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ON THE DOUBLET THEORY OF STRONG INTERACTIONS

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ON THE DOUBLET THEORY OF STRONG INTERACTIONS

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__ABSTRACT__: A general procedure for constructing interactions of baryons with bosons when various particles have different parities is given using a doublet scheme for baryons, in which the doublets are eigenstates of parity operator.

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I - INTRODUCTION:

Since the extension of the well known iso-spin concept for nucleons and pions to Hyperons and K mesons by Gell-Mann[1] and Nishijima[2] charge independence for the strong interaction of baryons with pions and K mesons has been discussed by many authors[3] [4]. In its earlier versions it has been customary to regard nucleons and Ξ hyperon to form isospinors while Σ hyperon to form an isovector and Λ an isoscalar under rotations in the three dimensional charge space. The pion is regarded likewise an isovector while K mesons are iso-spinors. Tiomno and, independently, Gell-Mann, Schwinger and Pais suggested that if we disregard the mass difference of Σ and Λ and assume the same parity for all the baryons we could, with the advantage of uniformity (fermions being iso-fermions and bosons being isobosons), as well, describe the eight baryons by four two-isospinors[4]:

$$N_1 = \begin{pmatrix} p \\ n \end{pmatrix} \quad N_2 = \begin{pmatrix} \Sigma_+ \\ Y_o \end{pmatrix} \quad N_3 = \begin{pmatrix} Z_o \\ \Sigma_- \end{pmatrix} \quad N_4 = \begin{pmatrix} \Xi_o \\ \Xi_- \end{pmatrix}$$

where $Y_o = (\Lambda^o - \Sigma_o)/\sqrt{2}$ and $Z_o = (\Lambda^o + \Sigma_o)/\sqrt{2}$. The charge independent interaction could then be easily written down similar to the case of nucleons (see section II). However, when the parities of various baryons are different (say Σ has parity opposite to that of Λ) the above doublets are hard to interpret. Moreover, we are required to use a γ_5 inside the doublet with the Σ or Λ in order to be able to use the doublets for constructing the interaction. The use of γ_5 is objectionable as it reverses the sign of energy and anticommutes with the baryon number and strangeness. In the next section we will give an alternative procedure using γ_4 (Parity operation) instead which does not take particle into its anti-particle. We will construct baryon doublets $N_i^{(\pm)}$ (i = = 1, 2, 3, 4) which are eigenstates of parity operator P:

$$P\, N_i^{(\pm)}\, P^{-1} = \eta N_i^{(\pm)} \qquad ; \quad \eta = (\pm)$$

and will use these doublets for constructing the interactions, both parity conserving and parity nonconserving ones. We will illustrate the procedure by considering the interaction with pseudoscalar pion, K meson and vector meson fields. The extension to other cases is obvious.

II - INTERACTION FOR SAME BARYON PARITIES.

The charge independent pion interaction in the doublet theory is written

$$[\pi] = \sum_{j=1}^{4} G_j\, \overline{N}_j\, \vec{\tau}\, i\, \gamma_5\, N_j \cdot \vec{\pi}$$

where $\vec{\tau}$ are the Pauli matrices. Written explicitly

$$[\pi] = i\, G_1 \left[\sqrt{2}(\overline{p}\, \gamma_5\, n\, \pi_+ + \overline{n}\, \gamma_5\, p\, \pi_-) + (\overline{p}\, \gamma_5\, p - \overline{n}\, \gamma_5\, n)\, \pi_o\right]$$

$$+ i\, G_4 \left[\sqrt{2}(\overline{\Xi}_o\, \gamma_5\, \Xi_-\, \pi_+ + \overline{\Xi}_-\, \gamma_5\, \Xi_o\, \pi_-) + (\overline{\Xi}_o\, \gamma_5\, \Xi_o - \overline{\Xi}_-\, \gamma_5\, \Xi_-)\, \pi_o\right]$$

$$+ i\, F_2 \left[(- \overline{\Sigma}_+\, \gamma_5\, \Sigma_o\, \pi_+ + \overline{\Sigma}_-\, \gamma_5\, \Sigma_o\, \pi_-) + h.c. + (\overline{\Sigma}_+\, \gamma_5\, \Sigma_+ - \overline{\Sigma}_-\, \gamma_5\, \Sigma_-)\pi_o\right]$$

$$+ i F_2\left[\bar{\Sigma}_+ \gamma_5 \Lambda_o \pi_+ + \bar{\Sigma}_- \gamma_5 \Lambda_o \pi_- + \bar{\Sigma}_o \gamma_5 \Lambda_o \pi_o\right] + h.c.$$

$$+ i F_3\Big|(- \bar{\Sigma}_+ \gamma_5 \Sigma_o \pi_+ - \bar{\Sigma}_- \gamma_5 \Sigma_o \pi_-) + h.c. + (\bar{\Sigma}_+ \gamma_5 \Sigma_+ + \bar{\Sigma}_- \gamma_5 \Sigma_-)\pi_o$$

$$- (\bar{\Sigma}_o \gamma_5 \Sigma_o + \bar{\Lambda}_o \gamma_5 \Lambda_o) \pi_o \Big]$$

$$+ i F_3\left[\bar{\Sigma}_+ \gamma_5 \Lambda_o \pi_+ - \bar{\Sigma}_- \gamma_5 \Lambda_o \pi_-\right] + h.c.$$

where $G_2 = (F_2 + F_3)$ and $G_3 = (F_2 - F_3)$.

The charge independent form in which Σ is iso-vector and Λ iso-scalar is obtained by putting $F_3 = 0$. The doublet form thus gives interaction more general than the usual charge independent form; the coefficient of F_3, however, is not charge independent in the usual iso-spin scheme for Σ and Λ. It is also worthwhile noting that the coupling constants for $\Sigma\Sigma\pi$ and $\Sigma\Lambda\pi$ interactions turn out to be the same both in the case when $F_3 = 0$, $F_2 \neq 0$ and $F_3 \neq 0$, $F_2 = 0$.

III - INTERACTION FOR DIFFERENT PARITIES OF BARYONS.

The above mentioned doublets become hard to interpret physically when the parities of the various baryons are different. We will now give a general procedure to be adopted in these cases, and which includes the above one (section II). Instead of the above doublets we will construct doublets which are eigenstates of parity operation and use them for building the interaction, both parity conserving and parity non-conserving.

We will illustrate the procedure by considering the special case when Λ and Ξ have the same parity as the nucleons while Σ's have opposite parity, e.g.

$$P\,\psi(\vec{x},t)\,P^{-1} = \eta\,\gamma_4\,\psi(-\vec{x},t)$$

where $\eta = +1$ for N, Λ and Ξ and $\eta = -1$ for Σ. It follows

$$P\,\psi_{Y_o}(\vec{x},t)\,P^{-1} = +\gamma_4\,\psi_{Z_o}(-\vec{x},t)$$

$$P\,\psi_{Z_o}(\vec{x},t)\,P^{-1} = +\gamma_4\,\psi_{Y_o}(-\vec{x},t)$$

where $\psi_{Y_o} = (\psi_{\Lambda_o} - \psi_{\Sigma_o})/\sqrt{2}$ and $\psi_{Z_o} = (\psi_{\Lambda_o} + \psi_{\Sigma_o})/\sqrt{2}$.

The parity operation thus takes Y_o to Z_o and vice versa.

It is clear that we can write every field as

$$\psi = \left[\psi^{(+)} + \psi^{(-)}\right]$$

where

$$P\,\psi^{(\pm)}(\vec{x},t) = \pm\,\eta\,\psi^{(\pm)}(-\vec{x},t)$$

with $\eta = +$ for N, Λ, Ξ.

$= -$ for Σ.

In fact

$$\psi^{(\pm)} = \frac{1}{2}\,(1 \pm \gamma_4)\,\psi$$

$$\gamma_4\,\psi^{(\pm)} = \pm\,\psi^{(\pm)}$$

In momentum space $\psi^{(\pm)}$ corresponds to the state symmetrized with respect to the two momentum directions ($\vec{p}$ and $-\vec{p}$) while $\psi^{(-)}$ to the antisymmetrized state; the energy of the particle remains fixed in sign a characteristic of the γ_4 operation which does not affect the time component of a four vector.

A similar separation can be affected for ψ_{Y_o} and ψ_{Z_o} as well. In the case under consideration we must clearly define them as follows:

$$\psi_{Y_o}^{(\pm)} = \left[\psi_{\Lambda}^{(\mp)} - \psi_{\Sigma_o}^{(\pm)}\right]/\sqrt{2} \quad ; \quad \psi_{Y_o} = \left[\psi_{Y_o}^{(+)} + \psi_{Y_o}^{(-)}\right]$$

$$\psi_{Z_o}^{(\pm)} = \left[\psi_{\Lambda}^{(\mp)} + \psi_{\Sigma_o}^{(\pm)}\right]/\sqrt{2} \quad ; \quad \psi_{Z_o} = \left[\psi_{Z_o}^{(+)} + \psi_{Z_o}^{(-)}\right]$$

in order to be able to use them with Σ hyperons to form doublets. We have

$$P\,\psi_{Y_o}^{(\pm)}(x)\,P^{-1} = \mp\psi_{Y_o}^{(\pm)}(-\vec{x},t) \quad ; \quad P\,\psi_{Z_o}^{(\pm)}(x)\,P^{-1} = \mp\psi_{Z_o}^{(\pm)}(-\vec{x},t)$$

same as in the case of Σ. Thus we can define the following doublets which are simultaneously the eigenstates of parity operator

$$N_1^{(\pm)} = \begin{pmatrix}\psi_p^{(\pm)}\\ \psi_n^{(\pm)}\end{pmatrix} \qquad N_2^{(\pm)} = \begin{pmatrix}\psi_{\Sigma_+}^{(\pm)}\\ \psi_{Y_o}^{(\pm)}\end{pmatrix} \qquad N_3^{(\pm)} = \begin{pmatrix}\psi_{Z_o}^{(\pm)}\\ \psi_{\Sigma_-}^{(\pm)}\end{pmatrix} \qquad N_4^{(\pm)} = \begin{pmatrix}\psi_{\Xi_o}^{(\pm)}\\ \psi_{\Xi_-}^{(\pm)}\end{pmatrix}$$

$$P\,N_{1,4}^{(\pm)}(\vec{x},t)\,P^{-1} = \pm N_{1,4}^{(\pm)}(-\vec{x},t)$$

$$P\,N_{2,3}^{(\pm)}(\vec{x},t)\,P^{-1} = \mp N_{2,3}^{(\pm)}(-\vec{x},t)$$

It may be remarked that $\psi_{Y_o}^{(\pm)}$ and $\psi_{Z_o}^{(\pm)}$ are not eigenstates of γ_4 and cannot be thus expressed in the above mentioned explicit form for pure fields. In fact

$$\gamma_4 \psi_{Y_o}^{(\pm)} = \mp \psi_{Z_o}^{(\pm)}$$

(a) - Pion Interaction

The charge independent strong interaction Hamiltonion for pseudo-scalar pion can then quite generally be written in terms of the above doublets as follows

$$[\pi]' = \sum_{j=1}^{4} \Big\{ G_j \Big[\overline{N}_j^{(+)} (i\gamma_5 + \xi_j) \vec{\tau} N_j^{(-)} + \overline{N}_j^{(-)} (i\gamma_5 + \xi_j) \vec{\tau} N_j^{(+)} \Big] \cdot \vec{\pi}$$
$$+ G_j \Big[\overline{N}_j^{(+)} (i\gamma_5 + \xi_j') \vec{\tau} N_j^{(+)} + \overline{N}_j^{(-)} (i\gamma_5 + \xi_j') \vec{\tau} N_j^{(-)} \Big] \cdot \vec{\pi} \Big\}$$

Here G_j represent the coupling constant for parity conserving interaction while G_j' those for Parity nonconserving terms. Using the fact

$$N_{1,4}^{(\pm)} = \frac{1}{2} (1 \pm \gamma_4) N_{1,4}$$

the interaction reduces to (considering only parity conserving part for convenience)

$$[\pi]' = G_1 \overline{N}_1 \, i\gamma_5 \vec{\tau} N_1 \cdot \vec{\pi} + G_4 \overline{N}_4 \, i\gamma_5 \vec{\tau} N_4 \cdot \vec{\pi}$$

$$+ G_2 \Big[\sqrt{2} \, (\overline{\Sigma}_+^{(+)} (i\gamma_5 + \xi_2) Y_o^{(-)} \pi_+ + \overline{Y}_o^{(+)} (i\gamma_5 + \xi_2) \Sigma_+^{(-)} \pi_-)$$

$$+ (\overline{\Sigma}_+^{(+)} \, i\gamma_5 \Sigma_+^{(-)} - \overline{Y}_o^{(+)} (i\gamma_5 + \xi_2) Y_o^{(-)} + ((+) \longleftrightarrow (-)) \Big]$$

$$+ G_3\Big[\sqrt{2}(\bar{Z}_o^{(+)}(i\gamma_5+\xi_3)\Sigma_-^{(-)}\pi_+ + \bar{\Sigma}_-^{(+)}(i\gamma_5+\xi_3)Z_o^{(-)}\pi_-)$$

$$+ (\bar{Z}_o^{(+)}(i\gamma_5+\xi_3)Z_o^{(-)} - \bar{\Sigma}_-^{(+)}(i\gamma_5+\xi_3)\Sigma_-^{(-)})\pi_o\Big]$$

If Σ also has the same parity as Λ, $[\pi]'$ reduces to the case discussed in the introduction *. However, in the present case $\bar{N}_2^{(+)} N_2^{(-)}$ and $\bar{N}_3^{(+)} N_3^{(-)}$ do not vanish and we obtain for the $\Sigma\Sigma\pi$ and $\Sigma\Lambda\pi$ interactions the form

$$g_2\Big[(-\bar{\Sigma}_+ i\gamma_5 \Sigma_o \pi_+ + \bar{\Sigma}_- i\gamma_5 \Sigma_o \pi_-) + h.c. + (\bar{\Sigma}_+ i\gamma_5 \Sigma_+ - \bar{\Sigma}_- i\gamma_5 \Sigma_-)\pi_o\Big]$$

$$+ g_2'\Big[\bar{\Sigma}_+ \Lambda_o \pi_+ + \bar{\Sigma}_- \Lambda_o \pi_- + \bar{\Sigma}_o \Lambda_o \pi_o\Big] + h.c.$$

$$+ g_3\Big[(-\bar{\Sigma}_+ i\gamma_5 \Sigma_o \pi_+ - \bar{\Sigma}_- i\gamma_5 \Sigma_o \pi_-) + h.c.$$

$$+ (\bar{\Sigma}_+ i\gamma_5 \Sigma_+ + \bar{\Sigma}_- i\gamma_5 \Sigma_-)\pi_o - (\bar{\Sigma}_o i\gamma_5 \Sigma_o + \bar{\Lambda}_o i\gamma_5 \Lambda_o)\pi_o\Big]$$

$$+ g_3'\Big[\bar{\Sigma}_+ \Lambda_o \pi_+ - \bar{\Sigma}_- \Lambda_o \pi_-\Big] + h.c.$$

with $G_2 = (g_2+g_3)$, $G_3 = (g_2-g_3)$, $G_2\xi_2 = (g_2'+g_3')$, $G_3\xi_3 = (g_2'-g_3')$ and a similar expression for the parity nonconserving part. The usual charge independent form is obtained by putting $g_3 = g_3' = 0$. It may be remarked that in this case the doublet

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* In this case $Y_o^{(\pm)} = \dfrac{(\Lambda_o^{(\pm)} - \Sigma_o^{(\pm)})}{\sqrt{2}}$ and $Z_o^{(\pm)} = \dfrac{(\Lambda_o^{(\pm)} + \Sigma_o^{(\pm)})}{\sqrt{2}}$.

theory allows for different coupling constants for the $\Sigma\Sigma\pi$ and $\Sigma\Lambda\pi$ interactions for both cases when $g_3 = g_3' = 0$ or when $g_2 = g_2' = 0$ in contrast to the case when Σ and Λ have the same parity.

(b) - K-meson interaction

The charge independent interaction with the K mesons (K is isoscalar in doublet scheme) is easily obtained (assuming K pseudoscalar for example)

$$[K] = \sqrt{2}\Big[f_1\, \overline{N}_1^{(+)}(i\gamma_5+\lambda_1)\, N_2^{(+)}\, K_o + f_2\, \overline{N}_1^{(+)}(i\gamma_5+\lambda_2)\, N_3^{(+)}\, K_+ +$$

$$+ f_3\, \overline{N}_4^{(+)}(i\gamma_5+\lambda_3)\, N_2^{(+)}\, \overline{K}_+ + f_4\, \overline{N}_4^{(+)}(i\gamma_5+\lambda_4)\, N_3^{(+)}\, \overline{K}_o + ((+)\leftrightarrow(-))\Big]$$

which can be rewritten as:

$$[K] = F_1\left[\overline{p}\, i\gamma_5\, \Lambda_o\, K_+ + \overline{n}\, i\gamma_5\, \Lambda_o\, K_o\right] + h.c.$$

$$+ F_2\left[\sqrt{2}\,(\overline{p}\, \Sigma_+\, K_o + \overline{n}\, \Sigma_-\, K_+) + \overline{p}\, \Sigma_o\, K_+ - \overline{n}\, \Sigma_o\, K_o\right] + h.c.$$

$$+ F_3\left[\overline{\Xi}_-\, i\gamma_5\, \Lambda_o\, K_+ - \overline{\Xi}_o\, i\gamma_5\, \Lambda_o\, \overline{K}_o\right] + h.c.$$

$$+ F_4\left[\sqrt{2}\,(\overline{\Xi}_o\, \Sigma_+\, \overline{K}_+ - \overline{\Xi}_-\, \Sigma_-\, \overline{K}_o) - \overline{\Xi}_o\, \Sigma_o\, \overline{K}_o - \overline{\Xi}_-\, \Sigma_o\, \overline{K}_+\right] + h.c.$$

$$+ F_1'\left[-\,\overline{p}\, i\gamma_5\, \Lambda_o\, K_+ + \overline{n}\, i\gamma_5\, \Lambda_o\, K_o\right] + h.c.$$

$$+ F_2'\left[\sqrt{2}\,(\overline{p}\, \Sigma_+\, K_o - \overline{n}\, \Sigma_-\, K_+) - \overline{p}\, \Sigma_o\, K_+ - \overline{n}\, \Sigma_o\, K_o\right] + h.c.$$

$$+ F_3'\left[\bar{\Xi}_-\, i\gamma_5\, \Lambda_o\, \bar{K}_+ + \bar{\Xi}_o\, i\gamma_5\, \Lambda_o\, \bar{K}_o\right] + h.c.$$

$$+ F_4'\left[\sqrt{2}\,(\bar{\Xi}_o\, \Sigma_+\, \bar{K}_+ + \bar{\Xi}_-\, \Sigma_-\, \bar{K}_o) + \bar{\Xi}_o\, \Sigma_o\, \bar{K}_o - \bar{\Xi}_-\, \Sigma_o\, \bar{K}_+\right] + h.c.$$

The usual case when K is an isospinor is obtained by putting $F_1' = F_2' = F_3' = F_4' = 0$ and a remark similar to the case of pion about the various coupling constants holds here too.

(c) - Interaction with Vector meson:

The interaction with vector meson field can similarly be written easily. The parity conserving electromagnetic interaction, for example, is given by ($P\,\vec{A}\,P^{-1} = -\vec{A}$, $P\,A_4\,P^{-1} = +A_4$, $\gamma_4\,\vec{\gamma}\,\gamma_4 = -\vec{\gamma}$)

$$H_{em} = -\,ie\left[\bar{N}_1^{(+)}\,\vec{\gamma}\left(\frac{1+\tau_3}{2}\right) N_1^{(-)}\cdot\vec{A} + \bar{N}_1^{(+)}\,\gamma_4\left(\frac{1+\tau_3}{2}\right) N_1^{(+)}\,A_4 + ((+)\longleftrightarrow(-))\right] + \ldots$$

which reduces to

$$H_{em} = -ie\left[\bar{N}_1\,\gamma_\mu\left(\frac{1+\tau_3}{2}\right) N_1 + \bar{N}_4\,\gamma_\mu\left(\frac{-1+\tau_3}{2}\right) N_4 + \bar{N}_2\,\gamma_\mu\left(\frac{1+\tau_3}{2}\right) N_2 + \bar{N}_3\,\gamma_\mu\left(\frac{-1+\tau_3}{2}\right) N_3\right] A_\mu = -\,ie\left[\bar{p}\,\gamma_\mu\, p + \bar{\Sigma}_+\,\gamma_\mu\,\Sigma_+ - \bar{\Sigma}_-\,\gamma_\mu\,\Sigma_- - \bar{\Xi}_-\,\gamma_\mu\,\Xi_-\right] A_\mu$$

DISCUSSION:

The doublets $N_i^{(\pm)}$ constructed in the section III thus serve as a convenient starting point for the construction of the interaction Lagrangian of baryons and mesons. Other invariance properties (charge conjugation, G conjugation, time reversal etc.) may as well be formulated in terms of the components $\psi^{(\pm)}$ in which any field can be decomposed.

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