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DECAY OF $\Lambda_o \longrightarrow n + \gamma$ THROUGH NON LOCAL FERMI INTERACTIONS DUE TO VECTOR MESONS*

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ABSTRACT

The decay of $\Lambda_o \longrightarrow n + \gamma$ through non local Fermi interactions due to vector mesons is calculated. It is found that the branching ratio of $\Lambda_o \longrightarrow n + \gamma$ with respect to experimental pionic decay mode $\Lambda_o \longrightarrow n + \pi_o$ can be of the order 5×10^{-4}.

INTRODUCTION

The V-A coupling theory for Fermi interaction proposed by Feynman and Gell-Mann [1], Sudarshan and Marshak [2], and others can be

* Supported in part by the Conselho Nacional de Pesquisas.

1. R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).

2. E. C. G. Sudarshan and R. E. Marshak - Padua Venice Conference on "Mesons and Recently Discovered Particles", September 1957.

regarded as due to an interaction of fermions with intermediate heavy charged vector mesons [3,4].

Such an interaction may give rise to certain decays of particles which would otherwise be of second order in the Fermi coupling constant [5,6]:

$$\mu \longrightarrow e + \gamma$$

$$\Sigma \longrightarrow p + \gamma .$$

Another possible decay is the following

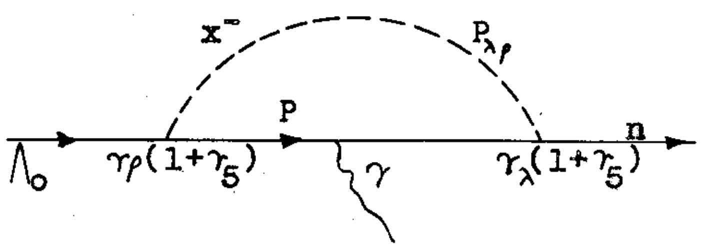
$$\Lambda_0 \longrightarrow n + \gamma$$

with the two graphs indicated in figure for the perturbation theoretical treatment in first order.

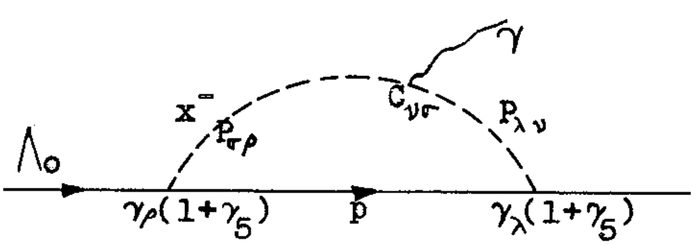
MATRIX ELEMENT

In graph (A) the photon is emitted by the virtual proton whereas in graph (B) the photon is emitted by the virtual meson:

(A)



(B)



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3. R.P. Feynman and M. Gell-Mann, See reference (1).

4. J. Leite Lopes - Nucl. Phys. Vol. 8(1958), nº2, 234.

5. G. Feinberg - Phys. Rev. 110, 1482 (1958).

6. P. Prakash and A.H. Zimmerman - Il Nuovo Cimento, serie X, vol. 11, 869 (1959).

The matrix elements are given by

$$M^{(A)} \sim eg^2 \int d^4q\ \bar{u}^1(p_1)\, \gamma_\lambda\, (1+\gamma_5)\, \frac{1}{i(q\cdot\gamma - k\cdot\gamma)+m_p}\, (\epsilon\cdot\gamma)\, \frac{1}{iq\cdot\gamma + m_p}\ \cdot \tag{1}$$

$$\cdot\, \gamma_\rho\ (1+\gamma_5)\ P_{\lambda\rho}\ (p_2 - q)\ u^2(p_2)$$

$$M^{(B)} \sim eg^2 \int d^4q\ \bar{u}^1(p_1)\, \gamma_\lambda\, (1+\gamma_5)\ P_{\lambda\nu}\ (p_1 - q)\ C_{\nu\sigma}\, P_{\sigma\rho}\, (p_2 - q)\ \frac{1}{iq\cdot\gamma + m_p}\ \cdot \tag{2}$$

$$\cdot\, \gamma_\rho\, (1+\gamma_5)\ u^2(p_2)\ ,$$

where the vectorial meson propagator $P_{\lambda\rho}$ is given by

$$P_{\lambda\rho}\ (p_2 - q) = \left(\delta_{\lambda\rho} + \frac{(p_2 - q)_\lambda\ (p_2 - q)_\rho}{M_x^2}\right)\cdot \frac{1}{(p_2 - q)^2 + M_x^2}$$

and

$$C_{\nu\sigma} = \left[(p_1 - q)\cdot\epsilon + (p_2 - q)\cdot\epsilon\right]\delta_{\nu\sigma} - \left[(p_1 - q)_\sigma\, \epsilon_\nu + (p_2 - q)_\nu\, \epsilon_\sigma\right]$$

and $p_2 = p_1 + k$

p_2, p_1 and q are the four-momenta of Λ_o, neutron and virtual proton of masses m_Λ, m_N and m_p respectively; k is the four-momentum of the emitted photon; M_x is the mass of the virtual meson.

The two matrix elements contain quadratic and logarithmic divergences which do not cancel when summed. Moreover the term with quadratic divergence is non gauge invariant:

$$-\frac{5}{2}\ \frac{(\epsilon\cdot\gamma)}{M_x^2} \int d^4q\ \frac{1}{q^2 + M_x^2}\ .$$

To remove the non-gauge invariant terms we use the regular ization method of Pauli and Villars [7,8].

The regularization method consists in adding and subtracting other nucleon fields of much larger masses m_i (m_0 is the mass of the intermediate proton). Since the infinities have the same structure it is possible to choose conveniently the number and the masses of the additional fields to make up the resultant matrix convergent.

The regularized matrix elements are given by

$$M_R^{(A)} \sim eg^2 \int d^4q \sum_{o}^{n}{}_i C_i \, \bar{u}^1(p_1)\gamma_\lambda (1 + \gamma_5) \frac{1}{i(q\cdot\gamma - k\cdot\gamma) + m_i}(\epsilon \cdot \gamma) \cdot \frac{1}{iq\cdot\gamma + m_i}\gamma_\rho (1 + \gamma_5) P_{\lambda\rho} (p_2 - q) u^2(p_2) \qquad (1')$$

$$M^{(B)} \sim eg^2 \int d^4q \sum_{o}^{n}{}_i C_i \, \bar{u}^1(p_1)\gamma_\lambda (1 + \gamma_5) P_{\lambda\nu} (p_1 - q) C_{\nu\sigma} P_{\sigma\rho} (p_2 - q) \cdot \frac{1}{iq\cdot\gamma + m_i} \gamma_\rho (1 + \gamma_5) u^2)p_2), \qquad (2')$$

where

$$\begin{cases} m_0 = m_p \\ m_i \longrightarrow \infty, \text{ if } i \neq 0 \end{cases} \quad \text{and} \quad \begin{cases} C_0 = 1 \\ C_i = \pm 1 \text{ if } i \neq 0, \end{cases}$$

and $P_{\lambda\rho}$ and $C_{\nu\sigma}$ are not modified.

The total matrix element may be reduced by usual technique (See Appendix I) to

$$M_R = M_R^{(A)} + M_R^{(B)} \sim eg^2 \, \bar{u}^1(p_1) (1 - \gamma_5) Q_R (1 + \gamma_5) u^2(p_2),$$ where

7. W. Pauli and F. Villars, Rev. Mod. Phys., Vol. 21, 434 (1949).
8. J. Steinberger - Phys. Rev., 76, nº 8, 1180 (1949).

$$Q_R = A \int d^4q \sum_{o}^{n}{}_i\, C_i \frac{1}{q^2 + M_x^2} + B \int d^4q \sum_{o}^{n}{}_i\, C_i\, m_i^2 \frac{1}{(q^2 + M_x^2)^2}$$

$$+ C \int d^4q \sum_{o}^{n}{}_i\, C_i \frac{1}{(q^2 + M_x^2)^2} + D \int_o^1 x\, dx \int_o^1 dy \sum_{o}^{n}{}_i C_i m_i^2 +$$

$$+ \text{convergent part (function of } m_i). \qquad (3)$$

A, B, C and D are functions of masses of particles.

Thus, to eliminate the non-gauge invariant quadratically divergent term we are required to impose $\sum_{o}^{n}{}_i C_i = 0$. To eliminate the logarithmic divergence we require $\sum_{o}^{n}{}_i\, C_i\, m_i^2 = 0$. The term with coefficient D also disappears.

Calculating the convergent part using the two conditions above ($\sum_{o}^{n}{}_i\, C_i = 0$ and $\sum_{o}^{n}{}_i\, C_i\, m_i^2 = 0$) a logarithmic divergence reappears when we make m_i $(i \neq 0)$ tend to infinity.

The final total matrix element is given by

$$M_R = M_R^{(A)} + M_R^{(B)} = - e \left(\frac{g^2}{M_x^2}\right) \bar{u}^1(p_1) \left\{ \sum_{1}^{n}{}_i\, C_i \lg \frac{M_x^2 - m_\Lambda^2}{m_i^2} \left[m_\Lambda (1 - \gamma_5) + \right.\right.$$

$$\left.\left. + m_N(1 + \gamma_5) \right] (k.\gamma)(\epsilon.\gamma) + F \right\} u^2(p_2)$$

where F is the finite part and does not tend to infinity when M_x does.

The divergence $\sum_{1}^{n}{}_i\, C_i \lg \dfrac{M_x^2 - m_\Lambda^2}{m_i^2}$ is eliminated imposing one more condition: $\sum_{o}^{n}{}_i\, C_i \lg m_i^2 = 0$. Then we obtain

$$\sum_{1}^{n}{}_i\, C_i \lg \frac{M_x^2 - m_\Lambda^2}{m_i^2} = - \lg \frac{M_x^2 - m_\Lambda^2}{m_P^2}.$$

The finite part is of the order of $\frac{1}{50}$ of the other term, if the mass of vector meson is much larger than that of neutron ($M_x \sim 10\ m_N$).

The matrix element is finally given by

$$M = e\left(\frac{g^2}{M_x^2}\right) \lg \frac{M_x^2 - m_\Lambda^2}{m_p^2}\ \bar{u}^1(p_1)\left[m_\Lambda(1-\gamma_5) + m_N(1+\gamma_5)\right](k \cdot \gamma)(\epsilon \cdot \gamma)\, u^2(p_2) \qquad (4)$$

and it is analogous to that obtained by Prakash and Zimerman [6] in the decay of

$$\Sigma \longrightarrow p + \gamma .$$

TRANSITION PROBABILITY

The transition probability for the decay is given by (See Appendix II):

$$P = \frac{e^2}{4\pi}\left(\frac{g^2}{M_x^2}\right)^2 \cdot \frac{m_\Lambda^5}{2^5(2\pi)^4} \cdot \left(\lg \frac{M_x^2 - m_\Lambda^2}{m_p^2}\right)^2 \left[1 + \frac{m_N^2}{m_\Lambda^2}\right]\left[1 - \frac{m_N^2}{m_\Lambda^2}\right]^3 \qquad (5)$$

The coupling constant g is related to the coupling constant G for the direct Fermi interaction by

$$\frac{g^2}{M_x^2} = \frac{G}{\sqrt{2}} .$$

Thus, the transition probability for our decay may be written as

$$P = \frac{e^2}{4\pi}\left(\frac{G}{\sqrt{2}}\right)^2 \frac{m_\Lambda^5}{2^5\,(2\pi)^4}\left[\lg \frac{M_x^2 - m_\Lambda^2}{m_p^2}\right]^2\left[1 + \frac{m_N^2}{m_\Lambda^2}\right]\left[1 - \frac{m_N^2}{m_\Lambda^2}\right]^3 . \qquad (6)$$

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1. P. Prakash and A. H. Zimerman, Il Nuovo Cimento, serie X, Vol. 11, 869 (1959).

The mean life of Λ_o for the decay $\Lambda_o \longrightarrow n + \gamma$ is found to be

$$\tau = \frac{1}{P} \cong 5{,}5 \times 10^{-7} \text{ sec}$$

and for the branching ratio with respect to experimental pionic decay mode we obtain

$$\frac{P(\Lambda_o \longrightarrow n + \gamma)_{th}}{\Lambda_o \longrightarrow n + \pi_o)_{exp}} \cong 5{,}4 \times 10^{-4} \; .$$

ACKNOWLEDGMENT

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APPENDIX I

To arrive at the total matrix element (Formula 3), we made the following development:

We had written $M_R^{(A)} \sim eg^2 \, \bar{u}^1(p_1) \left[Q_R^{(A)}\right] u^2(p_2)$ (7)

and after simple transformations we obtain

$$\left[Q_R^{(A)}\right] = (1 - \gamma_5) \int d^4q \sum_{o}^{n} i \, C_i \frac{N_1^{(A)}}{D^{(A)}} \circ (1 + \gamma_5) + \qquad (8)$$

$$+ \frac{1 - \gamma_5)}{M_x^2} \int d^4q \sum_{o}^{n} i \, C_i \frac{N_2^{(A)}}{D^{(A)}} \circ (1 + \gamma_5) \; ,$$

where

$$N_1^{(A)} = - 2 \, (m_i^2 + q^2)(\epsilon \circ \gamma) + 4 \, (q \circ \epsilon)(q \circ \gamma) + 2(q \circ \gamma)(k \circ \gamma)(\epsilon \circ \gamma)$$

$$N_2^{(A)} = - q^4(\epsilon \circ \gamma) + q^2 \, (\epsilon \circ \gamma)(q \circ \gamma)(p_2 \circ \gamma) + (p_2 \circ \gamma)(q \circ \gamma)(\epsilon \circ \gamma)(q \circ \gamma)(p_2 \circ \gamma) +$$

$$+ q^2 (p_2\cdot\gamma)(q\cdot\gamma)(\epsilon\cdot\gamma) + (p_2\cdot\gamma)(k\cdot\gamma)(\epsilon\cdot\gamma)(q\cdot\gamma)(p_2\cdot\gamma) =$$

$$= q^2 (p_2\cdot\gamma)(k\cdot\gamma)(\epsilon\cdot\gamma) + q^2 (q\cdot\gamma)(k\cdot\gamma)(\epsilon\cdot\gamma) + m_i^2 (p_2\cdot\gamma)(\epsilon\cdot\gamma)(p_2\cdot\gamma) -$$
$$-m_i^2 (p_2\cdot\gamma)(\epsilon\cdot\gamma)(q\cdot\gamma) - m_i^2 (q\cdot\gamma)(\epsilon\cdot\gamma)(p_2\cdot\gamma) + m_i^2 (q\cdot\gamma)(\epsilon\cdot\gamma)(q\cdot\gamma) =$$
$$= (q\cdot\gamma)(k\cdot\gamma)(\epsilon\cdot\gamma)(q\cdot\gamma)(p_2\cdot\gamma),$$

and $\dfrac{1}{D^{(A)}}\ \dfrac{1}{\left[(q-k)^2 + m_i^2\right]\left[q^2 + m_i^2\right]\left[(p_2 - q)^2 + M_X^2\right]}$.

Doing $\dfrac{1}{D^{(A)}}\ \dfrac{1}{abc}$, where $a = \left[(p_2 - q)^2 + M_X^2\right]$,

$b = \left[(q-k)^2 + m_i^2\right]$ and $c = \left[q^2 + m_i^2\right]$, and using the identity

$$\frac{1}{abc} = \int_0^1 2x\,dx \int_0^1 \frac{dy}{\left[axy + bx(1-y) + c(1-x)\right]^3}$$, we arrive

at $$\frac{1}{abc} = \frac{1}{D^{(A)}} = \int_0^1 2x\,dx \int_0^1 \frac{dy}{|(q-d)^2 + f^2|^3} , \qquad (9)$$

where

$$d = kx(1-y) + p_2\,xy - p_1\,x(1-y) + p_2\,x$$

$$f^2 = m_i^2 + (M_X^2 - m_\Lambda^2 - m_i^2)\,xy + (m_\Lambda^2 - m_N^2)\,x^2y + m_N^2\,x^2y^2 .$$

Substituting (9) in (8), we have

$$\left[Q^{(A)}_R\right] = (1 - \gamma_5) \int d^4q \int_0^1 2x\,dx \int_0^1 dy \sum_0^n {}_i\, C_i \frac{N_1^{(A)}}{\left[(q-d)^2+f^2\right]^3}\cdot$$

$$\cdot (1 + \gamma_5) + \frac{(1-\gamma_5)}{M_X^2} \int d^4q \int_0^1 2x\,dx \int_0^1 dy \sum_0^n {}_i\, C_i \frac{N_2^{(A)}}{\left[(q-d)^2+f^2\right]^3}\cdot$$

$$\cdot (1 + \gamma_5) .$$

We can resume the second matrix element (Formula (2')) as follows:

$$M_R^{(B)} \sim eg^2 \bar{u}^1(p_1) \left[Q_R^{(B)} \right] u^2(p_2) \quad . \tag{10}$$

After the usual procedure, we have:

$$-\left[Q_R^{(B)} \right] = (1-\gamma_5) \int d^4q \sum_{o}^{n}{}_i\, C_i \frac{N_1^{(B)}}{D^{(B)}} \cdot (1+\gamma_5) +$$

$$+ \frac{(1-\gamma_5)}{M_X^2} \int d^4q \sum_{o}^{n}{}_i\, C_i \frac{N_2^{(B)}}{D^{(B)}} \cdot (1+\gamma_5), \tag{11}$$

where $N_1^{(B)}$ and $N_2^{(B)}$ are expressions similar to the ones $\begin{cases} N_1^{(A)} \\ N_2^{(A)} \end{cases}$ and

$$\frac{1}{D^{(B)}} = \frac{1}{\left[q^2 + m_i^2\right] \left[(p_1 - q)^2 + M_X^2\right]\left[(p_2 - q)^2 + M_X^2\right]} \quad .$$

Using the same technique we have used before, we obtain for $\frac{1}{D^{(B)}}$:

$$\frac{1}{D^{(B)}} = \int_o^1 2x\, dx \int_o^1 dy \frac{1}{\left[(q-r)^2 + s^2\right]^3}$$

where

$$r = p_1\, x(1-y) + p_2\, xy = p_1\, x + k\, xy$$

$$s^2 = m_i^2 + (M_X^2 - m_N^2 - m_i^2)x + (m_N^2 - m_\Lambda^2)\, xy + (m_\Lambda^2 - m_N^2)\, x^2y + m_N^2\, x^2 \quad .$$

Substituting this expression for $\frac{1}{D^{(B)}}$ in (11), we obtain:

$$-\left[Q_R^{(B)}\right] = (1-\gamma_5) \int d^4q \int_o^1 2x\, dx \int_o^1 dy \sum_{o}^{n}{}_i\, C_i \frac{N_1^{(B)}}{\left[(q-r)^2 + s^2\right]^3} \cdot$$

$$\cdot (1+\gamma_5) + \frac{1-\gamma_5)}{M_X^2} \int d^4q \int_o^1 2x\, dx \int_o^1 dy \sum_{o}^{n}{}_i\, C_i \frac{N_2^{(B)}}{\left[(q-r)^2 + s^2\right]^3} (1+\gamma_5)$$

From now on we used the usual integral calculus procedures.

APPENDIX II

To arrive at the transition probability (Formula(5)), we made the following development:

The transition probability for the decay is given by

$$P = \iint \frac{(S_{if})^2}{TV} \, d^3k \, d^3p_1 ,$$

where [9]:

$$(S_{if})^2 = \langle |M|^2 \rangle \left\{ \frac{1}{\left[(2\pi)^{3/2}\right]^2} \left(\frac{m_\Lambda}{E_\Lambda}\right)^{\frac{1}{2}} \left(\frac{M_n}{E_N}\right)^{\frac{1}{2}} \left(\frac{1}{2\,k_o}\right)^{\frac{1}{2}} \left[(2\pi)^4\right]^3 . \right.$$

$$\left. \cdot \left[\frac{-i}{(2\pi)^4}\right]^3 \delta^4(p_2 - p_1 - k) \right\}^2 =$$

$$= \langle |M|^2 \rangle \frac{T\,V}{(2\pi)^{10}} \; \frac{m_N \, m_\Lambda}{2\, p_{2o}\, p_{1o}\, k_o} \; \delta^4(p_2 - p_1 - k) ,$$

$\langle |M|^2 \rangle$ being the average of $M^\dagger M$ over the inicial spin states, and sum over the final one and it is given by

$$\langle |M|^2 \rangle = \left(\frac{e^2}{2}\right) \left(\frac{g^2}{M_X^2}\right)^2 \pi^4 \left[\lg \frac{M_X^2 - m_\Lambda^2}{m_p^2} \right]^2 .$$

$$\text{Trace} \left[L' \Lambda_-(p_1) \, L \, \Lambda_-(p_2) \right] \quad \text{where} \quad L' = \gamma_4 \, L \, \gamma_4 =$$

$$= \gamma_4 \left\{ \left[(m_N + m_\Lambda) + (m_N - m_\Lambda)\gamma_5 \right] (\epsilon \cdot \gamma)(k \cdot \gamma) \right\}^\dagger \gamma_4 ,$$

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9. S.S. Schweber, H. A. Bethe and F. de Hoffman - "Mesons and Fields", Row, Peterson and Company, New York, 1955 - Vol. I, Section 19c.

$$\Lambda_-(p_1) = \frac{-i\ (p_1 \cdot \gamma) + m_N}{2m_N} \text{ and } \Lambda_-(p_2) = \frac{-i\ (p_2 \cdot \gamma) + m_\Lambda}{2m_\Lambda}.$$

We can easily arrive at the following expression of $\langle |M|^2 \rangle$:

$$\langle |M|^2 \rangle = \frac{e^2 \pi^4}{2} \left(\frac{g^2}{M_x^2} \right)^2 \left[\lg \frac{M_x^2 - m_\Lambda^2}{m_P^2} \right]^2 \frac{(m_N^2 + m_\Lambda^2)(m_N^2 - m_\Lambda^2)^2}{m_N\ m_\Lambda}$$

Thus,

$$\frac{(S_{if})^2}{T\ V} = e^2 \left(\frac{g^2}{M_x^2} \right)^2 \frac{\pi^4}{2} \left[\lg \frac{M_x^2 - m_\Delta^2}{m_P^2} \right]^2 \frac{(m_N^2 + m_\Lambda^2)(m_N^2 - m_\Lambda^2)^2}{2\, p_{2o}\ p_{1o}\ k_o\ (2\pi)^{10}} \ \delta^4(p_2 - p_1 - k)$$

and in the center-of-mass frame the transition probability is

$$P = e^2 \left(\frac{g^2}{M_x^2} \right)^2 \frac{\pi^4}{2} \left[\lg \frac{M_x^2 - m_\Lambda^2}{m_P^2} \right]^2 \frac{(m_N^2 + m_\Lambda^2)(m_\Lambda^2 - m_N^2)^2}{(2\pi)^{10}\ 2\ m_\Lambda^2} \int \frac{d^3k}{p_{1o}\ k_o} \delta^o(E_f - E_i),$$

where $\displaystyle\int \frac{d^3k}{p_{1o}\ k_o} \delta^o(E_f - E_i) = \int \frac{k_o\ d\Omega}{p_{1o}} \frac{dk_o}{dE_f} \delta^o\ (E_f - E_i)\ dE_f$

and $dE_f = dp_{1o} + dk_o$.

From now on we used the usual calculus procedures.