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WEYL INTEGRABLE SPACE TIME : A MODEL OF OUR COSMOS?

by

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Abstract

We introduce a Weyl Integrable Space Time as a consequence of a dynamical process.

We study the particular case of a vector field non minimally coupled to gravity. The consequences for cosmology are considered.

We find that it is possible to connect one Riemannian Space-time with another Riemannian space time by a series of Weyl integrable manifold.

It is well-known that Einstein's field equations are derivable from a Lagrangian $\mathcal{L}_E \equiv \sqrt{-g}\, R$, if one varies the metric $g_{\mu\nu}$. However, it is also possible to start from an arbitrary affine geometry and, using Palatini's method, varying independently the metric $g_{\nu\mu}$ and the affine object $\Gamma^{\alpha}_{\mu\nu}$ to obtain

$$g_{\mu\nu;\lambda} = g_{\mu\nu,\lambda} - \Gamma^{\varepsilon}_{\mu\lambda} g_{\varepsilon\nu} - \Gamma^{\varepsilon}_{\nu\lambda} g_{\mu\varepsilon} = 0 \qquad (1a)$$

$$R_{\mu\nu} = 0 \qquad (1b)$$

respectivelly.

The advantage of this procedure is to introduce the dependence of the structure of the geometry on dynamics. In general, for the full Lagrangian of geometry and matter, we have that L is a simple sum

$$L = L_E + L_M \qquad (2)$$

If L_M contains any kind of fields coupled minimally to gravity variation ã la Palatini implies the same relation (1a) and consequently we assume the riemannian structure of space-time by means of a prescribed dynamic process.

Elsewhere [1] we have already pursued the idea of a modified Lagrangian, where electromagnetism and gravitation are coupled non-minimally. The Lagrangian we considered reads

$$L = \sqrt{-g}\, \left[- \frac{1}{4} f_{\mu\nu} f^{\mu\nu} - R A_{\mu} A^{\mu} - \frac{1}{k} R \right] \qquad (3)$$

We have shown there that an interesting (eternal) universe without singularity is a possible solution of the equations which follow from the variation of $g_{\mu\nu}$ ã la Einstein.

Here we pursue the idea that the variation is to be performed ã la Palatini. In this case

$$g_{\mu\nu;\lambda} = g_{\mu\nu,\lambda} - \Gamma^{\varepsilon}_{\mu\lambda} g_{\varepsilon\nu} - \Gamma^{\varepsilon}_{\nu\lambda} g_{\mu\varepsilon} = -\frac{(A^2)_{,\lambda}}{A^2} g_{\mu\nu} \tag{4}$$

where

$A^2 \equiv A_\mu A^\mu$

Here $A_\mu = (A_0, A_1, A_2, A_3)$ is an eletromagnetic four-vector potential and in the Riemannian case we found that "longitudinal photons $(A_0 \neq 0, A_i = 0)$ are capable of curving space-time, unifying thereby electromagnetism and gravity [2].

In the Palatini case we obtain a Weyl Integrable Space Time (WIST), since equation (4) reads now

$$g_{\mu\nu;\lambda} = -\nabla_\lambda \phi \, g_{\mu\nu} \tag{5}$$

with $\phi := \ell n\, A^2$, which is a necessary and sufficient condition for a WIST. This WIST is conformally Riemannian, i.e., we can write

$$ds^2_{WIST} = \Omega^2(x)\, ds^2_{RIEM} \tag{6}$$

Besides equation (4) the remaining set of equations obtained from (3) are:

$$f^{\mu\nu}{}_{\|\nu} = -R\, A^\mu \tag{7a}$$

in which double bar means covariant derivative in the riemannian part of the connection.

$$\left[\frac{1}{k} + A^2\right] G_{\mu\nu} = -E_{\mu\nu} - R A_\mu A_\nu - T^*_{\mu\nu} \tag{7b}$$

in which $E_{\mu\nu} \equiv f_{\mu\alpha} f^{\alpha}{}_{\nu} + \frac{1}{4} g_{\mu\nu} f_{\alpha\beta} f^{\alpha\beta}$ and $T^*_{\mu\nu}$ is the energy-momentum tensor for the matter (minimally coupled to gravity). We note that in order for the space time to be Riemannian we must have $A^2 =$ constant.

This case has been considered before in a completely differente context (and independently of each other) by Dirac [3] and Nambu [4,5].

Dirac suggested a new classical theory of the electron. The main argument rests or the fact that breaking the gauge symmetry of electrodynamics enables us to use the superfluous variables to describe electric charges. Set $A_\mu A^\mu \equiv A^2 = \beta^2$. Defining $V_\mu = \frac{1}{\beta} A_\mu$ we define a velocity field. Dirac argues then that its physical significance is nothing but the velocity which any charged particle flow has if its introduced in such an electromagnetic field.

Nambu examined the quantum case and arrived at the conclusion that $A^2 =$ constant implies a spontaneous breakdown of Lorentz invariance, which then could help in the interpretation of the photon as a Goldstone boson, being the massless excitations associated to the break of symmetry.

Both authors found that the requirement for consistency with experiments leads us to set $\beta = \frac{m}{e}$. Our equations of motion (which follow from $T^{\mu\nu}{}_{;\nu} = 0$) modify Dirac-Nambu in only one essential point: it introduces a cosmological rest mass for the photon.

Therefore our theory agrees completely with standard electrodynamics as far as laboratory experiments are concerned. We note further that our theory incorporates one essential aspect of some recent work of Canuto and co-workers: atomic clocks and gravitational clocks are related in different places by A^2. Looking at equation (7b) we see that the gravitational constant is renormalized in our new theory and this is exactly the reason why non-singular cosmological solutions arise in this theory.

We note, furthermore, that it is possible to obtain assymptotic regimes in which the norm A^2 remains constant. In these regions the structure of space time is Riemannian. Thus it is possible that through a sequence of Weyl Integrable Space Times a Riemannian space-time becomes related assymptotically to a different Riemannian space-time.

This is along the lines of some recent work of Zel'dovich [6], Guth [7] and others who also tried to avoid the Big Bang singularity by introducing a sufficiently large cosmological "constant" Λ which however is not constant as it is thought to be zero or very small to-day (and initially represented the pseudo energy momentum tensor of fluctuations of empty space-time). We arrive here at a somewhat different scheme by enlarging the mathematical structure of space time. Already the most trivial solution of our set of field equations contains a free function, which can be used to make the "age" of our universe arbitrarily large and provides thereby possibly a viable model of a cosmos of biological age. Further study of the subject is in progress.

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