}{4} \int f_1^{ep}(x)dx; \\
\int \{s(x) + \bar{s}(x)\}dx &= -\frac{3}{4} + \frac{3}{2} \int f_1^{ep}(x)dx;
\end{aligned} \qquad (7)$$

If we write in eqs. 6 all integrals e.g. at the l.h.s. then infinities due to $\frac{1}{x}$ behaviour if they s h o u l d occur will exactly cancel each other. In the case of eqs. 7 one must explicitly assume that the $\frac{1}{x}$ behaviour is changed for $x \to 0$, indepently of but in accord with our considerations above (see ref. 9).

Eqs. 6 and 7 together with our basic eq. 3 now allow us to calculate average multiplicities in the current fragmentation region.

We first study the inclusive process $ep \to epX$. With the definition of ref. 1 we get for $x > x_o$ (where x_o is chosen such that all but the contributions from the u(x) and d(x) quarks can be neclegted, see ref. 10 for details):

$$f_1^{ep}(x)N_{ep}^{p}(x,z) = \frac{4}{9}\ u(x)D_u^p(z) + \frac{1}{9}\ d(x)D_d^p(z) \quad ; \tag{8}$$

From this follows by z- integration:

$$f_1^{ep}(x) < n_{ep}^{p}(x) > = \frac{4}{9}\ u(x) \int u(z)dz + \frac{1}{9}d(x) \int d(z)dz \ ; \tag{9}$$

where eq. 3 has been used.

Using now eqs. 6 and expressing u(x) and d(x) by neutrino structure functions[1] we obtain

$$<n_{ep}^{p}(x)> = \frac{1}{36 f_1^{ep}(x)} \Big[5\ \{f_1^{\nu n}(x) - f_3^{\nu n}(x)\} + \frac{1}{4}\ \{f_1^{\nu p}(x) - f_3^{\nu p}(x)\} + 3\ (\{f_1^{\nu n}(x) - f_3^{\nu n}(x)\} + \frac{1}{4}\ \{f_1^{\nu p}(x) - f_3^{\nu p}(x)\}) . \int f_1^{ep}(x)dx \Big] \tag{10}$$

The further results simplify very much if we take $x \simeq 1$. This will be done in the rest of the paper, if not stated otherwise. It is clear how to obtain the results for any x. For $x \simeq 1$ only the u-quark contributes[1] and we get from (8) and the corresponding definitions of N_{ep}^{n}, N_{en}^{p} and N_{en}^{n} the relations:

$$< n_{ep}^{p} > = < n_{en}^{n} > = \frac{1}{4}\ \{5 + 3 \int f_1^{ep}(x)dx\ \} \ ;$$

$$< n^{n}ep > = < n_{en}^{p} > = \frac{1}{4}\ \{1 + 3 \int f_1^{ep}(x)dx\ \} \ ; \tag{11}$$

Experimentally the integral $\int_0^1 f_1^{ep}(x)dx$ will be known only with a lower end of integration $\varepsilon > 0$. So there is always an extrapolation of the data or a prejudice on how much the very small x values contribute. The following relations are independent of $\int f_1^{ep}(x)dx$ (and would also be valid if an $\frac{1}{x}$ divergence were there, leading to e.g. $\langle n_{ep}^{p}(x,q^2)\rangle = \frac{5}{4} + \frac{3}{4}\gamma_0 \ln q^2$ with γ_0 determined by deep inelastic data).

$$\langle n_{ep}^{p} \rangle - \langle n_{ep}^{n} \rangle = \langle n_{en}^{n} \rangle - \langle n_{en}^{p} \rangle = 1 \; ; \qquad (12)$$

In the case of a non-SU(3) singlet Pomeron[9] we get from eqs. 7 ($x \simeq 1$).

$$\langle n_{ep}^{p} \rangle = \langle n_{en}^{n} \rangle = \frac{3}{4}\left\{\frac{3}{2} + f_1^{ep}(x)dx\right\} \; ;$$

$$\langle n_{ep}^{n} \rangle = \langle n_{en}^{p} \rangle = \frac{3}{4}\left\{\frac{1}{2} + f_1^{ep}(x)dx\right\} \; ; \qquad (13)$$

and

$$\langle n_{ep}^{p} \rangle - \langle n_{ep}^{n} \rangle = \langle n_{en}^{n} \rangle - \langle n_{en}^{p} \rangle = \frac{3}{4} \; ; \qquad (14)$$

So in both cases we get more p than n from ep, and more n than p from en, which means that it is easiest to accelerate the target matter in the direction of the current!

If the fragmentation process is assumed to be SU(3) symmetric one obtains additional relations e.g.

$$\langle n_{ep}^{\Sigma^+} \rangle = \langle n_{ep}^{p} \rangle \; ; \qquad \langle n_{ep}^{\Xi^0} \rangle = \langle n_{ep}^{n} \rangle \; ; \qquad (15)$$

For neutrino reactions e.g. $\nu p \to \bar{\mu} pX$, $\bar{\nu}p \to \mu pX$ we obtain from the definitions

$$f_1^{\nu p}(x)\; N_{\nu p}^{p}(x,z) = 2\;\{d(x)D_u^p(z) + \bar{u}(x)D_d^{\bar{p}}(z)\} \quad ; \tag{16}$$

$$f_1^{\bar{\nu}p}(x)\; N_{\bar{\nu}p}^{\bar{p}}(x,z) = 2\;\{u(x)D_d^p(z) + \bar{d}(x)D_u^{\bar{p}}(z) \quad ; \tag{16'}$$

using eqs. 3 and 6

$$\begin{aligned} < n_{\nu p}^{p}(x) > &= \frac{1}{4} - \frac{f_3^{\nu p}(x)}{f_1^{\nu p}(x)} + \frac{3}{4}\int f_1^{ep}(x')dx' \quad ; \\ < n_{\bar{\nu}p}^{\bar{p}}(x) > &= -\frac{1}{4} - \frac{f_3^{\bar{\nu}p}(x)}{2f_1^{\bar{\nu}p}(x)} + \frac{3}{4}\int f_1^{\nu p}(x')dx' \quad ; \end{aligned} \tag{17}$$

or

$$< n_{\nu p}^{p}(x) > - < n_{\bar{\nu}p}^{\bar{p}}(x)> = \frac{1}{2} + \frac{1}{2}\;\frac{f_3^{\bar{\nu}p}(x)}{f_1^{\bar{\nu}p}(x)} - \frac{f_3^{\nu p}(x)}{f_1^{\nu p}(x)} \quad ; \tag{18}$$

These eqs. are supposed to hold in the whole x region, the last one being independent of the properties of the integral $\int f_1^{ep}(x)dx$.

For the case of a non-SU(3)-singlet we get correspondingly:

$$\begin{aligned} < n_{\nu p}^{p}(x) > &= \frac{1}{4} - \frac{7}{8}\frac{f_3^{\nu p}(x)}{f_1^{\nu p}(x)} + \frac{3}{4}\int f_1{}^{ep}(x')dx' \quad ; \\ < n_{\bar{\nu}p}^{\bar{p}}(x) > &= -\frac{1}{4} - 5\;\frac{f_3^{\bar{\nu}p}(x)}{f_1^{\bar{\nu}p}(x)} + \frac{3}{4}\int f_1^{ep}(x')dx' \quad ; \end{aligned} \tag{19}$$

or

$$< n^{p}_{\nu p}(x)> - <n^{\bar{p}}_{\nu p}(x)> = \frac{1}{2} - \frac{7}{8}\frac{f_3^{\nu p}(x)}{f_1^{\nu p}(x)} + 5\frac{f_3^{\bar{\nu} p}(x)}{f_1^{\bar{\nu} p}(x)} \qquad (20)$$

At $x \simeq 1$ we have from u-quark dominance $f_3^{\bar{\nu}p}/f_1^{\bar{\nu}p} = -1$, and thus get from eqs. 11, 17 and 19.

$$< n^{\bar{p}}_{\nu p} > + 1 = < n^{p}_{ep} > \quad ; \qquad (21)$$

valid with an SU(3) singlet Pomeron and

$$< n^{\bar{p}}_{\nu p} > + \frac{7}{8} = < n^{p}_{ep} > \qquad (22)$$

valid with the non-SU(3) singlet Pomeron of ref. 9.

It is now easy to obtain more relations of the kind presented here.

III - MESON IN FINAL STATE

The case of the mesons is much simpler than that of baryons because according to ref. 7 all quark densities can be expressed, if duality is imposed, by one single function $a(x)^{(2)}$ in the following way:

$$d_u^{\pi^+}(x) = d_{\bar{d}}^{\pi^+}(x) = d_{\bar{u}}^{\pi^-}(x) = d_d^{\pi^-}(x) = \frac{1}{2}\,a(x) \quad ;$$

$$d_u^{\pi^0}(x) = d_{\bar{u}}^{\pi^0}(x) = d_d^{\pi^0}(x) = d_{\bar{d}}^{\pi^0}(x) = \frac{1}{4}a(x) \; ; \qquad (23)$$

All quark densities not listed here and not obtainable from the ones listed here by isospin charge conjugation or U-spin invariance vanish[(2),(7)].

From the work of ref. 7 one easily derives the integral of a(x) to be:

$$\int_0^1 a(x)dx = \frac{1}{4} \quad ; \tag{24}$$

Thus by eq. 23 the integrals over all mesonic quark densities are known. We need not discuss an $\frac{1}{x}$ difficulty. So in this case there is no plateau of length $\ln q^2$. Hence in the meson case there are long range correlations, due to the combination of parton and hole into a meson bound state plus anything. From these correlations no baryon can be produced.

We start with the process $ep \to e\pi^+X$, and first derive an inequality for $\langle n_{ep}^{\pi^+}(x)\rangle$ for $x \gg x_o$[(10)] and then give its value at $x \simeq 1$.[(10)]

At $x > x_o$ [(10)] we have

$$f_1^{ep}(x)\, N_{ep}^{\pi^+}(x,z) = \frac{4}{5}\, u(x) D_u^{\pi^+}(z) + \frac{1}{9}\, d(x) D_d^{\pi^+}(z) \quad ; \tag{25}$$

From eqs. 3 and 23 the last term vanishes identically and we get

$$f_1^{ep}(x).\ \langle n_{ep}^{\pi^+}(x)\rangle = \frac{4}{9}\, u(x) \int d_u^{\pi^+}(z)dz \tag{26}$$

Eqs. 23, 24 give, if we express u(x) again by suitable structure functions

$$\langle n_{ep}^{\pi^+}(x)\rangle = \frac{1}{72}\, \frac{f_1^{\bar{\nu}p}(x)}{f_1^{ep}(x)} \left(1 - \frac{f_3^{\bar{\nu}p}(x)}{f_1^{\bar{\nu}p}(x)}\right) \tag{27}$$

Using the inequality[7]

$$3 \leq \frac{f_1^{\bar{\nu}p}(x)}{f_1^{ep}(x)} \leq \frac{9}{2} \quad ; \tag{28}$$

we derive from this

$$\frac{1}{24}\left(1 - \frac{f_3^{\bar{\nu}p}(x)}{f_1^{\bar{\nu}p}(x)}\right) \leq \langle n_{ep}^{\pi^+}(x)\rangle \leq \frac{1}{16}\left(1 - \frac{f_3^{\bar{\nu}p}(x)}{f_1^{\bar{\nu}p}(x)}\right) \tag{29}$$

At $x \simeq 1$ we have $f_1^{ep}(x) = \frac{4}{9}\,u(x)$ and thus get from (26):

$$\langle n_{ep}^{\pi^+}\rangle = \frac{1}{8} \quad ; \tag{30}$$

Looking at $ep \to e\pi^- X$ we get in the same way:

$$f_1^{ep}(x)\, N_{ep}^{\pi^-}(x,z) = \frac{1}{9}\, d(x) d_u^{\pi^+}(z) \tag{31}$$

for $x > x_o$, from which follows

$$\langle n_{ep}^{\pi^-}(x)\rangle = \frac{1}{288}\, \frac{f_1^{\nu p}(x)}{f_1^{ep}(x)} \left(1 - \frac{f_3^{\nu p}(x)}{f_1^{\nu p}(x)}\right); \tag{32}$$

Using the inequality[7]

$$\frac{f_1^{\nu p}(x)}{f_1^{ep}(x)} \leq 6 \tag{33}$$

we arrive at

$$<n_{ep}^{\pi^-}(x)> \;\leq\; \frac{1}{48}\left(1 - \frac{f_3^{\nu p}(x)}{f_1^{\nu p}(x)}\right) \tag{34}$$

From (31) for $x \simeq 1$ and using $\left.\frac{d(x)}{u(x)}\right|_{x=1} = 0$[11], we get:

$$<n_{ep}^{\pi^-}(x)> \underset{x \to 1}{=} 0 \tag{35}$$

Furthermore for en-processes one derives in the same way:

$$<n_{en}^{\pi^+}(x)> \underset{x > x_o}{=} \frac{1}{72\, f_1^{en}(x)} \left(f_1^{\nu p}(x) - f_3^{\nu p}(x)\right) \quad ;$$

$$<n_{en}^{\pi^+}(x)> \underset{x \to 1}{=} 0; \tag{36}$$

$$<n_{en}^{\pi^-}(x)> \underset{x > x_o}{=} \frac{1}{288\, f_1^{en}(x)} \left(f_1^{\bar{\nu} p}(x) - f_3^{\bar{\nu} p}(x)\right) \quad ;$$

$$<n_{en}^{\pi^-}(x)> \underset{x \to 1}{=} \frac{1}{8} \quad ;$$

So for $x \simeq 1$ there are π^- from ep, no π^+ from en and the same number of π^+ from ep as π^- from en.

The important point on these relations is that we are able to calculate definite values for multiplicities not only ratios.

We now derive a relation for neutral pions in final state.

The following relation holds for all x:

$$f_1^{ep}(x)\, N_{ep}^{\pi^o}(x,z) - f_1^{en}(x)\, N_{en}^{\pi^o}(x,z) = \{f_1^{ep}(x) - f_1^{en}(x)\}\;\; d_u^{\pi^o}(z); \qquad (37)$$

With eqs. 23 and 24 get:

$$\frac{1}{4} \leq \frac{\frac{1}{16} - \langle n_{ep}^{\pi^+}(x)\rangle}{\frac{1}{16} - \langle n_{en}^{\pi^o}(x)\rangle} \leq 1\;; \qquad (38)$$

where

$$\frac{1}{4} \leq \frac{f_1^{en}(x)}{f_1^{ep}(x)} \leq 1\;; \qquad (39)$$

was utilized[12]

From eq. 38 also follows:

$$\langle n_{en}^{\pi^o}(x)\rangle \leq \frac{1}{16}\;;$$

$$\langle n_{ep}^{\pi^o}(x)\rangle \leq \frac{1}{16}\;;$$

for all x.

For $x \simeq 1$ we have

$$\langle n_{en}^{\pi^o}\rangle = \langle n_{ep}^{\pi^o}\rangle = \frac{1}{16}\;;$$

So there are only half as many neutral pions than charged pions produced for $x \simeq 1$ (This is qualitatively clear because one needs the double number of quarks to produce a π^0 rather than a $\pi^{\pm}$).

Requiring SU(3) symmetry our equations are easily carried over to K and η mesons. We list some relations for K's:

$$\langle n_{ep}^{K^+}(x)\rangle \underset{x \to 1}{=} \frac{1}{8} \quad ;$$

$$\langle n_{en}^{K^-}(x)\rangle \underset{x \to 1}{=} 0 \quad ; \tag{42}$$

So the K^+/K^- assymetry at $x \simeq 1$ should be much larger than the π^+/π^- assymetry. We could easily calculate it in terms of structure functions for any x.

$$\langle n_{en}^{K^+}(x)\rangle \underset{x>x_o}{=} \langle n_{en}^{\pi^+}(x)\rangle \quad ;$$

$$\langle n_{ep}^{K^+}(x)\rangle \underset{x\to 1}{=} 0 \quad ; \tag{43}$$

$$\langle n_{ep}^{K^-}(x)\rangle \underset{x\to 1}{=} 0$$

So at $x \approx 1$ only K^+ in ep will be significantly produced.

For (en| $\overset{(-)}{K_o}$) we derive:

$$\langle n_{ep}^{\pi^-}(x)\rangle = \langle n_{ep}^{K_o}(x)\rangle \underset{x\simeq 1}{=} 0 \quad ; \tag{44}$$

$$\langle n_{ep}^{\overline{K}_o}(x)\rangle \underset{x \geq x_o}{=} 0 \quad ;$$

$$\langle n_{en}^{\overline{K}_o}(x)\rangle \underset{x > x_o}{=} 0 \quad ; \tag{44}$$

$$\langle n_{en}^{\pi^-}(x)\rangle = \langle n_{en}^{K_o}(x)\rangle \underset{x \simeq 1}{=} \frac{1}{8} \quad ;$$

Integrating the normalized distribution(3)

$$G_{e^+e^-}^{h}(z) = \frac{1}{\frac{1}{2}\sum_i Q_i^2} \cdot \sum_i Q_i^2 \; D_i^h(z) \quad ; \tag{45}$$

over z and using SU(3) symmetry we furthermore find:

$$\langle n_{e^+e^-}^{\pi^+}\rangle = \langle n_{e^+e^-}^{\pi^-}\rangle = \langle n_{e^+e^-}^{\pi^o}\rangle =$$

$$= \langle n_{e^+e^-}^{K^+}\rangle = \langle n_{e^+e^-}^{K^-}\rangle = \frac{5}{48} \; ; \tag{46}$$

$$\langle n_{e^+e^-}^{K_o}\rangle = \langle n_{e^+e^-}^{\overline{K}_o}\rangle = \frac{1}{24} \quad ;$$

Finally we list our results for neutrino reactions. The definition of the quantities $N_{\nu N}^{h}(x,z)$ is completly analogous to $N_{eN}^{h}(x,z)$ (See ref. 1 and 3). For the average multiplicities we obtain (We recall that the quantities $N_{\nu N}^{\pi}$ are x-independent(10).):

for all x:

$$\langle n^{\pi^+}_{\bar{\nu}n} \rangle = \langle n^{\pi^-}_{\nu n} \rangle = \langle n^{\pi^-}_{\bar{\nu}p} \rangle =$$

$$= \langle n^{\pi^+}_{\nu p} \rangle = \frac{1}{8} \; ; \qquad (47)$$

$$\langle n^{\pi^-}_{\nu p} \rangle = \langle n^{\pi^-}_{\bar{\nu}n} \rangle = \langle n^{\pi^+}_{\nu n} \rangle = \langle n^{\pi^+}_{\bar{\nu}p} \rangle = 0;$$

$$\langle n^{\pi^o}_{\nu p} \rangle = \langle n^{\pi^o}_{\bar{\nu}p} \rangle = \langle n^{\pi^o}_{\nu n} \rangle = \langle n^{\pi^o}_{\bar{\nu}n} \rangle = \frac{1}{16} \; ;$$

$$\langle n^{K^+}_{\nu p}(x) \rangle = \langle n^{K^+}_{\bar{\nu}n}(x) \rangle = \langle n^{K_o}_{\bar{\nu}p}(x) \rangle =$$

$$= \langle n^{K_o}_{\nu n}(x) \rangle = \frac{1}{16} \left(1 - \frac{f_3^{\nu p}(x)}{f_1^{\nu p}(x)}\right) \; ;$$

$x > x_o$:

$$\langle n^{K^+}_{\nu p}(x) \rangle = \langle n^{K^+}_{\bar{\nu}n}(x) \rangle = \langle n^{K_o}_{\bar{\nu}p}(x) \rangle =$$

$$= \langle n^{K_o}_{\nu n}(x) \rangle = \frac{1}{8} \; ; \qquad (47)$$

for all x :

$$<n^{K^+}_{\bar{\nu}p}(x)> \;=\; <n^{K^+}_{\nu n}(x)> \;=\; <n^{K_o}_{\bar{\nu}n}(x)> \;=$$

$$= \; <n^{\bar{K}_o}_{\nu p}(x)> \;=\; 0 \;;$$

$$<n^{K_o^-}_{\nu p}(x)> \;=\; <n^{K^-}_{\bar{\nu}n}(x)> \;=\; <n^{\bar{K}}_{\bar{\nu}p}(x)> \;= \qquad (48)$$

$$= \; <n^{\bar{K}_o}_{\nu n}(x)> \;=\; 0 \;;$$

$$<n^{\bar{K}_o}_{\nu p}(x)> \;=\; <n^{K^-}_{\bar{\nu}p}(x)> \;=\; <n^{K^-}_{\nu n}(x)> \;=$$

$$= \; <n^{\bar{K}_o}_{\bar{\nu}n}(x)> \;=\; \frac{1}{16}\left(1 + \frac{f_3^{\nu p}(x)}{f_1^{\nu p}(x)}\right) \;;$$

For $x > x_o$ we have $\frac{\bar{u}(x)}{d(x)} \to 0$ and thus obtain from the last set of relations:

$$<n^{\bar{K}_o}_{\nu p}(x)> \;=\; <n^{K^-}_{\bar{\nu}p}(x)> \;=\; <n^{K^-}_{\nu n}(x)> \;=$$

$$= \; < n^{\bar{K}_o}_{\bar{\nu}n}(x) > \;=\; 0; \qquad (48')$$

Fom eqs. 46, 47 and 48 one obtains:

$$<n^{K^+}_{\nu p}(x)> \;=\; <n^{\bar{K}_o}_{\nu p}(x)> \;=\; <n^{\pi^+}_{\nu p}> \;\; \left(=\frac{1}{8}\right) \;; \qquad (49)$$

$$<n^{\pi^0}_{e^+e^-}> \; = \frac{5}{6} \cdot <n^{\pi^+}_{\nu p}> \quad (= \frac{5}{48}) \quad ; \qquad (50)$$

Comparing with ref. 3 one realizes that eqs. 49 and 50 exactly saturate corresponding SU(3) bounds. This is a general property of mos of our results, which originates from eqs. 23 in conjunction with the basic equation 3, as can also be seen by comparing with the multiplicity ratios listed in ref. 11, which where calculated on the basis of duality constraints.

IV - CONCLUSION

In writing eq. 3 one must be careful in the wee region. In the Breit frame(1),(3) the variable z is wee for $z \sim \frac{1}{xp}$ (x fixed) and x is wee for $x \sim \frac{1}{p}$ (P is the proton's Breit frame momentum). As the functions D^h_i and d^h_i are expected to behave characteristically in their respective wee regions(1) eq. 3 simplicitly contains a consistency requirement, which we have tacitly assumed to be fulfilled, because x and z are not wee at the same time.

Form meson multiplicities we have made rather stringent predictions e.g. $<n^{\pi^+}_{ep}(x)> = 0$ for $x \simeq 1$. These are due to the fact that the meson case, contrary to the baryon case, is rather sharply constrained by duality(7). So if the data will point in a different direction one could think of relaxing these conditions in the meson case. At present data are only available for $0.2 \leq z \leq 0.7$(13),(14). So we cannot make a reliable fit in the whole z area to calculate the integrals $<n^{\pi}_{ep}(x)>$. The available data however are encouraging: $<n^{\pi^+}_{ep}(x \simeq 1)>$ will presumably below 0.2 (if we tentatively assume that the region $\frac{1}{z}$ changes to an integrable behaviour does contribute negligibly), because there is a sharp decrease for $x \to 1$(14), and $<n^{\pi^-}_{ep}(x \simeq 1)>$ must be considerably amaller than $<n_{ep}^{\pi^+}(x \simeq 1)>$. For a precise comparision we need data for all $0 \leq z \leq 1$.

If it should turn out that the data require a correction of a l l data by a c o m m o n factory γ_0 , then this simply means that the hadronic and the current plateau do not have equal height, but are otherwise completly equivalent, hence $\psi(z) = \gamma_0$ in eq. 1.

On the other hand one should repeat the analysis when $\psi(z)$ is measured according to the suggestion of ref. 2.

We finally mention that we cannot verify within the framework of this paper Feynman's conjecture that the average quantum numbers in the fragmentation region are those of the quarks.

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