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Unitary Corrections to Current Algebra Versus Chiral Perturbation Calculations in Kaon-Pion Scattering

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Abstract

We compare, in kaon-pion scattering, chiral perturbation method and the unitarization program of current algebra. As occured in pion-pion, we show in this paper that, even having different number of free parameters, both methods lead to the same analytic structure for the amplitudes. The main difference between the two approaches resides in phase-shift definition. We reproduce in this paper a three parameter fit of experimental $K\pi$ S and P phase- shifts of the quasi-unitarized amplitude.

Key-words: Kaon-pion interaction; Chiral symmetry; Current algebra; Unitary.

Introduction

The low energy structure of Quantum Chromodynamics (QCD) is a basic problem for meson physics. We will consider two methods that aim to help in its understanding.

The method called chiral perturbation theory (Ch P T) consists in expanding the Green's functions of QCD in powers of momenta and of quark masses. As chiral symmetry implies a set of Ward identities which link the various Green's functions, it is possible to interrelate the expansion coefficients. To analyse the low energy structure of QCD, ChPT considers the non-linear sigma model coupled with external fields [(1)].

Even ignoring the underlying theory, in the early sixties, the chiral current algebra method implied a set of Ward identities that could be solved under suitable assumptions like saturation of axial divergences with meson poles [(2)].

It has been shown that tree level ChPT calculations are equivalent to the well known current algebra low-energy theorems. In order to go beyond threshold for meson processes, one must calculate ChPT in next-to-leading order, including loop diagrams. On the other hand unitarity corrections to soft-meson amplitudes allows one to access the resonance region for meson-meson scattering. As both of them follows from chiral symmetric Ward identities it is interesting to compare the results obtained by these two methods .

Consider, for instance, pion-pion scattering. One of us compared [(3)] the analysis made in ChPT context [(4)] with the result of current algebra unitarization method [(5)]. The conclusion is that both of them has the same analytic structure. The only difference is that our quasi-unitarized amplitude has three parameters and fits low energy experimental phase shifts whereas ChPT can not fit the experimental data with its two free constants that are not constrained by symmetry requeriments.

In this letter we are considering kaon-pion scattering. We conclude that the kaon-pion scattering amplitude derived by Bernard, Kaiser and Meißner [(6)] has the same analytic structure as the quasi-unitarized result, published long ago [(7)] .

We also show that, the polynomial parts of these amplitudes are different. Whereas ChPT has six free parameters , current algebra has only three parameters that are adjusted to fit low energy experimental $K\ \pi$ phase-shifts . The different phenomenological consequences of the two approaches are due to the fact that in ref.(6) the phase-shifts were defined from the real part of the amplitude while we addopted the usual definition. We will discuss this point in the conclusion.

II Comparison between the two methods

Current Algebra quasi-unitarized amplitude

We applied our current algebra unitarization method to kaon pion scattering. The starting point in our derivation was an exact expression for the four current correlation function with the quantum numbers of kaon and pion in terms of three- and two- point functions.

From this expression, by using vertex and propagators estimates, we could re-obtain the so-called soft meson Weinberg result[8] that, as expected, coincides with the tree level approximation named $T^{(2)}$ in formula 3.13 of ref. (7) .

The remaining of the amplitude is the equation 2.14 of ref.(7), namely:

$$
\begin{aligned}
&\overline{T}_{\alpha\beta\gamma\delta}(s,t,u) = -\frac{C^2_{K_A} C^2_{A_1}}{F^4} t_c(s,t,u) + d_{\alpha\gamma\epsilon} d_{\epsilon\beta\delta} f^\sigma_K(t)\Delta^\sigma(t) f^\sigma_\pi(t) + \\
&f_{\alpha\gamma\epsilon} f_{\epsilon\beta\delta}(s-u)\Big[\Big(\frac{1}{2F^2}S + 1 - f^v_K(t)\Big)\Delta^\rho(t)\Big(\frac{1}{2F^2} + 1 - f^v_\pi(t)\Big) - \frac{1}{4F^4}S\Big] + \\
&\Big\{-f_{\alpha\beta\epsilon} f_{\epsilon\gamma\delta}\Big[(q-p)\Big(\frac{1}{2F^2}S + 1 - f_+(s)\Big) + \\
&(q+p)\Big(f_-(s) + \frac{D^\kappa}{\Delta^\kappa}\tilde{f}_0(s) - \frac{1}{2F^2}(m^2_K - m^2_\pi)\tilde{D}^\kappa(s)\Big)\Big]\frac{1}{\overline{\Delta}^{K^*}(s)} \\
&\Big[(q'-p')\Big(\frac{1}{2F^2}S + 1 - f_+(s)\Big) + (q'+p')\Big(f_-(s) + \frac{D^\kappa}{\Delta^\kappa}\tilde{f}_0(s) - \frac{1}{2F^2}(m^2_K - m^2_\pi)\tilde{D}^\kappa(s)\Big)\Big] + \\
&\frac{1}{4F^4} f_{\alpha\beta\epsilon} f_{\epsilon\gamma\delta}\Big[(u-t)S + (m^2_K - m^2_\pi)^2 D^\kappa(s)\Big] + \tilde{f}^0(s)\Delta^\kappa(s)\tilde{f}^0(s) + (s \leftrightarrow u)\Big\}, \qquad (II.1)
\end{aligned}
$$

where $f_{\pm}$ are the K_{l3} form factors, $\tilde{f}^0$ and f^σ_m are the scalar form factors of the meson m, Δ^m are current propagators and S is the Schwinger term.

The current algebra unitarization program, proposed by one of us and applied to pion-pion and to kaon-pion scattering, consists in estimating the behaviour of form-factors and propagators at low energy. In this way we have assumed that f_+ and the eletromagnetic form factors are, near threshold, of the same order of magnitude as current algebra amplitudes while other functions are comparably smaller at low energies.

For example, equation 2.15 of ref (7) establishes this assumption,since we write, at $x \simeq (m_K + m_\pi)^2$, the K_{l3} form factors as $f_+(x) \simeq 1 + f_+^{(1)}(x)$ and $f_-(x) \simeq f_-^{(1)}(x)$.
All functions denoted by a superscript (1) are of the order $(m_K + m_\pi)^2/X^2$, X being of the order of magnitude of the vector meson mass in a vector dominance approximation and so, at low energies, they can be considered as corrections to the soft meson limit.

The main point in the construction of the amplitude is that, since current algebra gives real amplitudes, the corrected partial-wave has an imaginary part that is known to first order of the calculation. Namely,

$$Im T_{\ell I}^{(1)}(s) = \frac{1}{16\pi}\rho(x) T_{\ell I}^{CA}(s)^2,$$

where $T_{\ell I}^{CA}$ is the partial-wave ℓ , with isospin I soft meson current algebra amplitude obtained from eq. 2.8 of ref.(7).

The absorptive part of partial-wave amplitudes comes from elastic unitarity used in a peculiar way for form-factors and propagators. For instance,

$$Im f_+^{(1)}(x) = \frac{1}{16\pi}\rho(x) T_{11/2}^{K\pi}(x)$$

where $\rho(x) = \frac{1}{x}\left[x - (m_K + m_\pi)^2\right]^{1/2}\left[x - (m_K - m_\pi)^2\right]^{1/2}$.

Collecting first order corrected form-factors and propagators determined via dispersion relation technique, we get the final expression for the isospin 3/2 kaon-pion scattering amplitude presented in the appendix of ref. (7):

$$T^{(1)}_{3/2}(s,t,u) = \frac{1}{2F^2}(M^2-s) + \frac{1}{4F^4}(s-M^2)\left[(s-M^2)G(s) + \frac{s}{32\pi^2}\frac{M^4}{m^4}\right] +$$
$$\frac{1}{F^4}\xi_1(t-2m_K^2)(t-2m_\pi^2) + \frac{1}{F^4}(\xi_2-\xi_3)(s-M^2)^2 + \frac{1}{F^4}(\xi_2+\xi_3)(u-M^2)^2 +$$
$$\frac{1}{4F^4}(u-s)\left[(2m_K^2-t)g_K(t) - \frac{t}{96\pi^2}\right] +$$
$$\frac{1}{12F^4}(u-M^2)\left[(u-M^2)G(u) + \frac{u}{32\pi^2}\frac{M^4}{m^4}\right]$$
$$\frac{1}{32F^4}(t-s+\frac{m^4}{u})\left[(u-2M^2+\frac{m^4}{u})G(u) + \frac{u}{32\pi^2}(\frac{3}{2}\frac{M^4}{m^4}+\frac{1}{3}) - \frac{M^2}{32\pi^2}\right] +$$
$$\frac{1}{96\pi^2}(5u-2M^2-3\frac{m^4}{u})\left[(5u-2M^2-3\frac{m^4}{u})G(u) + \frac{u}{32\pi^2}(\frac{7}{2}\frac{M^4}{m^4}-1) + \frac{3}{32\pi^2}M^2\right] \quad (II.2)$$

In this expression ξ_1 , ξ_2 and ξ_3 are "seagull" free parameters, F is the pion decay constant, here considered equal to the kaon decay constant,

$M^2 = m_K^2 + m_\pi^2, \quad m^2 = m_K^2 - m_\pi^2$ and

$$16\pi^2 G(x) = -\rho(x) ln\frac{x - M^2 + x\rho}{2m_K m_\pi} + (\frac{M^2}{m^2} - \frac{m^2}{x}) ln\frac{m_K}{m_\pi} + 1 + i\pi\rho(x).$$

Chiral perturbation calculation

Meson-meson transition amplitudes to second-order in the momenta and quark masses can be evaluated by expanding a non-linear sigma-model Lagrangian L_σ in powers of the fields. The tree diagrams derived in this way give rise to the current algebra predictions up to order $O(p^2, m_m^2)$, with p denoting an external momentum and m_m the meson masses.

To go further, as required by unitarity, $O(p^4, p^2 m_m^2, m^4)$ corrected amplitudes are to be found. These have different sources, namely tadpole graphs and loop diagrams with vertices from L_σ as well as higher order in field derivative terms tree graph .

The arbitrary coupling constants allow one to absorb all divergences of one loop diagrams. This is a very important point firstly conjectured by Weinberg[(9)] .

The application of ChPT to kaon-pion scattering performed in ref. (6) follows exactly these lines. The corrections to current algebra come from loop diagrams, tadpole and higher order couplings tree graphs. The six renormalized couplings, denoted by L^r , as well as tadpole contributions, depend on a renormatization scale μ .

The T matrix calculated from the effective action can be written in terms of physical masses and of physical decay constants. In the following we present the isospin 3/2 $K\pi$ amplitude obtained in ref. (6) in a form that is convenient for us in order to compare with expression $(II.2)$.

$$
\begin{aligned}
T_{3/2}(s,t,u) = &\frac{1}{2F^2}(M^2-s)+\frac{1}{4F^4}(s-M^2)^2\overline{J}(s)+\\
&\frac{2}{F^4}(L_1^r+L_3^r)(t-2m_K^2)(t-2m_\pi^2)+\frac{1}{F^4}L_2^r(s-M^2)^2+\frac{2}{F^4}(2L_2^r+L_3^r)(u-M^2)^2+\\
&\frac{1}{32F^4}\left[(t-s+\frac{m^4}{u})(u-2M^2+\frac{m^4}{u})-(5u-2M^2-3\frac{m^4}{u})\frac{m^4}{u}+(11u^2-12M^2u+4M^4)\right]\overline{J}(u)+\\
&\frac{1}{24F^4}\left[(u-s)(t-4m_\pi^2)+3t(2t-m_\pi^2)\right]\overline{J}_\pi(t)+\frac{1}{48F^2}\left[(u-s)(t-4m_K^2)+9t^2\right]\overline{J}_K(t)+\\
&\frac{2}{F^4}L_4^rM^2t-\frac{1}{F^4}L_5^r(s+m^2)+\frac{2}{F^4}(2L_6^r+L_8^r-4L_4^r)(M^4-m^4)+\\
&\frac{1}{16F^2}\left[\mu_K(4M^2-4s+m^2)+2\mu_\pi(5s-5M^2-m^2)\right]+\\
&\frac{1}{16F}(\mu_k-\mu\pi)\left[(2s-3u-14t+2M^2-2m^2)+su-(11u^2-12M^2u+4M^4)\right]+\\
&\frac{1}{768\pi^2F^2}\left[(t-s+\frac{m^4}{u})(u-3M^2)+2t(u-s)\right] \qquad (II.3)
\end{aligned}
$$

The comparison of the last expression with eq (II.2) leads us to make the following observations :

i) We can identify $\overline{J}$ as the real part of the function $G(x)$ and $\overline{J}_K(t)=2\ Re\ g_K(t)$.

ii) We have omitted the terms corresponding to loop diagrams including $\eta\eta$ and $K\eta$ intermediate states because, as checked in ref. 6, they introduce corrections up to 3 % .

iii) We can relate the low-energy parameters of the two amplitudes:

$\xi_1 = 2(L_1^r + L_3^r) \quad 2\xi_2 = 2L_3^r + 5L_2^r \quad$ and $2\xi_3 = 3L_2^r + 2L_3^r$.

iv) The remaining parameters as well as the scales μ_K and μ_π have no place in our final amplitude.

v) The coefficients of $\overline{J}(u)$ and $G(u)$ in these expressions are the same and this is a consequence of the correct crossing properties of amplitudes derived from Ward identities.

vi) In our final expression the t-channel exchanges are somewhat incorrect for we did not include the pion and kaon electromagnetic form-factors that appear explicitly in formula (II.1). We claim that this error has minor consequence on experimental data fitting. For this we present in the figures the results including t-channel exchanges of ref. (6) and the results corresponding to no t-channel exchanges at all.

vii) The polynomial part of the quasi-unitarized amplitude comes from the regularization of dispersion relation integrals whereas that of chiral perturbation are due to tadpole calculation. We also see that the number of free parameters of ChPT exceeds that of quasi-unitarized amplitude in three.

We would like to emphasize that the basic structure of the amplitudes is the same. The main difference between the two approaches resides in the definition of phase-shifts δ . As elastic unitarity is not satisfied in either of the amplitudes, the definition of partial-wave phase-shift is quite arbitrary.

We have adopted for partial-wave ℓ , isospin I phase-shift $\delta_{\ell I}$ the definition

$$\tan \delta_{\ell I} = \frac{Im\ T_{\ell I}}{Re\ T_{\ell I}},$$

the quasi-unitarized amplitude satisfies $\quad Im\ T_{\ell I}^{(1)} = \rho\ |T_{\ell I}^{CA}|^2$.

The authors of ref. (6) prefered adopting the definition $\delta_{\ell I} = \rho\ Re\ T_{\ell I}$, which is valid only for very small values of the phase-shift.

Conclusion

In this paper we have extended to kaon-pion scattering the comparison between chiral perturbation calculations and the unitarization program of current algebra .

Exactly as in the previous analysis made in pion-pion scattering, we have shown that both methods lead to the same analytic structure for the amplitudes, in spite of having different number of free parameters.

In respect to phase-shift definition, we would like to emphasize that current algebra gives real amplitudes. Thus any method, intending to go beyond threshold for meson processes, must explore the imaginary part it develops. One of the advantages of the phase-shift definition used in the current algebra unitarization program is the possibility of exploring the resonance region. It is interesting to note that our approach leads to a good fit for K^* resonance and S-wave using only three parameters.

One of us (J. Sá Borges) would like to thank CBPF for hospitality.

Figure Captions

Fig. 1. Isospin 1/2 P-wave phase-shifts. Solid line is our result, dot-dash line is our result without any contribution to t-channel and short-dash line corresponds to the inclusion in our amplitude of t-channel exchange of ref. 6 . The experimental points are from ref. 10.

Fig. 2. Isospin 1/2 and 3/2 S-wave phase-shifts with the same convention as in Fig. 1 for line drawing. The experimental data are indicated by circles (ref. 11) and triangles (ref. 12).

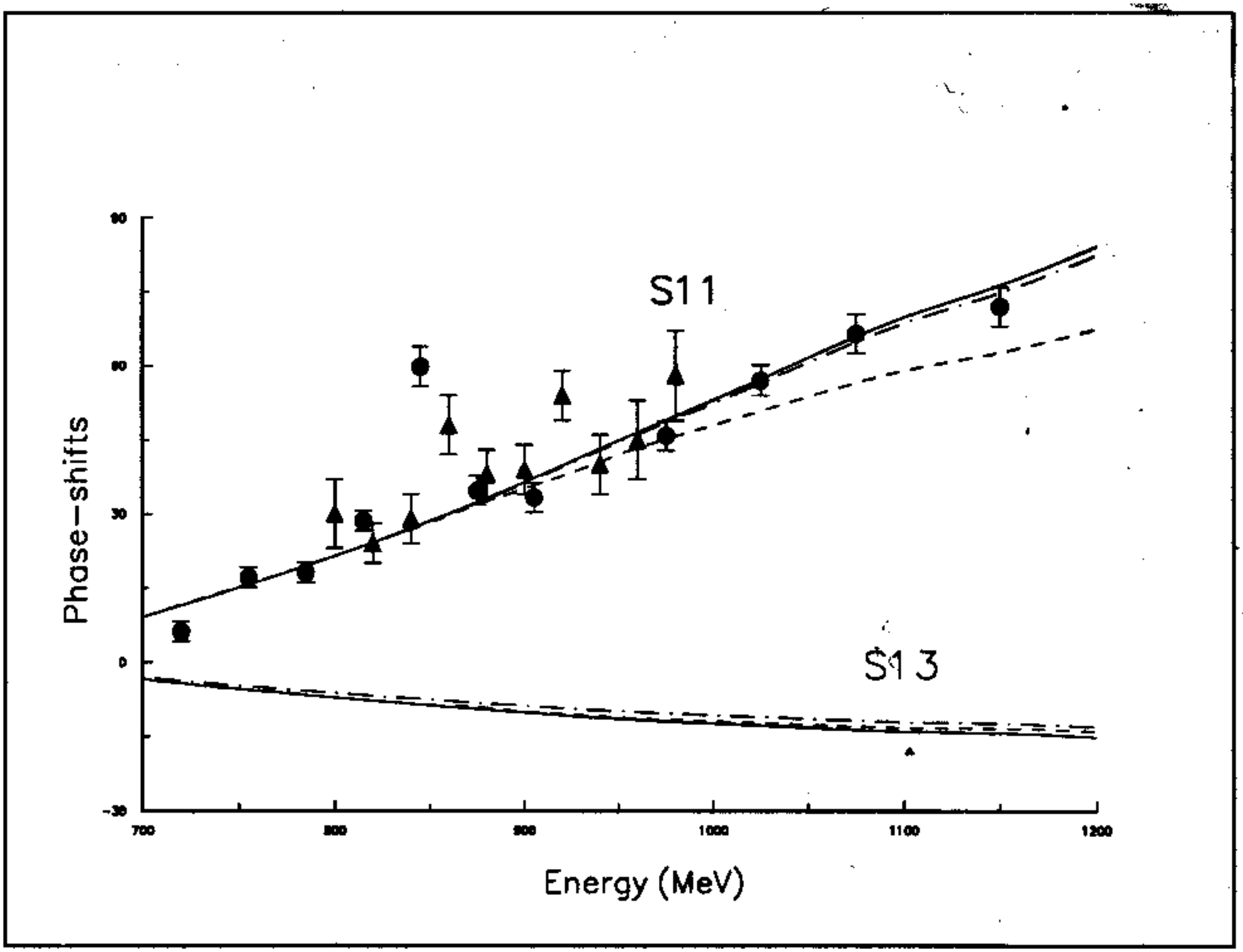
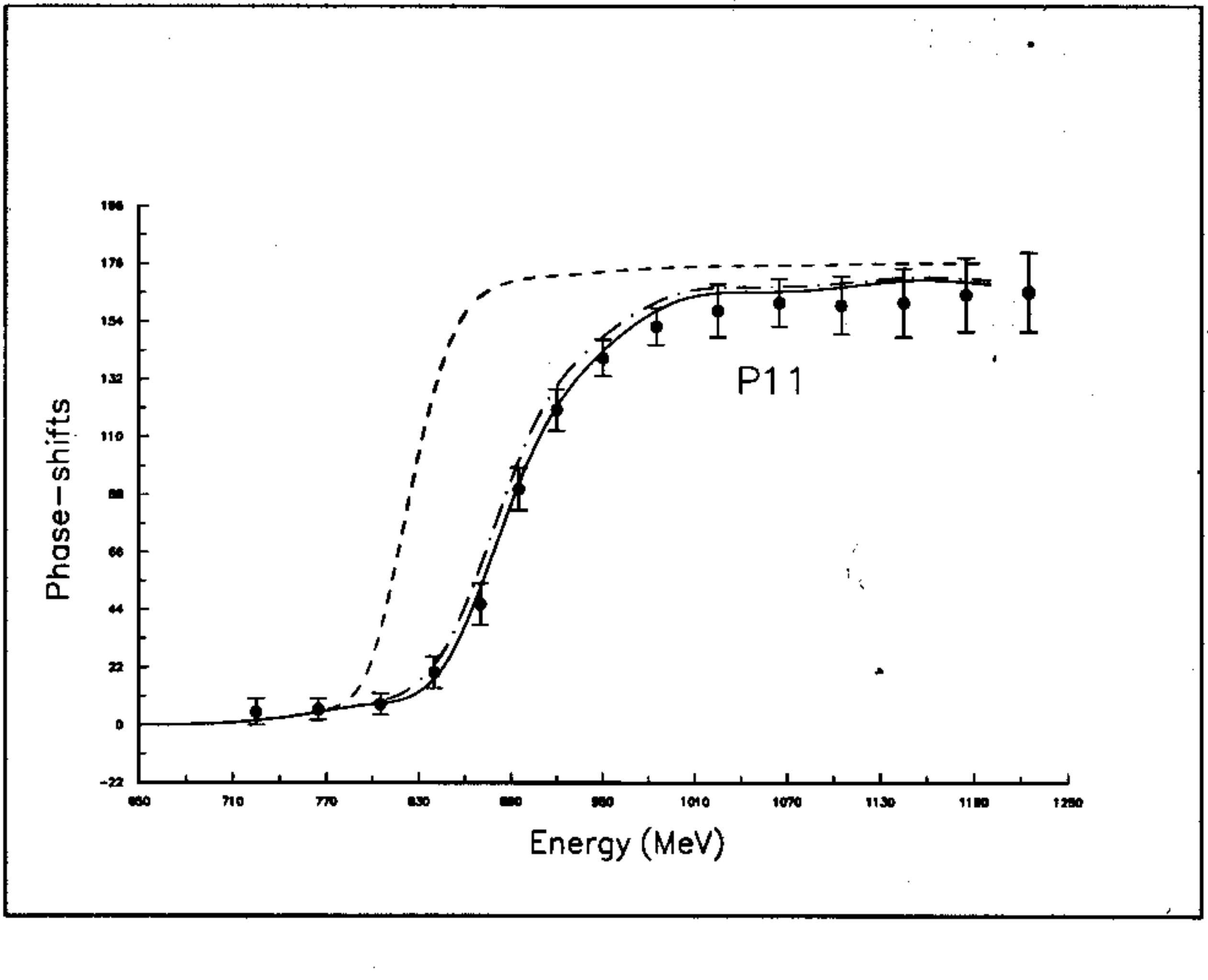

90
60
30
0
-30
700
800
900
1000
1100
1200
S11
S13
Phase-shifts
Energy (MeV)

FIG. 1


Phase-shifts
Energy (MeV)
P11

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