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BARGMANN-WIGNER THEORY FOR PARTICLES OF SPIN 3/2 *

by

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Abstract. A development of the formalism of Bargmann and Wigner for the case of a free spin-3/2-particle is presented. A comparison with the Rarita-Schwinger is discussed and we find that the theories are formally equivalent.

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INTRODUCTION

The relativistic wave-equations for arbitrary spin have beem considered by several authors. The first theory was developed by Dirac [1], Fierz and Pauli [2], and has been simplified by the spin-tensor formulation of Rarita and Schwinger [3]. Another well known formalism is due to Bargmann and Wigner [4] (hereafter referred to a BW) in which the wave function that describes a particle of spin $n/2$ is a n-th rank symmetric spinor. In the present note, we are interested in the development of this theory for the case of a free spin-3/2-particle, in terms of an antisymmetric spin--tensor field and a spin-vector.

It is shown that the spin-tensor actually has only 20 independent components, and so can be interpreted as a new BW-wave-function. Then, the corresponding equations of motion follow from the original BW-equations. In the last section, a comparison with Rarita-Schwinger theory shows that both formalisms are formally equivalent, as it would be expected.

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BW-WAVE-FUNCTION

The BW-wave-function for a particle of spin 3/2, $\psi_{\alpha\beta\gamma}$, is a third rank symmetric spinor with 20 independent components.*

However, for the purpose of the present note, we introduce a new BW-wave-function which may be constructed by considering the following ten symmetrical Dirac operators,

$$(-B\,\gamma_5\,\gamma_i) = (C^{-1}\gamma_i)$$

$$(-B\,\gamma_5\,\gamma_i\,\gamma_j) = (C^{-1}\,\gamma_i\,\gamma_j) \tag{1}$$

where

$$C = \gamma_5\,B^{-1}, \quad B^T = -B, \quad \gamma_i^T = -C^{-1}\,\gamma_i\,C$$

With the operators (1) and $\psi_{\alpha\beta\gamma}$ we obtain a spin-vector and an antisymmetric spin-tensor as follows:

$$\phi_\gamma^i = (C^{-1}\gamma^i)_{\beta\alpha}\,\psi_{\alpha\beta\gamma} \tag{2}$$

$$F_\gamma^{ij} = (C^{-1}\gamma^i\,\gamma^j)_{\beta\alpha}\,\psi_{\alpha\beta\gamma} \tag{3}$$

with 16 and 24 components respectively. However, because of

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* The spinor and vector indices are denoted by greek and italic letters, respectively.

For the γ-matrices we use the notation of Schweber-Bethe-de Hoffmann book, Mesons and Fields, Vol. I, sec. 3. The metric is:

$$g_{ik} = -\delta_{ik} \quad \text{for} \quad i,k = 1, 2, 3; \quad g_{oo} = 1$$

$$J_{\alpha\sigma} G_{\gamma\beta} = \frac{1}{4} \sum_A \gamma^A_{\gamma\sigma} (J \gamma^A G)_{\alpha\beta} \tag{4}$$

which holds for two arbitrary four-dimensional matrices, [5] such as J and G, we find:

$$\phi^i = - \gamma_j F^{ij} \tag{5}$$

$$\gamma_i \gamma_j F^{ij} = 0 \tag{6}$$

and so

$$\gamma_i \phi^i = 0 \tag{7}$$

Equation (5) means that F^{ij}_γ is actually the unique independent one and (6) that is a 20-components wave function, since (6) represents four conditions on it. Hence, F^{ij}_γ is our BW-wave function.

The inverse transformation of (2) and (3) is

$$\psi_{\alpha\beta\gamma} = \frac{P}{12} \left[- (\gamma_i C)_{\alpha\beta} (\gamma_j F^{ij})_\gamma - \frac{1}{2} (\gamma_j \gamma_i C)_{\alpha\beta} F^{ij} \right] \tag{8}$$

where P is the symmetrization operator which acts on the spinor indices.

This expression is a generalization of a wave function for spin 1 given by Leite Lopes. [6]

<u>BW-EQUATIONS</u>

The BW equations for a free particle of spin 3/2 can be written simply:

$$(i\ \gamma^{i}_{\alpha'\alpha}\ \partial_{i} - m\ \delta_{\alpha\alpha'})\psi_{\alpha\beta\sigma} = 0 \tag{9}$$

Then, from (9) and (3) we get:

$$\partial^{i}\ \gamma_{j}\ F^{kj} - \partial^{k}\ \gamma_{j}\ F^{ij} = m\ F^{ik} \tag{10.1}$$

$$(i\partial\!\!\!/ - m)\ F^{ij} = 0 \tag{10.2}$$

$$i\ \partial_{j}\ F^{ij} = m\ \gamma_{j}\ F^{ij} \tag{10.3}$$

which are the equations of motion for F^{ij}. They constitute together with relation (6) the spin-tensor form of the BW-equations.

<u>Equivalence of Bargmann-Wigner and Rarita-Schwinger theories</u>

We shall make now a comparison with the RS-formalism for particles of spin 3/2. In this theory, the equations of motion are:

$$(i\partial\!\!\!/ - m)\ \phi^{i} = 0 \tag{11.1}$$

$$\partial_{i}\ \phi^{i} = 0 \tag{11.2}$$

and the supplementary condition is:

$$\gamma_{i}\ \phi^{i} = 0 \tag{11.3}$$

where ϕ^{i} is a spin-vector with 16 independent components.

It can be seen that, they may be obtained from the above BW-equations (10) and the subsidiary condition (6) if ϕ^{i} is defined by (5). Thus the BW-equations (10) imply the RS equations (11). On the other hand, introducing definition (5) into (10.1) one obtains:

$$F^{ij} = \frac{1}{m}\ (\partial^{i}\ \phi^{j} - \partial^{j}\ \phi^{i}) \tag{12}$$

which from the point of view of RS formalism may be considered as a definition of F^{ij} in terms of ϕ^{i}. Now, one can show that F^{ij} defined by (12) satisfy the BW-equations (10) if ϕ^{i} is a solution of equations (11). Thus the RS equations also imply the BW ones, i.e., the two sets of equations are equivalent for free particles.

We remark that the Lagrangians of the two theories are however not identical. Therefore, when electromagnetic interactions are taken into account the two formalisms are no longer equivalent.

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