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CHIRAL SYMMETRY, SUPERCONVERGENCE AND SUM RULES FOR SCATTERING AMPLITUDES*

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ABSTRACT

Generalizing an idea of chiral symmetry being exact in an asymptotic limit, we obtain superconvergent sum rules for suitable combinations of scattering amplitudes. The predictions of the sum rules are compared with experiment.

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In a recent letter Das, Mathur and Okubo [1] have obtained some interesting sum rules, including two sum rules obtained earlier by Weinberg [2], by postulating that the chiral symmetry SU(2) × × SU(2), which is broken in nature, becomes exact in a certain asymptotic limit. They have considered the Fourier transforms, $\Delta^V_{\mu\nu}(q)$ and $\Delta^A_{\mu\nu}(q)$ of the vacuum expectation values of the time-ordered product of two vector and axial-vector currents, respectively, and assumed that the difference of these Fourier transforms tends to zero as $q^2 \to \infty$. The sum rules they have obtained on this assumption seem to be well satisfied. [3]

The idea of the chiral symmetry being exact, in an asymptotic limit, admits, however, of a wider generalization. Since exact chiral symmetry implies in some sense conserved current operators, one can assume with equal validity that not only the differences between Fourier transforms of vacuum expectation values but also the differences between more general matrix elements of the time-ordered product of two vector and axial-vector currents tend to zero asymptotically. In this letter we want to report on such a generalization by considering the matrix elements of T-product of two vector and axial-vector currents between one-pion states and the sum rules which can be deduced from it. The relevant Fourier transforms are essentially the scattering amplitudes of a pion and a vector or axial-vector meson. We are here regarding the vector and axial-vector currents as sources of vector and axial-vector fields, respectively. The asymptotic limit in this case is the limit of the centre-of-mass energy becoming infinite. We are assuming, in short, that the difference of two suitable invariant amplitudes representing the scattering of a vector particle against some target and the scattering of a "correspond ing" axial-vector against the same target satisfies a super-convergent dispersion relation and gives rise to a sum rule.

Following the notation of Alfaro et al.,[4] the appropriate Fourier transforms are written as

$$T^{V}_{\mu\nu} = \int d^4x\; e^{iq\cdot x} \langle \pi(p_2)|T(V_\mu(x),\, V_\nu(0))|\pi(p_1)\rangle \tag{1a}$$

$$T^{A}_{\mu\nu} = \int d^4x\; e^{iq\cdot x} \langle \pi(p_2)|T(A_\mu(x),\, A_\nu(0))|\pi(p_1)\rangle \; . \tag{1b}$$

These Feynman amplitudes can be decomposed into invariant amplitudes as follows:

$$T^{V(A)}_{\mu\nu} = \varepsilon_1\cdot P\; \varepsilon_2\cdot P\; A^{V(A)} + \frac{1}{2}\,(\varepsilon_1\cdot P\; \varepsilon_2\cdot Q + \varepsilon_1\cdot Q\; \varepsilon_2\cdot P)B^{V(A)}$$

$$+\; \varepsilon_1\cdot Q\; \varepsilon_2\cdot Q\; C_1^{V(A)} + \varepsilon_1\cdot\varepsilon_2\; C_2^{V(A)} \; . \tag{2}$$

We assume that the A_1-meson is the axial counterpart of ρ-meson in the sense that A_1-meson dominates the axial-vector current just as ρ-meson does for the vector current. Then A^V, B^V, etc., refer to ρ-π scattering and A^A, B^A, etc., to A_1-π scattering [5].

It can be shown by arguments based on unitarity that asymptotically $C_{1,2} \sim S^{\varepsilon}$, $B \sim S^{-1+\varepsilon}$ and $A \sim S^{-2+\varepsilon}$ as $s \to \infty$, where ε is a real positive number. Our postulate about chiral symmetry being exact when the c.m. energy ($\sqrt{s}$) becomes infinite implies that the difference of any one of these invariant amplitudes for $\rho-\pi$ scattering, and the corresponding one for $A_1-\pi$ scattering, becomes convergent or superconvergent depending upon ε. However, in the absence of a definite knowledge of ε, it would be reasonable to assume that the amplitudes $(B^V(s)-B^A(s))$ and $(A^V(s)-A^A(s))$ are superconvergent in any isotopic spin state even though A^V, B^V or A^A, B^V may not be. The resulting sum rules may be symbolically written as

$$\int \mathrm{Im}\left[A_I^V(s') - A_I^A(s')\right] ds' = 0 \tag{3a}$$

$$\int \mathrm{Im}\left[B_I^V(s') - B_I^A(s')\right] ds' = 0 \tag{3b}$$

where the suffix I denotes the isotopic spin state in the s-channel. In exploiting these sum rules we assume as usual that the integrals in eq.(3) are saturated by the contributions of a few low mass states. In deriving the relations between coupling constants implied by the sum rules (3a) and (3b) it would be simpler to consider specific reactions. The reactions we consider are: $\pi^{\pm} + \rho^o(A_1^o) \rightarrow \pi^{\pm} + \rho^o(A_1^o)$ and $\pi^o + \rho^o(A_1^o) \rightarrow \pi^o + \rho^o(A_1^o)$. The important poles that contribute to the first set of reactions are π, A and ρ, while for the second set these are ω_o, f_o and $f_o'(1500)$. We evaluate the sum rules in the forward direction. With the intermediate states mentioned above, the contributions of s- and u-channel pole terms cancel for the sum rule (3a) which is therefore satisfied trivially. The non-trivial sum rules, following from (3b), as specialized to the set of above reactions, are as follows [6]:

$$4g^2_{\rho\pi\pi} = \frac{m^2_{A_1} - m^2_\rho}{m^2_\rho} \left(\frac{k}{m_{A_1}}\right)^2 \left(g^2_L - 1.5\, g^2_T\right) m^2_{A_1} \tag{4}$$

$$g^2_{f_oA_1\pi} \left[-5/3 + \frac{1}{m^2_{f_o}} \left\{ - m^2_\pi + \frac{4}{3} \left(m^2_\pi + \nu_{f_o} \right) \right\} + \right.$$

$$\left. + \frac{4}{3} \left(\frac{m^2_\pi + \nu_{f_o}}{m^2_{f_o}} \right)^2 \right]$$

$$+ g^2_{f'_oA_1\pi} \left[- 5/3 + \frac{1}{m^2_{f'_o}} \left\{ - m^2_\pi + \frac{4}{3} \left(m^2_\pi + \nu_{f'_o} \right) \right\} + \right.$$

$$\left. + \frac{4}{3} \left(\frac{m^2_\pi + \nu_{f'_o}}{m^2_{f'_o}} \right)^2 \right]$$

$$+ 2\, g^2_{\omega\rho\pi} \left(2\nu_\omega + m^2_\rho \right) = 0 \tag{5}$$

where $\nu_{f_o,f'_o} = \frac{1}{2}\left(m^2_{f_o,f'_o} - m^2_{A_1} - m^2_\pi\right)$, $\nu_\omega = \frac{1}{2}\left(\nu^2_\omega - m^2_\rho - m^2_\pi\right)$;

k is c.m. momentum in A_1-decay;

g_L and g_T refer to longitudinal and transverse couplings of A_1 to ρ and π. We also give here the relation of the width of A_1-decay into $\rho\pi$ in terms g_L and g_T which is given by

$$\Gamma_{A_1 \to \rho\pi}/m_{A_1} = \frac{1}{12\pi}\left(\frac{m_{A_1}}{m_\rho}\right)^2\left(\frac{k}{m_{A_1}}\right)^5\left(g_L^2 + 2\,g_T^2\right)m_{A_1}^2 \,. \qquad (6)$$

Gilman and Harari [6] find from superconvergence relations in $\rho - \pi$ scattering that $g_T = 0$. If we assume $g_T = 0$ and calculate the width of A_1 using eqs. (4) and (6), we get $\Gamma_{A_1 \to \rho\pi} \sim 70$ MeV, which is to be compared with the experimental value of $\Gamma_{A_1 \to \rho\pi} = 130 \pm 40$ MeV. [7] Considering the large error in the experimental value, the calculated value seems to be consistent with experiment. [8] Alternatively, treating eqs. (4) and (5) as a simultaneous set to determine g_L and g_T in terms of the experimental value (130 MeV), we get $g_L^2/g_T^2 = 5.8$, a value which can be checked experimentally by measuring the ratio of longitudinal to transverse polarization of the ρ-meson in A_1-decay. Further, if we take $g_{f_0 A_1 \pi} \sim g_{f_0' A_1 \pi}$, eq. (5) gives a value $\Gamma_{f_0 \to A_1\pi} \sim 2$ MeV, which is consistent with experiment. In addition we find $\Gamma_{f_0' \to A_1\pi} \sim 50$ MeV, which is probably large to be consistent with experiment. But by assuming an $f_0 - f_0'$ mixing angle one can substantially reduce the value of the width $\Gamma_{f_0' \to A_1\pi}$ we obtained.

The successful predictions coming out of our sum rules lend considerable validity to our generalization of chiral symmetry being exact at high energies. Other applications of the same idea to various other reactions will be reported elsewhere.

The above analysis of chiral symmetry can immediately be extended to strangeness-changing currents. Assuming that K^* and K_A dominate the strangeness-changing vector and axial-vector currents respectively and considering the reactions $\pi^{\pm} + K^{*o}(K_A^o) \longrightarrow \pi^{\pm} + K^{*o}(K_A^o)$ we get a sum rule similar to (4), namely

$$4\, g^2_{K^*K\pi} = \frac{\left(m^2_{K_A} - m^2_{K^*}\right)}{m^2_{K^*}} \left(\frac{k'}{m_{K_A}}\right)^2 \left(g'^2_L - 1.5\, g'^2_T\right) m^2_{K_A} \qquad (7)$$

where k' is the c.m. momentum in K_A-decay into $K^*\pi$ and g'_L, g'_T are appropriate coupling constants of K_A to K^* and π. Taking $g'_T = 0$, the width of K_A decay into $K^*\pi$ comes out to be 67 MeV, which is to be compared with the experimental value 80 ± 20 MeV, which is an overlap of two modes $K^*\pi$ and $K\rho$. As the phase-space favours the former mode, our result is consistent with experiment.

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