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by
Prem P. Srivastava

CENTRO BRASILEIRO DE PESQUISAS FÍSICAS
Av. Wenceslau Braz, 71
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Prem P. Srivastava *
Centro Brasileiro de Pesquisas Físicas
Rio de Janeiro - Brazil

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ABSTRACT

The ratio F_K/F_π is derived using Weinberg's first spectral function sum rule in SU(3) x SU(3) while the second sum rule is used only in SU(2) x SU(2).

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* This work was accomplished while the author was at CERN - Geneva.

In recent papers [1], the ratio between the decay constants F_K and F_π, the decay constants in the leptonic decays of K and π mesons respectively, has been evaluated using the two spectral function sum rules derived by Weinberg [2] and their SU(3) generalizations given by Das, Mathur and Okubo [3]. The derivations in Ref. [1] make use of both of the sum rules and thus necessarily imply, in the pole approximation considered, the exact SU(3) × SU(3) result $m_\rho = m_{K^*}$, $m_{A_1} \simeq m_{K_A}$. We derive here the above-mentioned ratio by using the first sum rule [4] in SU(3) × SU(3) while the second sum rule is used only in the SU(2) × SU(2) subgroup. The argument [5] for the restriction on the use of the second sum rule derives from the successful predictions [2, 3] $m_{A_1} \simeq \sqrt{2}\, m_\rho$, $m_{K_A} \simeq \sqrt{2}\, m_{K^*}$ and the disappearance of the unpleasant feature mentioned above.

The first spectral function sum rule gives rise to the following relations [2, 3]:

$$\frac{G_\rho^2}{m_\rho^2} + \frac{G_{A_1}^2}{m_{A_1}^2} = F_\pi^2$$

$$\frac{G_{K^*}^2}{m_{K^*}^2} - \frac{G_{K_A}^2}{m_{K_A}^2} = F_K^2$$

$$\frac{G_\rho^2}{m_\rho^2} = \frac{G_{K^*}^2}{m_{K^*}^2}$$

The second spectral function sum rule applied in the subgroup SU(2) × SU(2) leads to the relations:

$$G_\rho = G_{A_1} \quad \text{and} \quad G_{K*} = G_{K_A}$$

These relations then lead to

$$\frac{F_K}{F_\pi} = \frac{m_{A_1}}{m_{K_A}} \sqrt{\frac{m_{K_A}^2 - m_{K*}^2}{m_{A_1}^2 - m_\rho^2}}$$

which [6] gives $(F_K/F_\pi) \simeq 1.07$ against the value ~ 1.16 found in Ref. [1]. Also it follows:

$$\frac{G_{A_1}^2}{m_{A_1}^2} \simeq \frac{G_{K_A}^2}{m_{K_A}^2}$$

The Cabibbo angles θ_A and θ are related through F_K/F_π by the relation

$$\frac{F_K}{F_\pi} = \frac{\tan\theta_A}{\tan\theta}$$

which can be rewritten [7] as

$$\frac{1}{F_+(0)}\left(\frac{F_K}{F_\pi}\right) - \frac{\sqrt{1 - F_+^2(0)\sin^2\theta}}{F_+(0)\sin\theta} \cdot \tan\theta_A$$

Here $F_+(0)$ is the $K-\pi$ form factor normalized to unity in exact SU(3) limit. Now θ_A can be obtained from the branching ratio of K_{μ_2} and π_{μ_2} decays while $\left[F_+(0)\sin\theta\right]$ is determined from K_{ℓ_3} decays. Using for the right-hand side the value [8] ~ 1.28, we obtain for the form factor, $F_+(0) \simeq 0.84$. On the other hand, if we assume $F_+(0) = 1$, the calculated value of the Cabibbo angle turns out to be $\tan\theta \simeq 0.26$ which is about 18% higher than the value quoted in Ref. [8].

In conclusion, it is of interest to note the implications of our results on the branching ratio $\Gamma_{\rho \to \pi\pi} / \Gamma_{K^* \to K\pi}$. Assuming pole dominance in the pion form factor we can derive

$$G_\rho \, g_{\rho\pi\pi} = \sqrt{2}\, m_\rho^2$$

The K_{ℓ_3} form factor F_+ under the assumption of K^* pole dominance leads to analogous relation:

$$G_{K^*} \, g_{K^*K\pi} = \frac{F_+(0)}{\sqrt{2}} \, m_{K^*}^2$$

We obtain, thereby, (using $G_{K^*}/m_{K^*} = G_\rho/m_\rho$):

$$\frac{g_{\rho\pi\pi}}{g_{K^*K\pi}} = 2 \left(\frac{m_\rho}{m_{K^*}} \right) \frac{1}{F_+(0)}$$

and

$$\frac{\Gamma_{\rho^+ \to \pi\pi}}{\Gamma_{K^{*+} \to K\pi}} = \frac{4}{3} \left(\frac{k^\rho_{C.M}}{k^{K^*}_{C.M}} \right)^3 \frac{1}{(F_+(0))^2} \simeq \frac{2.455}{(F_+(0))^2}$$

For $F_+(0) \simeq 1$ this implies $\Gamma_{\rho \to \pi\pi} = 122$ MeV for $\Gamma_{K^* \to K\pi} = 49.6$ MeV. For $F_+(0) \simeq 0.84$ it implies $F_{\to \pi\pi} \simeq 173$ MeV. However, due to great uncertainties in the experimental decay widths of the particles involved it is not possible to draw definite information on $F_+(0)$ from this piece of experimental data.

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H. T. Nieh - Phys. Rev. Letters **19**, 43 (1967).

2. S. Weinberg - Phys. Rev. Letters **18**, 507 (1967).

3. T. Das, V. S. Mathur and S. Okubo - Phys. Rev. Letters **18**, 761 (1967); Phys. Rev. Letters **19**, 470 (1967). We use the notation of their papers.

4. The first spectral function sum rule reads: [i, j being the SU(3) indices]

$$\int_0^\infty \left[\rho_V^i(\mu^2) - \rho_A^j(\mu^2)\right] \frac{d\mu^2}{\mu^2} = F_j^2$$

while the second sum rule is:

$$\int_0^\infty \left[\rho_V^i(\mu^2) - \rho_A^j(\mu^2)\right] d\mu^2 = 0 \qquad \text{etc.}$$

5. Similar doubts about the applicability of the second sum rule in SU(3) x SU(3) have also been raised by J. Sakurai - Phys. Rev. Letters **19**, 803 (1967).

6. For $m_{A_1} = \sqrt{2}\, m_\rho$ and $m_{K_A} = \sqrt{2}\, m_{K*}$ it gives $F_K = F_\pi$ implying no renormalization effect due to SU(3) breaking. We have used $m_{K_A} = 1309$, $m_{K*} = 892.4$, $m_{A_1} = 1058$ and $m_\rho = 774$.

7. We make use of the fact that $F_+(0) \simeq 1$ and θ is a small angle. The author is indebted to Dr. A. Sirlin and Dr. N. Brene for a discussion on this point.

8. N. Brene, L. Veje, M. Roos and C. Cronstrom - Phys. Rev. 149, 1288 (1966); L. B. Auerbach, J. Mac G. Dobbs, A. K. Mann, W. K. Mc Farlane, D. H. White, R. Cester, P. T. Eschstruth, G. K. O'Neill and D. Yount - Phys. Rev. 155, 1505 (1967).
See, however, the footnote (9) in Ref. [1] (Glashow et al.), where they mention that the value quoted for θ should be $\sim 10\%$ higher.