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SEARCH FOR THE ELECTRONIC DECAY OF THE POSITIVE PION

by

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SEARCH FOR THE ELECTRONIC DECAY OF THE POSITIVE PION[+][§]

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I. INTRODUCTION

The normal decay of the charged pion is into a muon and a light neutral particle, presumed to be a neutrino. An alternative possibility, the decay into an electron instead of the muon, has never been observed. This seems particularly strange in view of the fact that the muon appears to have just those properties which would be expected of a more massive electron. Thus, the scattering of muons by nu-

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clei is mostly an effect of the Coulomb field. What anomalies have been observed in the scattering seem to find their explanation in the inelastic processes which can be induced by the electromagnetic behavior of the muon[1]. That this can hardly be different from that of an electron of muonic mass has been demonstrated in several ways. The energy levels of the μ-mesic atoms bear the correct relationship to those of an electron with augmented mass[2]. In fact, the electric charge distribution of the heavy nuclei as determined from the μ-mesic X-ray measurements is just that which is found from the scattering of high energy electrons. Moreover, observations of the procession of polarized muons in a magnetic field have confirmed the fact that their gyromagnetic ratio is, to within 0.24%, not different from that of the electron[3]. In addition, the production of μ-meson pairs by high energy γ-rays seems to proceed with the Bethe-Heitler cross-section calculated for a heavy electron[4].

While these evidences point to an identical electromagnetic behavior of the muon and the electron and to the absence of other strong interactions, it is remarkable that also in the weak interaction which it exhibits with the neutrino field the behaviour of the muon is closely the same as that of the electron. Thus, Godfrey[5] was able to demonstrate the close identity of the coupling strength involved in the μ capture reaction

$$\mu^- + {}^{12}C \rightarrow B^{12} + \nu$$

and the β-decay process

$$^{12}B \rightarrow {}^{12}C + e^- + \nu$$

involving the same nuclear levels. This experiment, among many others, has served to reinforce the idea that a single "universal" Fermi interaction sufficed to explain not only μ capture, β-decay[6], but also pion decay as well.

The inclusion of the pion decay in this class of processes follows from its strong interaction with the nucleon field. The decay is supposed to proceed as a two step process, in the first of which the pion transforms into a nucleon-antinucleon pair, which then annihilate in a typical β-decay process according to the scheme

$$\pi^+ \rightarrow p + \bar{n} \quad e^+ + \nu. \tag{1}$$

Since the pion is pseudoscalar, this transition is forbidden except for the pseudoscalar and the axial vector β-couplings. This will be the case provided that the order in which the particles are written in the interaction term is the conventional one (pneν)[7]. The smallness of the electronic decay of the pion appears, from this point of view, as evidence for the smallness of these couplings. Analyses of the data of β-decay have shown, in fact that the dominant couplings are the scalar and tensor. In his review[8] Michel quotes values for vector and axial vector couplings zero or small, with no evidence to show that the pseudovector coupling strength is not zero also. These analyses are, however, open to re-examination in view of the recent discovery that parity is not always conserved in these processes[9].

Unfortunately, there is no satisfactory method for calculating the transition rate in accordance with the scheme (1). The calculations of Steinberger[10] and of Ruderman and Finkelstein[11] involve divergent integrals and can give results differing by several orders of magnitude depending on the cut-off procedure used. Insofar as it can be supposed that the only difference between the muon and the electron is the mass, the calculation of the relative transition probability $f = \pi \to e + \nu / \pi \to \mu + \nu$ may be made independent of the field theoretic uncertainties; only the masses of the particles enter. For pseudoscalar coupling of the fermions these authors obtain $f = 5.4$, a result which is roughly equal to the ratio of phase space available and certainly not in accord with the observations. The axial vector coupling, on the other hand, discriminates strongly against low mass particles giving $f = 12 \cdot 10^{-5}$. The experiments have been directed at finding the electron and determining the value of f.

In a previous attempt to find an electronic decay of the pion, Friedman and Rainwater[12] examining pion endings in photographic emulsion could report zero or one such decay in 1419 muonic decays. This was sufficient to rule out the pseudoscalar coupling but could not be decisive for the axial vector case. Subsequently, Lokanathan and Steinberger[13] using counters sensitive to the higher energy and shorter lifetime of the π-decay electrons to discriminate against those from μ-decay, obtained $f = (-3 \pm 9) \cdot 10^{-5}$. The axial vector coupling appeared unlikely in view of this result, but only to the extent of 1.7 standard deviations, not really enough to be decisive. Moreover, with the addition of a suitable amount of pseudoscalar to the axial vector coupling, an arbitrarily small value of f could be explained. However, as Treiman and Wyld[14] have shown, even if by chance this happened to be the case, there would still remain the difficulty of explaining why the radiative decay process $\pi \to e + \nu + \gamma$ should thus far have escaped detection[15].

The present experiment was undertaken to pursue even more exhaustively the search for the π-e-decay. The chief experimental problem was to distinguish the electrons from π-decay from those of the μ-decay. Since the electrons from a two body decay of the pion would have a unique energy, well above the end point of the continuous μ-e spectrum, the magnetic spectrometer is the detector of choice. This frees the method from the fundamental limitation of the counter telescope of Lokanathan and Steinberger, which retains some sensitivity for the μ electrons at all usable absorber thicknesses through the detection of their bremsstrahlung. Our electronics were arranged to be particularly selective to those events which had a time dependence appropriate to the π-decay. This made it possible to include in our search for the π-e decay, a search for the π-e-γ process as well.

II. SPECTROMETER

We adapted for our use the magnetic spectrometer described by Stoker, Hok, Haan and Sizoo[16] for the analysis of β-spectra. This is a shaped field, iron core, double focussing spectrometer of the Siegbahn-Svartholm[17] type characterized by high transmission and good resolution. In this instrument the electrons are focussed in the central part, normally kept evacuated. The coils are external to the central region, and the iron magnetic return circuit is external to the coils. To preserve the high transmission of the design of Stoker et al.,[16] we retained their pole shape but extended the range by increasing very much the amount of copper and current in the coils, as well as the amount of iron in the magnetic return circuit. In this way we were able to reach a momentum of 100 MeV/c without encountering serious saturation effects. Since we were dealing with high energy electrons, we required baffles and shields several radiations length in thickness.

A sectional view of the spectrometer is given in Fig. 1. A plan view, showing the shielding and baffling and indicating the extreme usable orbits is given in Fig. 2. The central poles were machined from two Armco iron disks 1.52 m in diameter. This included a central portion to accomodate the brass cylinder which determined the spacing of the poles, a shaped field region which followed a template cut to the prescriptions of Stoker et al[16], a place for the stainless steel cylinder which, with its rubber gasket, formed the external vacuum envelope, then a recess for the coils and, finally, a flat section for the magnetic return circuit and bolting holes. Additional iron in the form of low carbon steel plates were added to reduce saturation in the iron and to keep low the stray fields external to the instrument. Total iron amounted to about 9 metric tons.

The coils were made up from double pancake spirals following the scheme used for the Chicago synchrocyclotron[18]. Here, a total of 120 turns of 1.22 cm square copper tubing with a 0.63 cm diameter hole for water cooling were employed in each of the two coils. With 1300 amperes we could obtain, without excessive overheating, a field of 11.7 kG at the central orbit (30.5 cm radius) capable of focussing electrons of momentum 109.5 MeV/c.

Holes were provided through the poles at the source and detector positions. Referring to Fig. 1, the lower right hand hole was used for the entrance of the pion beam into the instrument. This had to pass through a monitor counter (no. 2), the pion counter (no. 3), an external polyethylene moderator, a thin aluminum window which served for vacuum seal, an internal polyethylene moderator with thickness adjusted so that in the end most of the pions came to rest in the source counter (no. 4), sometimes called the target. The target was a strip of plastic scintillator 2.54 cm wide, 1.9 cm thick and 7.2 cm long. Held at an angle of 45° to the axis of the pion beam its axial extension in the spectrometer was only 5.08 cm. This scintillator was viewed with the help of a lucite light pipe extending through the lower left hand hole by means of an RCA 14 stage photomultiplier, which was thereby removed from the presence of strong magnetic fields. Further magnetic shielding of the photomultiplier was afforded by means of a double mu-metal shield inside a 6 mm thick iron container. The end of the light-pipe on which the source counter was supported was made hollow, the better to define the volume in which the pions came to rest and from which electrons could emerge and enter proper spectrometer orbits.

The detector consisted of two plastic scintillators, each 25 mm wide and 50 mm long, e.g. of the same radial and axial extension as the source, set inside a single lucite light-pipe which extended across the spectrometer through the upper holes. Aluminum reflectors were fitted so that the light from the first scintillator was recorded in counter no. 5 while that from the second in no. 6. Since the scintillators were rather thick, 19 mm, particles with range smaller than this would be recorded as a coincidence in these two counters. The possible background from low energy electrons, pions and protons was greatly reduced thereby.

The isolation of the detector from the direct radiation from the source, required in this experiment, was provided by 5 cm thick lead shields surrounding both the source and the detector. In addition to the openings in the lead shields, three baffles were provided to define the location of the extreme usable orbits. These were made of 13 mm thick lead and served to limit the entrance of unwanted particles into the detector. Lead was chosen because the need to keep low the background due to the entrance of unwanted electrons to the detector outweighed the need to keep low the effects of scattering. Since we were interested in finding an electron with energy higher than any of those normally present we were not greatly troubled by the energy loss which can occur in scattering, the principal effect of this being to allow a high energy electron to be mistaken for one having lower energy. More important the multiple Coulomb scattering, particularly that which takes place at the openings in front of the source and detector. The principal effect of this was to broaden somewhat the effective size of the source and detector. This made difficult a precise analysis of the μ-e spectrum, but did not diminish the detectability of the π-electrons.

III. CALIBRATION

The range of the spectrometer was great enough to permit its use with many α-particle emitters. We used the α-particles of ^{239}Pu to determine the transmission, resolution, and absolute energy calibration of the instrument. These α-particles have an energy of 5.15 MeV, corresponding to a magnetic rigidity, expressed in terms of the momentum of a singly charged particle, of 97.96 MeV/c. A source of the α-particles having negligible thickness but ample intensity was kindly prepared for us by Mr. Ray Barnes and Mr. Paul Fields of the Argonne National Laboratory. This was distributed rather uniformly on an aluminum backing over an area 2.54 cm wide by 7.2 cm long. When located at the median plane of the source of our experiment, it corresponded quite closely to the electron source it was supposed to simulate. Moreover, we found by the use of a 6 mm diameter α source of this type, that the response of the spectrometer was rather independent of the radial and axial positioning, provided this remained within the 25 mm by 50 mm area defined by the detector. Outside this area the response fell rapidly. Thus, we suppose that the response of the spectrometer found with these α-particles would correspond quite closely to that which would obtain for the electrons, once due allowance was made for the loss of energy of the electrons in emerging from the interior of the source.

Our plastic scintillation detector was not sensitive to these α-particles, so we replaced it with a thallium activated cesium iodide crystal, obtained from the Harshaw Chemical Co., having the same surface dimensions but of thickness 3 mm. With this it was easy to observe the α-particles focussed by the spectrometer. From the counting rate observed as a function of magnet current we obtained the response curve given in Fig. 3. In this curve the response is plotted, with its maximum normalized to unity, as a function of $(p-p_o)/p_o$, the relative deviation of the particle momentum p from that of the momentum setting p_o of the spectrometer. In plotting this curve the small deviations from linearity in the relation between magnet current and magnetic field were taken into account. The resolution full width at half maximum is seen to be $3.0^o/o$, a value not very different from that which is due to the radial extension of the source and the detector. Although the resolution curve is not as symmetrical as it should be, the cut-off is rather sharp for momenta lower than that for which the spectrometer is set. This helps discriminate against the μ electrons when the instrument is set above the end-point of this spectrum. The rather appreciable resolution width of our spectrometer was needed to accomodate to the spread in energy brought about because the electrons came from various depths in the source. This point will be discussed in more detail below.

The maximum response occurred at a current of 1138 amperes, thereby establishing one point of the momentum calibration. We obtained the remainder by measuring the magnetic field at the central orbit as a function of current by means of a rotating coil type fluxmeter made by the Rawson Instrument Co. Thus, the calibration of the spectrometer relied only on the linearity of the fluxmeter response but not on the absolute calibration supplied by the manufacturer. A check of the calibration was provided by the observation of the end point of the μ-e spectrum which is at 52.85 MeV/c. Because of this we could be confident that our setting of the spectrometer could be set on the π-e peak to within 1/2 %. The full calibration curve is given in Fig. 4. The departure from linearity is less than 1 % to 900 amperes (79 MeV/c), at 1140 amperes (98 MeV/c) it is only 3%. These evidences indicate that saturation effects can have only a minor influence on the present experiment.

The transmission of the spectrometer was obtained from the maximum counting rate, 12500 counts per minute, and the knowledge of the number, $70 \cdot 10^4$ disintegrations per minute, of the α-particle source. The latter number was obtained by counting the α-particle tracks in emulsion exposed for a known time directly above the source. From these numbers we deduced that the transmission of our spectrometer was about 1.8%, a value somewhat poorer than the figure 2.8 % given by Stoker et al[16]. This loss in intensity of 35 % was more than compensated for by the four times larger source area which we used.

IV. ARRANGEMENT

The positive pions for this experiment were obtained from the Chicago synchrocyclotron. The most convenient beam seemed to be the one at 75 MeV. This was first focussed by a pair of quadrupole magnets of 10 cm aperture, then passed through a suitably oriented hole in the 3.7 m steel shield and then deflected through 90° by means of a wedge magnet arranged to give an approximate vertical and horizontal focussing of the beam at the source position of the spectrometer. The intensity of positive pions from the cyclotron is a maximum near 75 MeV yet this energy is low enough to allow the pions to be brought to rest within a suitably small volume without excessive losses due to the effects of multiple scattering and nuclear collisions. The normal intensity of the beam obtained in this way, after passing through a brass collimator which limited its cross-section to 38 mm x x 64 mm, was $1.2 \cdot 10^6$ per minute.

The manner of introducing the pions into the spectrometer may be seen in Fig. 1. The idea was to have the pions come to rest and decay inside a block of scintillation plastic at the source position of the spectrometer. By viewing this with a photomultiplier (counter no. 4) we could observe the pions entering the source as well as the electrons leaving. The spectrometer accepted electrons from the source emerging at an angle of 90° with respect to the direction of the entering pions, the emergence of the electrons being from the same face as the entrance of the pions. Electrons of the proper momentum were brought to focus at the detector so that a triple coincidence T_{456} served to identify electrons originating at the source and reaching the detector. The electrons from μ-decay were readily detected in this way and served for the adjustment of the counters and as a means of standardizing the measurements.

The observations were monitored with the doubles coincidence count D_{24} which measures pions traversing no. 2 and reaching the source (counter no. 4). To avoid an excessive counting rate the sensitive area of no. 2 was reduced by using two 6.3 mm diameter disks of 6 mm thick scintillator plastic near the center of the beam, these being imbedded in insensitive lucite. With this composite counter all the pions passed through the same thickness of material, but the counting rate was reduced by a factor of 12 to $1.2 \cdot 10^4$ per minute under normal conditions.

V. ADJUSTMENT

To bring the pions to rest in the target we used as moderator, polyethylene plastic disks, the optimum thickness of which was determined from a range curve. This was obtained from a measurement of the ratio of the double coincidences D_{34}/D_{23} as a function of polyethylene thickness. Counter no. 3 was a full area counter, able to count all the pions in the beam, so that the ratio D_{34}/S_3 gave directly the fraction of pions in the beam which entered the target and is the quantity plotted in Fig. 5. The arrow points to the thickness of polyethylene (12.8 g/cm^2) used throughout the experiment. From the curve we estimated that the number of pions stopping in the target was 10^5 per minute.

We also observed directly the π-μ endings in emulsion exposed at various depths inside a simulated scintillator block. A dummy target was made up of 3.2 mm layers of scintillator plastic so that 200 μm emulsion could be sandwiched within at various depths. Several exposures were made, exposing in each case no more than two emulsion layers at one time so as to disturb as little as possible the density distribution of the material in the dummy target. The result is given in Fig. 6. From the integral under the curve we obtained the figure 70% as the fraction of pions entering the target which come to rest within. This was in agreement with what was estimated with somewhat less confidence from the range curve.

These results enabled us to account for the counting rates observed for the electrons from μ-decay. Under normal operating conditions we had 10^5 pions per minute coming to rest within the target. Virtually all of these decayed into muons. Since the range of the muons is only 1.4 mm in scintillator plastic, most of these, about 95%, came to rest and decayed within the target. Of the electrons which were emitted in this process 1.8% were within the acceptance angle of the spectrometer. From the shape of the spectrum, Fig. 7, and the resolution curve, Fig. 3, we were able to deduce the momentum fraction of the electrons accepted by the spectrometer as being 4.7 % at the 450 ampere setting (39.9 MeV/c). Thus, we could expect a counting rate for T_{456} of 82 per minute. We can understand the observed rate of 65 per minute if we suppose that the counters were each about 92% efficient, as seems reasonable.

VI. μ-ELECTRON SPECTRUM

Following T_{456} over the range of the spectrometer gave the spectrum of the μ-electrons. The data given in Fig. 7 were obtained with counter 3 in anticoincidence with 4,5,6 in order to reduce the accidental coincidences between singles in 4 and doubles in 5,6. Since this would reduce the sensitivity to π-electrons the anticoincidence was removed for the points above 56 MeV/c. The general features of the μ-electron spectrum are easily recognized, the intensity falling rather abruptly by a factor of about 400 in the vicinity of the expected end point. Scattering may account for some of the counts observed above 60 MeV/c, but the larger part must have been due to accidentals. We write

$$A_{4,56} = 2TDS_4D_{56} \qquad (2)$$

With the pulse width $T = 10^{-8}$s, a duty factor of the cyclotron $D = 150$, a singles rates in 4 $S_4 = 3.10^5$ per minute, and a doubles rate in 5, 6 due to fast neutrons from the cyclotron $D_{56} = 10$ per minute, we find for the accidentals rate $A_{4,56} = 0.15$ counts per minute, rather close to that observed.

The curve given in Fig. 7 is not useful for a precision determination of the shape of the μ-electron spectrum, mainly because of the scattering which the electron may suffer, particularly from the lead at the openings near the source as well as near the detector. The effect depends on the energy of the electrons and is difficult to estimate in a precise way. The dominant effect, multiple Coulomb scattering from the walls of the openings, increases the apparent width of the source and detector, making the resolution somewhat wider than that observed with α-particles. The shape of the upper end of the spectrum depends on this as well as on the loss of energy which the electrons undergo in emerging from the source.

VII. ENERGY LOSS OF ELECTRONS

We were able to measure directly this energy loss by observing the height of the pulse in no. 4 which produced the triple coincidence T_{456}. For this, we photographed the traces appearing on the screen of a Tectronix 517 fast scope with equipment kindly lent us by Dr. Leona Marshall and Mr. Joseph Fischer of our laboratory. Observing, in this way, the pulse height distribution of the electrons recorded by the spectrometer at each of some 9 settings between 44 and 52 MeV/c, it became possible to obtain the spectrum near the end point corrected for the energy loss in the source. The loss due to bremsstrahlung is missed but this amounts to less than 0.2 MeV on the average and may be neglected for the purpose here.

Comparing this curve with that obtained by folding in our resolution function, Fig. 3, with the theoretical μ-electron spectrum having $\rho_{\text{Michel}} = 0.65$, we obtained a good agreement once some allowance for the effects of scattering were made. The position of the end point is rather independent of the uncertainties here and was found to be within 1/2 % of the accepted value. We could, therefore, set with some confidence the current in the spectrometer at which we could expect to observe the π-electrons.

We used the energy loss data to obtain the expected π-electron energy distribution from our source. This is plotted in Fig. 8. The fact that electrons losing less than 0.6 MeV would not be recorded by our coincidence circuits was automatically taken into account in this curve. The dotted curve in the same figure is a replot of the momentum response of the spectrometer obtained from Fig. 3 but corresponding to a setting at 760 amperes. The overlap of the two curves shows that 55% of the π-electrons would be recorded at this current setting. At 750 amperes the overlap was 52%. Both numbers would be somewhat higher if the resolution broadening due to scattering were taken into account.

Because of their broader distribution in momentum the spectrometer measures 1/12 as many of the μ-electrons at 450 amperes as it does of π-electrons at 760 amperes. A simple subtraction, performed using the highest four points of the data shown in Fig. 7 gives a preliminary value of $f = (4 \pm 4)\cdot 10^{-5}$, a result not much better and hardly more useful than that of Lokanathan and Steinberger[13].

VIII. DELAYED COINCIDENCE MEASUREMENTS

A considerable reduction in the accidental rate was obtained in the observations made with counter 3 in delayed coincidence with the simultaneous triples count in 4,5,6. The pulses from all counters were shaped by means of clipping lines at the anodes of the photomultiplier tubes to have a width fairly close to 10 mμs. We were able to verify, by means of delay curves, that the channel width for observing T_{45} was indeed twice this amount. A typical delay curve obtained by delaying the pulses from no. 4 with respect to those from 5,6 is given in Fig. 9. All the timing of the pulses was set from delay curves of this type. In particular, the delay in no. 3 was set by observing the coincidences due to pions traversing no. 3 and no. 4, then delaying the pulse in no. 3 until D_{34} fell to the accidentals rate. Typically, this delay amounted to 20 mμs, although it was varied somewhat from this value in accordance with changes in the delay curves observed from time to time, particularly when changes in the photomultipliers and the voltages at which they were operated were made.

The fraction of π-e events which would be recorded in a channel whose time response as a function of the delay t is A(t), was given by

$$a_1 = \int_{-\infty}^{\infty} A(t) \exp\left[-\left(\frac{t+t_0}{c}\right)\right] dt. \qquad (3)$$

Here, where the quantity A(t) is obtained from the delay curves and normalized such that $\int_{-\infty}^{\infty} A(t)dt = 1$, t_0 is the delay added to the pion pulse (in counter no. 3) and τ is the lifetime of the pion decay, taken to be 25.5 mμs[19]. For $t_0 = 0$, a_1 turned out, variously, to be between 1.03 and 1.12. In a particular case (Run III) we used t_o=20 mμs and had a_1 = 0.455. To catch an additional fraction we added 27.9 mμs in a second channel for which we then had a_1 = 0.164. A further 28 mμs delay was added to the pion pulse arriving in a third channel so that a further fraction amounting to 0.055 would be recorded there. The total fraction of π-e events which would be recorded in all three channels was, thus, 0.744. With this arrangement, a comparison of the counting rates in the three channels gives some possibility for distinguishing the π-decay electrons from other processes having a different time relationship.

The arrangement used is indicated in Fig. 10. Five quadruple coincidence circuits were constructed, as nearly alike as possible. Circuit A was used to record the monitor count D_{24}, circuit B for the triples count T_{456}, while circuits C, D, F were used for the delayed coincidence quadruples Q_{3456}. In various runs we interchanged the order of C, D and F, so as to reduce somewhat the effect of differences in the response of these. The circuits were similar to some used previously in this laboratory[20] as modified by Davidon and Frank[21]. A preferable procedure would have been to form separately D_{34} and T_{456} and then taking the delayed coincidence of these two; except for the difficulty of forming a fast enough triggered output from the first set of coincidence circuits.

With our arrangement the accidentals rate due to singles S_3 and triplets T_{456} is given by,

$$A_{3,456} = (T' + T'')\, DS_3 T_{456}. \qquad (4)$$

At 450 amperes T_{456} is due to μ-electrons and was 65 per minute under normal conditions (D_{24} = $13 \cdot 10^3$ per minute). Taking the duty factor D = 150, $S_3 = 12 \cdot 10^5$ per minute, and the sum of the pulse widths T' from no. 3 and T" from the smallest of either 4, 5 or 6, to be 20 mμs, we see that we could expect $A_{3,456}$ about 4 counts per minute. The true quadruples counts due to the π-μ-e decay can only be about 0.6 counts per minute (less, in the first channel) due to the narrow width of the channels, here 1/100 of the lifetime of the μ-decay. Thus, the observed quadruples rate at 450 amperes is primarily due to accidentals and can be taken as an experimental measure of the coefficient by which T_{456} should be multiplied to give the number of accidentals expected at some other setting of the spectrometer. The accidental quadruples rate is seen to be about 1/16 of the triples rate. Since, as we have seen, most of the triples counts above 60 MeV/c are themselves accidentals we could expect a somewhat smaller ratio. We found, in fact, counting rates of the order of 1 in 100 minutes so that it was necessary to count for many hours to take advantage of the improved sensitivity of our arrangement.

IX. SEARCH FOR π-ELECTRONS

We looked for the electrons from pion decay by setting the spectrometer at 760 amperes (67.3 MeV/c) and also at 750 amperes (66.4 MeV/c. These measurements were always interlaced with measurements of the μ-electrons at 450 amperes (39.9 MeV/c) and sometimes with measurements at 830 amperes (73.3 MeV/c), above the π-electron position, at and 650 amperes (57.6 MeV/c), below the π-electron position, above the end point of the μ-electron spectrum, but where might be found electrons from the process π→e + ν + γ. The results are summarized in Tables I and II.

TABLE I

Total counts obtained in the search for electrons from the decay of the pion.

RUN	I	I	II	II	II	II	II	II
Amperes	450	750	450	760	450	650	700	750
MeV/c	39.9	66.4	39.9	67.3	39.9	57.6	62.0	66.4
Configuration	FDC	FDC	FDC	FDC	CFD	CFD	CFD	CFD
Minutes	297.6	2512.8	21.7	363.6	31.4	74.5	311.7	75.5
A D_{24}	$2.5\cdot10^{6}$	$23.6\cdot10^{6}$	$0.3\cdot10^{6}$	5.10^{6}	$0.4\cdot10^{6}$	1.10^{6}	4.10^{6}	1.10^{6}
B T_{456}	11953	358	1644	57(a)	1644(b)	21(c)	32(d)	14(e)
C Q_{3456}	1044	21	68	1	164	1	0	0
D Q_{3456}	1078	35	75	2	88	1	1	0
F Q_{3456}	1600	34	231	11	192	0	7	1

RUN		II	II	II	III	II	III	III	III
Amps.	760	450	650	760	830	450	650	760	830
Mev/c	67.3	39.9	57.6	67.3	73.3	39.9	57.6	67.3	73.3
Config.	DCF	DCF	DCF	DCF	DCF	DCF	DCF	DCF	DCF
Min.	400.6	46.5	155.3	358.6	285.7	57.4	741.5	616.8	621.0
A D_{24}	5.10^{6}	$0.6\cdot10^{6}$	2.10^{6}	$4.5.10^{6}$	$3.6\cdot10^{6}$	$0.7\cdot10^{6}$	8.10^{6}	7.10^{6}	7.10^{6}
B T_{456}	71(f)	3161	92	87	76	3748	404	132	132
C Q_{3456}	4	172	4	2	4	252	13	16	4
D Q_{3456}	5	208	2	3	4	275	11	6	8
F Q_{3456}	5	186	4	2	1	277	6	5	8

(a) For $3.5\cdot10^{6}$ D_{24} (c) For $0.5\cdot10^{6}$ D_{24} (e) For $0.5\cdot10^{6}$ D_{24}

(b) For $0.3\cdot10^{6}$ D_{24} (d) For $2.5\cdot10^{6}$ D_{24} (f) For $4.0\cdot10^{6}$ D_{24}

TABLE II

Counts obtained per 10^6 D_{21}.

RUN	I - FDC	I-FDC	II-FDC	II-FDC	II-FDC	II-CFD	II-CFD	II-CF
Amperes	450	750	450	760	450	650	700	750
Minutes	111.8	106.7	72.3	72.3	78.5	74.5	77.9	75.5
B T_{456}	4781	15.2 ± 0.8	5480	16.3 ± 2.2	5480	42 ± 9	12.8 ± 2.3	28 ±
C Q_{3456}	417 ± 13	0.89 ± 0.20	277 ± 27	0.2 ± 0.2	410 ± 32	1 ± 1	0 ± 0.25	0 ± 1
D Q_{3456}	341 ± 13	1.49 ± 0.25	250 ± 29	0.4 ± 0.3	220 ± 23	1 ± 1	0.3 ± 0.3	0 ± 1
F Q_{3456}	640 ± 16	1.44 ± 0.25	770 ± 51	2.2 ± 0.7	480 ± 35	0 ± 1	1.8 ± 0.7	1 ± 1

RUN	II-CFD	II-DCF	II-DCF	II-DCF	II-DCF	III-DCF	III-DCF	III-DCF	III-DCF
Amps.	760	450	650	760	830	450	650	760	830
Min.	80.1	77.5	77.7	79.7	81.6	82.0	92.7	88.1	88.7
B T_{456}	17.7 ± 2.1	5268	46 ± 5	19.3 ± 2.1	21.1 ± 2.5	5354	50.5 ± 2.5	26.0 ± 1.9	18.9 ± 1.6
C Q_{3456}	0.8 ± 0.4	287 ± 22	2 ± 1	0.45 ± 0.31	1.11 ± 0.57	360 ± 23	1.62 ± 0.45	2.28 ± 0.57	0.57 ± 0.2
D Q_{3456}	1.0 ± 0.5	347 ± 24	1 ± 0.7	0.67 ± 0.38	1.11 ± 0.57	393 ± 24	1.37 ± 0.42	0.86 ± 0.35	1.14 ± 0.
F Q_{3456}	1.0 ± 0.5	310 ± 23	2 ± 1	0.45 ± 0.31	0.28 ± 0.28	396 ± 24	1.87 ± 0.50	0.71 ± 0.32	1.14 ± 0.

In the first run we concentrated the search at a single current 750 amperes, hoping to reveal the presence of the π-electrons by the decreasing counting rates observed in the channels arranged in the order FDC of increasing delays. When nothing significant was found, we carried out further runs changing the order of delays in the channels and changing the current setting of the spectrometer. There was no evidence whatever for a positive effect. The problem which remained was to give a measure of the uncertainty to be associated with our lack of success.

X. DATA REDUCTION

The reduction of the data was carried out by the method of least squares. We illustrate the procedure by taking the case of Run III. In this, besides the measurement at 450 amperes, used to standardize the run, we made measurements at each of three currents; below, at and above the π-electron position. In each case the delayed quadruple count was observed in each of three non overlapping delayed channels giving a total of nine measurements to be accounted for. We considered that each count m observed with statistical error n arose from two contributions. One part due to the electrons which might be expected at the given setting of the spectrometer, and a second due to accidentals. The equations are of the form

$$ax + cz = m \pm n, \qquad (5)$$

where x is the number of π-electrons which would enter the acceptance angle of the spectrometer under conditions in which 5 000 μ-electrons would be recorded as the triples T_{456}, with the current at 450 amperes. This is multiplied by a product of factors $a = a_1 a_2 a_3$; such that a_1 is the time acceptance of the channel as given by (3); a_2 is the sensitivity of the channel, being its triples count $T_{456}/10^6\ D_{24}$ measured at 450 amperes, divided by 5000; a_3 is the momentum acceptance of the spectrometer given by

$$a_3 = \int_0^\infty Q(p)F\left(\frac{p-p_o}{p_o}\right)\,dp \tag{6}$$

previously referred to as the overlap. Q(p) is the distribution in momentum of the electrons emitted from the source, normalized so that

$$\int_0^\infty Q(p)\,dp = 1.$$

$F\left((p - p_o)/p_o\right)$ is the momentum response of the spectrometer plotted in Fig. 3. For the π-electrons as we could expect to find them at 760 amperes, these functions are plotted together in Fig. 8 and give $a_3 = 0.55$, as has already been mentioned. At 650 amperes there is some uncertainty about the distribution function, so we rewrite the first terms as $\underline{b}\ \underline{y}$ with $\underline{b} = a_1a_2$ and $\underline{y} = a_3x$. At 830 amperes $a_3 = 0$ so the first term in (5) does not appear at all.

The accidentals are estimated in the factor $\underline{c}$ from the accidentals observed at 450 amperes. Subtracting a small correction Q_T of true quadruples expected from the π-μ-e decay from the observed quadruples $Q_{3456}(450)$ observed at 450 amperes, we multiply by the ratio triples count at current setting S to that at 450 amperes, thus,

$$c = \left[Q_{3456}(450) - Q_T\right]\left[T_{456}(S)\right]/T_{456}(450) \tag{7}$$

By itself, $\underline{c}$ does not give a correct estimate of the accidentals rate for currents S above the μ-electron end point. This is because the triples count obtained there arises, at least in part, from the accidental coincidences between the counts in no.4 and those in 56, in accordance with estimates given above. Such accidental triples are somewhat less effective in producing an accidental quadruple than are true triples.

For this reason we multiply c by a proportionality factor z whose value may then be adjusted to give a best fit to the data by the least squares criterion. Here we have supposed that the same factor z serves for all the circuits at all the currents used in a given run, and that any differences in response to accidentals will have been already included in the factor c.

For Run III, with configuration DCF we obtained the following nine equations:

				check
650 A	circuit	C	$0.172y + 2.56z = 1.62 \pm 0.45$	(1.59)
		D	$0.455y + 2.84z = 1.37 \pm 0.42$	(1.37)
		F	$0.057y + 2.86z = 1.87 \pm 0.50$	(1.98)
760 A		C	$0.090x + 1.39z = 2.28 \pm 0.57$	(0.96)
		D	$0.250x \quad 1.54z = 0.86 \pm 0.35$	(0.98)
		F	$0.031x + 1.55z = 0.71 \pm 0.32$	(1.11)
830 A		C	$1.00z = 0.57 \pm 0.29$	(0.72)
		D	$1.11z = 1.14 \pm 0.40$	(0.80)
		F	$1.12z = 1.14 \pm 0.40$	(0.81)

The solution of this set of equations by the method of least squares gave the following results:

$$x = -0.53 \pm 1.6$$

$$y = -1.51 \pm 1.2$$

$$z = 0.72 \pm 0.11$$

Reinserting these values in the left hand side of the above equations gave the values shown in the parentheses. These serve to check the goodness of the fit and the statistical significance of the data.

For Run I with configuration FDC the equations are;

			check
750 A circuit	C	$0.027x + 1.23z = 0.89 \pm 0.20$	(1.06)
	D	$0.089x + 1.28z = 1.49 \pm 0.25$	(1.06)
	F	$0.272x + 1.97z = 1.44 \pm 0.25$	(1.55)

with solutions:

$$x = -0.61 \pm 1.7$$

$$z = 0.87 \pm 0.18$$

For Run II with configuration FDC the equations are:

760 A circuit	C	$0.026x + 0.51z = 0.2 \pm 0.2$	(0.20)
	D	$0.069x + 0.58z = 0.4 \pm 0.3$	(0.55)
	F	$0.247x + 2.11z = 2.2 \pm 0.7$	(1.96)

with solutions;

$$x = 8.0 \pm 7.1$$

$$z = 0.0 \pm 0.7$$

For Run II with configuration CFD the equations are:

			check
650 A circuit	C	$0.424y + 2.87z = 1 \pm 1$	(0.7)
	D	$0.042y + 1.25z = 1 \pm 1$	(0.4)
	F	$0.150y + 3.44z = 0 \pm 1$	(1.1)
700 A	C	$0.424y + 0.84z = 0 \pm 0.3$	(0.0)
	D	$0.042y + 0.37z = 0.3 \pm 0.3$	(0.1)
	F	$0.150y + 1.00z = 1.8 \pm 0.7$	(0.3)
750 A	C	$0.223x + 1.89z = 0 \pm 1$	(0.79)
	D	$0.022x + 0.82z = 0 \pm 1$	(0.30)
	F	$0.079x + 2.26z = 1 \pm 1$	(0.83)
760 A	C	$0.234x + 1.12z = 0.8 \pm 0.4$	(0.52)
	D	$0.024x + 0.49z = 1.0 \pm 0.5$	(0.18)
	F	$0.082x + 1.34z = 1.0 \pm 0.5$	(0.51)

In writing the equations for the run at 700 amperes we neglected any difference in the yield of electrons which there might be at this setting compared with 650 amperes. We obtained the following solutions:

$$\begin{aligned} x &= 0.56 \pm 1.7 \\ y &= -0.60 \pm 0.65 \\ z &= 0.35 \pm 0.17 \end{aligned}$$

For Run II configuration DCF the equations are:

					check
650 A	circuit	C	$0.151y + 2.03z = 2$	± 1	(1.3)
		D	$0.386y + 2.55z = 1$	± 0.7	(2.2)
		F	$0.053y + 2.23z = 2$	± 1	(1.1)
760 A		C	$0.084x + 1.83z = 0.45$	± 0.31	(0.79)
		D	$0.213x + 1.04z = 0.67$	± 0.38	(0.48)
		F	$0.029x + 0.91z = 0.45$	± 0.31	(0.45)
830 A		C	$1.89z = 1.11$	± 0.57	(0.81)
		D	$1.12z = 1.11$	± 0.57	(0.48)
		F	$0.98z = 0.28$	± 0.28	(0.41)

with solutions:

$$\begin{aligned} x &= 0.15 \pm 2.0 \\ y &= 2.9 \pm 2.0 \\ z &= 0.42 \pm 0.13\ . \end{aligned}$$

The overall fit of the data may be judged from the point of view of the χ^2 criterion. In all, 36 equations have been fitted with 13 variables yielding $\chi^2 = 29$, a value not excessively greater than the number of degrees of freedom 23. In our equations we used as the error assigned to the observed count, only the uncertainty based on the total number of counts recorded. In a counting experiment involving rather complicated electronic equipment used over a long period of time, this must certainly be an underestimate. Drifts and changes in the sensitivity of the equipment occur and in spite of the use of interlacing of the measurements and careful monitoring, some additional uncertainty enters. In view of this, we believe that the fit obtained is good and that our method for reducing the data is justified.

Some increase in the stated error seems indicated. We choose this to be the amount which brings χ^2 down to its expectation value. A 12 % increase applied uniformly to all the errors as given above has just this effect and should serve to improve the statistical significance of our final result. This has been done in giving the weighted averages obtained from the above solutions,

$$\bar{x} = -0.04 \pm 0.92 \tag{8}$$

$$\bar{y} = -0.77 \pm 0.62 \tag{9}$$

We do not observe a positive effect, either for $\pi \rightarrow e + \nu$ or for $\pi \rightarrow e + \nu + \gamma$.

XI. -π-e DECAY

The fraction f of π-e to π-μ decays may be obtained from the value of $\bar{x}$, provided the momentum fraction G_μ of μ-electrons accepted by the spectrometer is known. Since our analysis for $\bar{x}$ refers to a counting rate $R_\mu = 5\,000$, we write,

$$f = \frac{1}{5000} G_\mu \bar{x} r, \tag{10}$$

where the subscript μ refers to values at 450 amperes. The factor r is the fraction of μ which do not escape from the source before decaying and is about 0.95. We may obtain G_μ from the observed μ-electron spectrum, as follows:

For the counting rate with the spectrometer set at p_o we write,

$$R(p_o) = N\int_o^\infty Q'(p)\, F\left(\frac{p-p_o}{p_o}\right) dp = NG(p_o) \qquad (11)$$

The distribution function of the μ-electrons is supposed normalized so that $\int_o^\infty Q'(p)dp = 1$. Neglecting the variation of Q' over the range of F we may take it outside the integral in (11). Since $H = \int_o^\infty F((p-p_o)/p_o)\, dp/p_o$ is a constant for the spectrometer, independent of p_o, we have

$$J = \int_o^\infty R(p_o)\ dp/p_o = NH\int_o^\infty Q'(p_o)dp_o = NH\ . \qquad (12)$$

Thus,

$$G_\mu = R_\mu\ H/J\ . \qquad (13)$$

From the data of Fig. 7 we obtain $R_\mu/J = 1.27$, while from Fig. 3, $H = 0.037$, giving $G_\mu = 0.047$. This should be increased by about 10 % to take account of the broadening due to electron scattering since this affects the μ-electron response somewhat more than that of the π-electrons. Finally,

$$f = 0.98 \cdot 10^{-5}\ \overline{x}\ . \qquad (14)$$

Our result for the fraction of π→e + ν decays is

$$f = (-0.4 \pm 9.0)\cdot 10^{-6}\ .$$

This appears to be statistically significant and thereby allows only a 1 % probability that f could be greater than $2.1\cdot 10^{-5}$.

XII. - π-e-γ DECAYS

We can obtain some information about the process π → e + γ + ν from our value of $\overline{y}$. We write

$$f_\gamma = \frac{r}{5000}\ \frac{G_\mu}{G_\gamma}\ \overline{y}\ , \qquad (15)$$

where G_γ, in analogy with G_μ, is the fraction of π-γ electrons accepted by the spectrometer at 650 amperes. We need to know the distribution function $Q''(p)$ of these electrons. Then,

$$G_\gamma = \int_0^\infty Q''(p)\, dpF\left(\frac{p-p_\gamma}{p_\gamma}\right) = Q''(p_\gamma) \cdot H, \quad (16)$$

where p_γ = 60.3 MeV/c, account having been taken of the 3.0 MeV average energy loss in the source.

The value of f_γ will depend on what is taken for the distribution function. Since we are dealing with a three body decay of relativistic particles the distribution may be expected to be similar to that of the μ-electrons. We obtain an order of magnitude by taking $G_\gamma = G_\mu$. Then $f_\gamma = (-1.5 \pm 1.2)\cdot 10^{-4}$. The result is not very different if the distribution given by Eguchi[22] assuming tensor interaction is taken, from which $G_\mu/G_\gamma = 1.36$ and we would then have

$$f_\gamma = (-2.0 \pm 1.6)\cdot 10^{-4}.$$

Recently, Cassels[15] has reported a much lower limit than this by looking for e-γ coincidences in the π-decay, giving $f_\gamma = (3 \pm 7)\cdot 10^{-6}$ assuming tensor interaction and the e-γ correlations predicted thereby. A lower limit than ours has also been reported by Lokanathan[15].

XIII. CONCLUSIONS

The non-occurence of any kind of electronic decay of the pion is now established well below the limits set by the explanations thus far offered in terms of an effect of mass alone. We may conclude that there is a more essential difference which distinguishes the electron from the muon in its interaction with the neutrino field. Within the framework of the idea that the electron interacts with the pion through the intermediary of a nucleon antinucleon pair, our result implies that not only the pseudoscalar, but also the axial vector coupling must be quite small.

Recently, Morpurgo[7] has proposed a way of understanding the behavior of the pion within the framework of the universal Fermi interaction. His idea is to forbid the process $\pi \rightarrow e + \nu$ by supposing that only the scalar and tensor (possibly also the vector) couplings intervene. For the π-μ decay, on the other hand, he supposes that the order in which the factors must be written in the Fermi interaction is different than for the electron. With a suitable choice the π-μ decay becomes allowed. With this scheme $\underline{f}$ may be made arbitrarily small and the value of $\underline{f}_\gamma \simeq 10^{-5}$, not inconsistent with present experimental limits.

We received a great deal of help in carrying through this experiment from our students, especially Mr. Richard Miller and Mr. Tadao Fujii, whose skill, patience and intelligence were invaluable to us. We are grateful to Mr. Paul Fields and Mr. Ray Barnes of the Argonne National Laboratory for the plutonium source they prepared and loaned to us. Dr. Leona Marshall and Mr. Joseph Fischer placed kindly at our disposal their pulse height analyzer. We also want to credit Mr. Clovis Bordeaux for the long steady operation of the cyclotron. One of us, (H. L. A.) profited from interesting discussions with Dr. G. Morpurgo and wishes to thank the Guggenheim Foundation for a Fellowship and the Institute of Physics "Guglielmo Marconi", Rome, for the hospitality which made the completion of this work possible.

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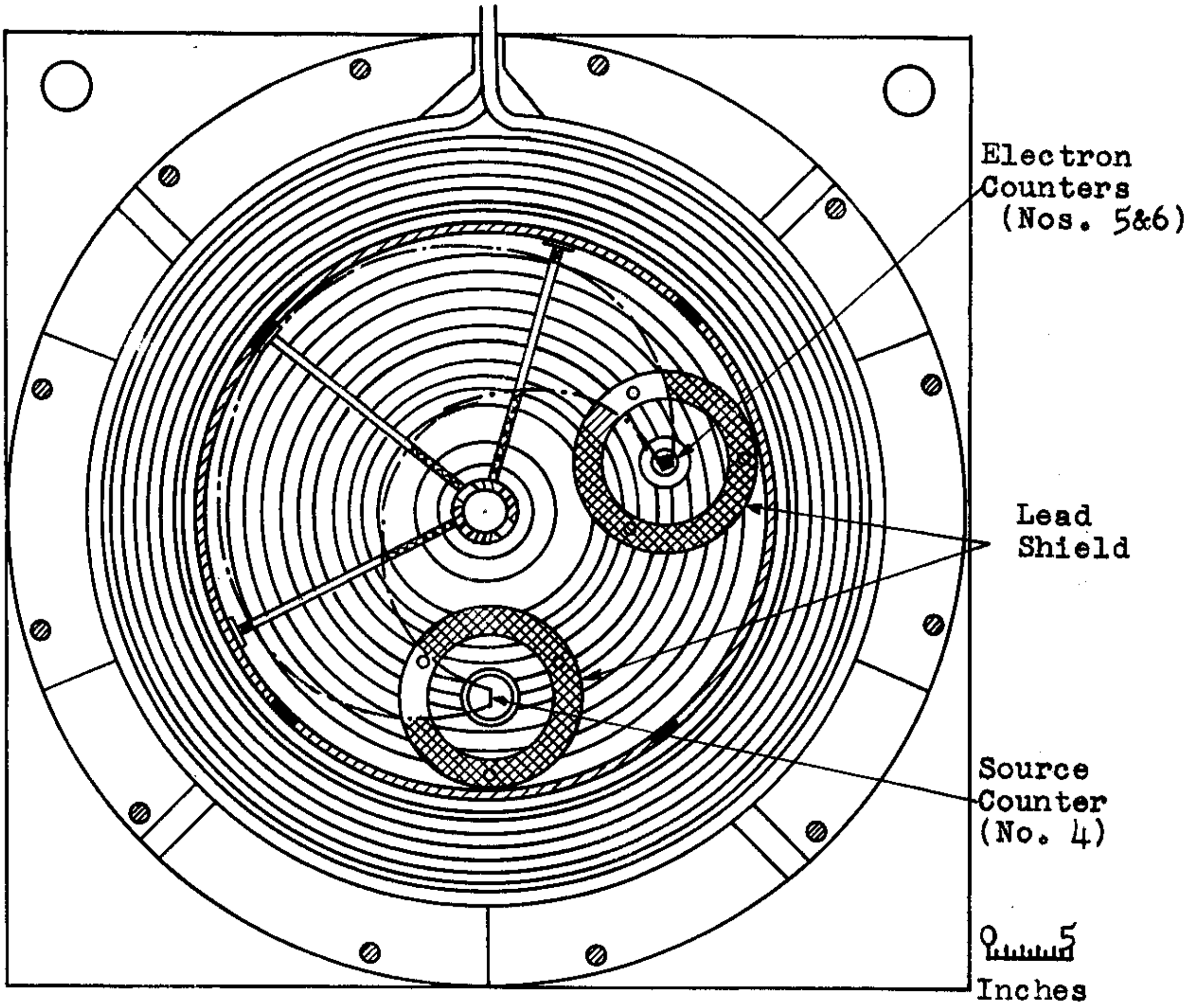


Fig. 1

Sectional view of the spectrometer. Pions enter through the brass collimator, traverse counters no. 2 and no. 3, are slowed by the polyethylene moderator and come to rest in counter no. 4. Electrons emerging from counter no. 4 are brought to focus and detected by means of a triple coincidence of 4, 5 and 6.

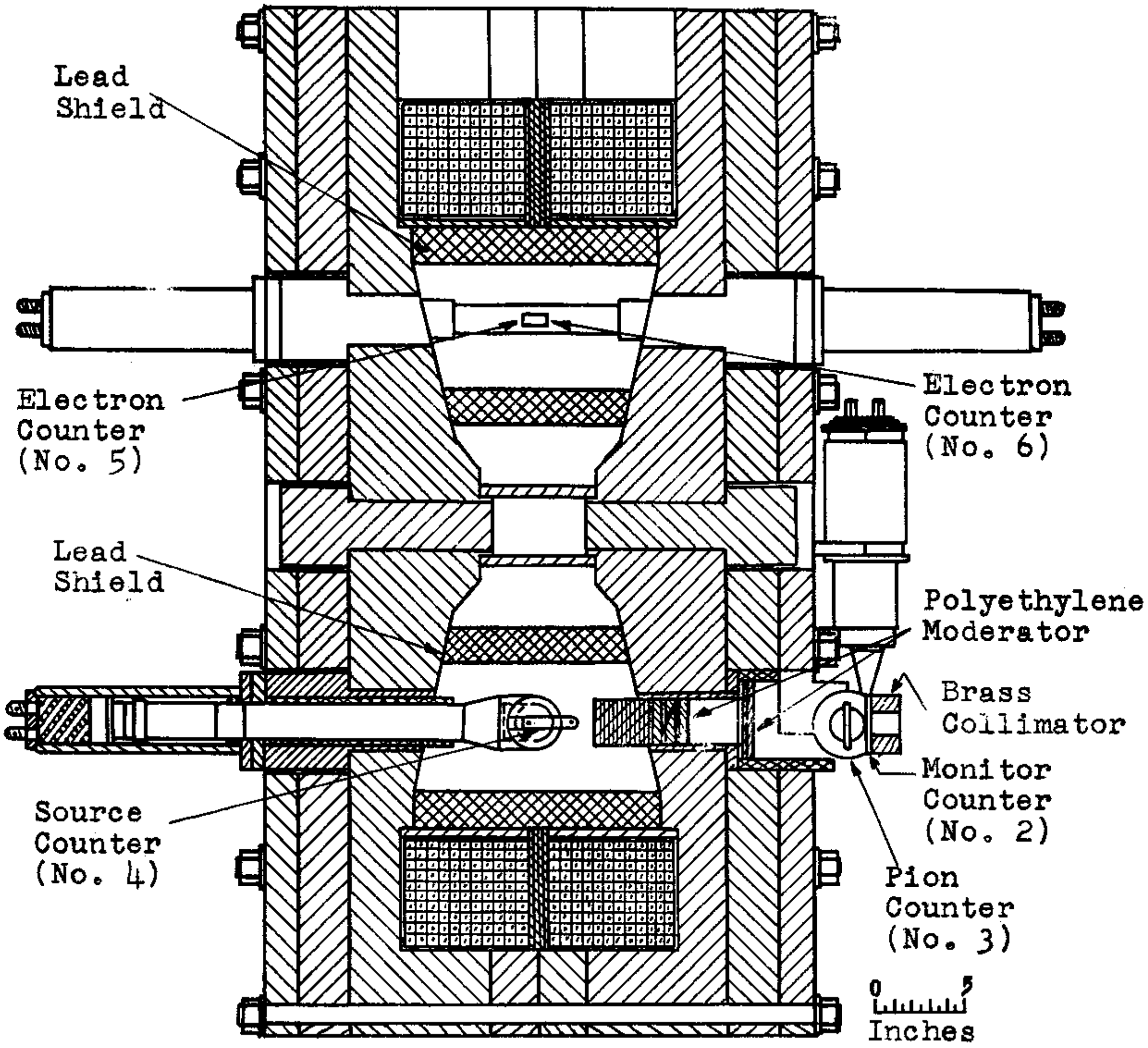


Fig. 2

Sectional plan view of the spectrometer showing the extreme electron trajectories.

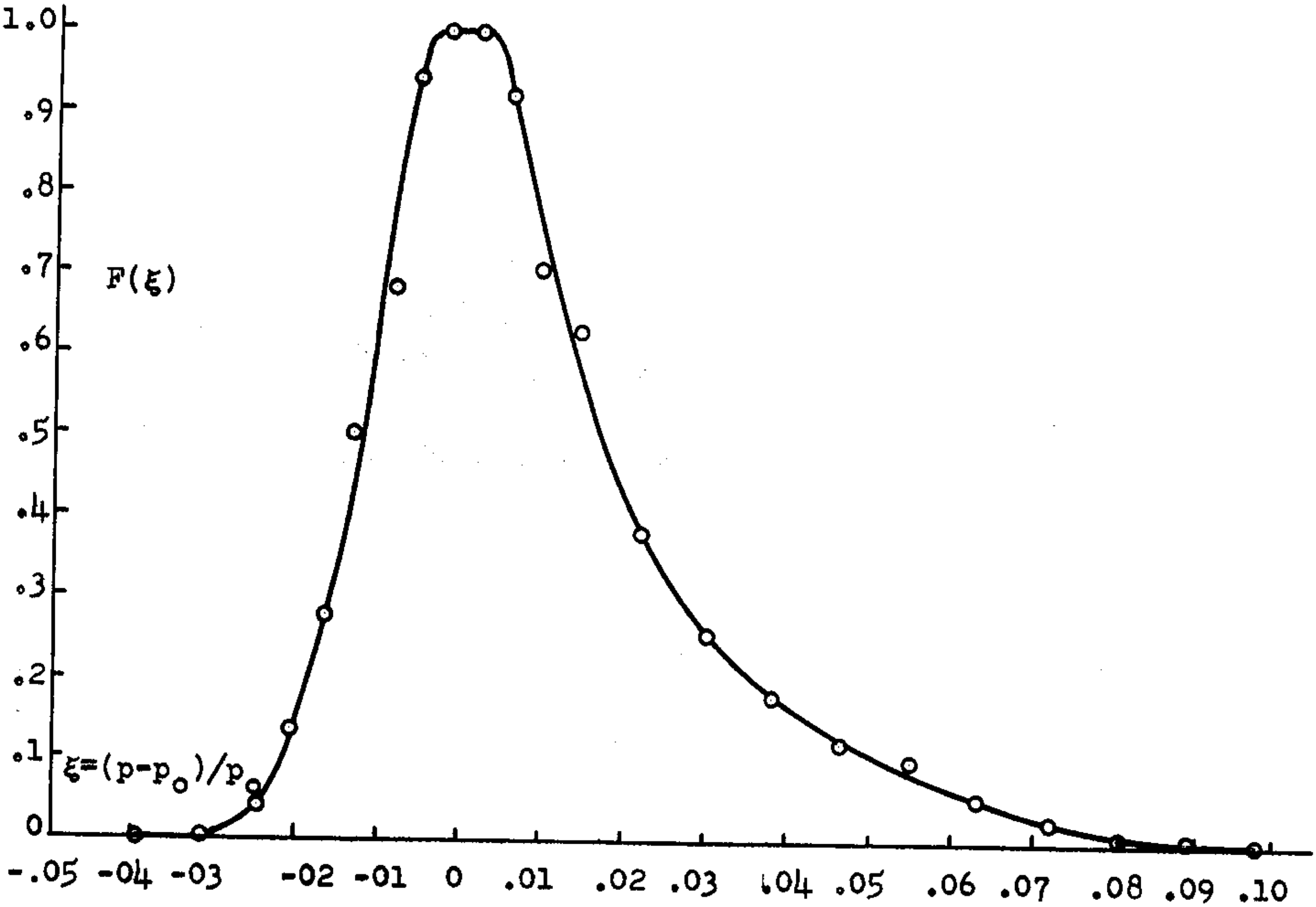


Fig. 3

Momentum response of the spectrometer as obtained by observing α-particles of ^{239}Pu (97.96 MeV/c).

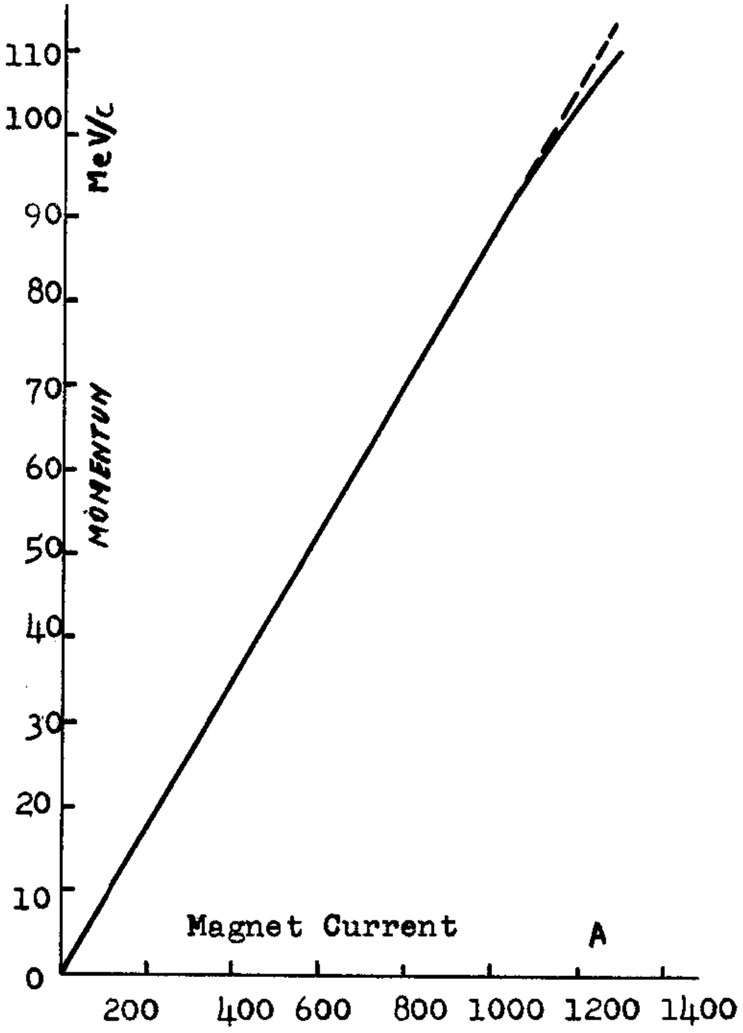


Fig. 4

Calibration curve of the spectrometer. The dotted line is the linear extension of the lower part to show the effect of saturation.

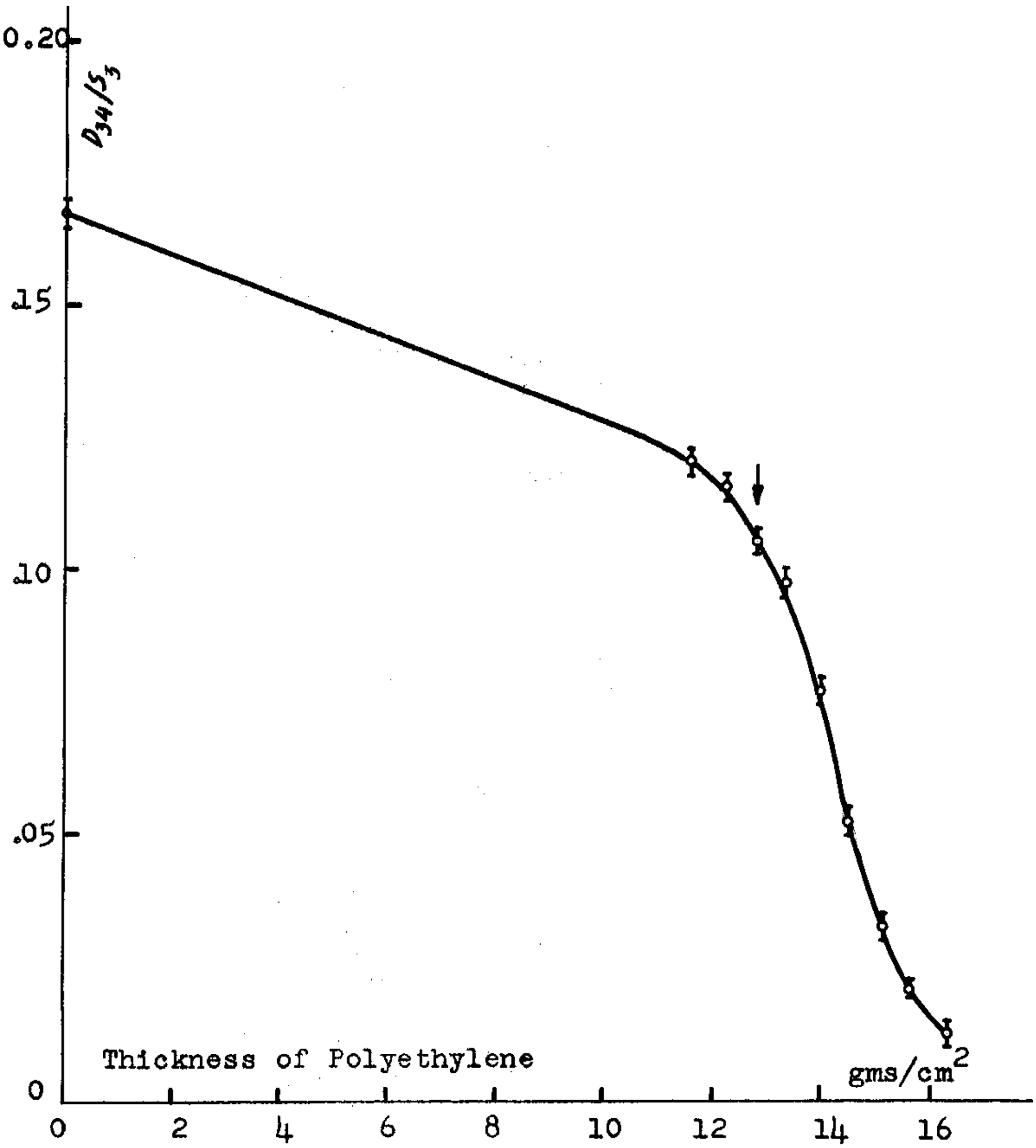


Fig. 5

Range curve of the pions in polyethylene. The arrow points to the thickness of polyethylene used in the experiment to bring the pions to rest in the target.

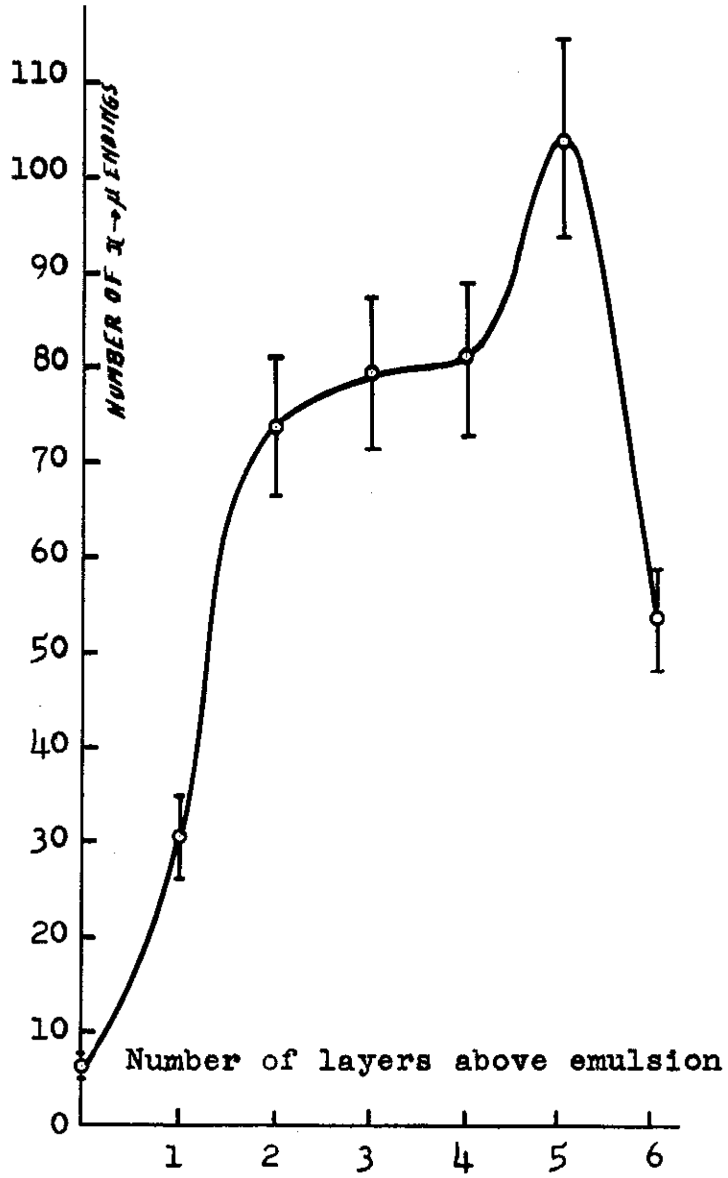


Fig. 6. Distribution of $\pi \rightarrow \mu$ endings in the target. Plotted as ordinate is the number of endings in 1 cm^2 of 200 μm emulsion per 25000 counts of D_{23}, for various numbers n of 3.18 mm thick layers of scintillator placed above and at an angle of 45° to the pion beam.

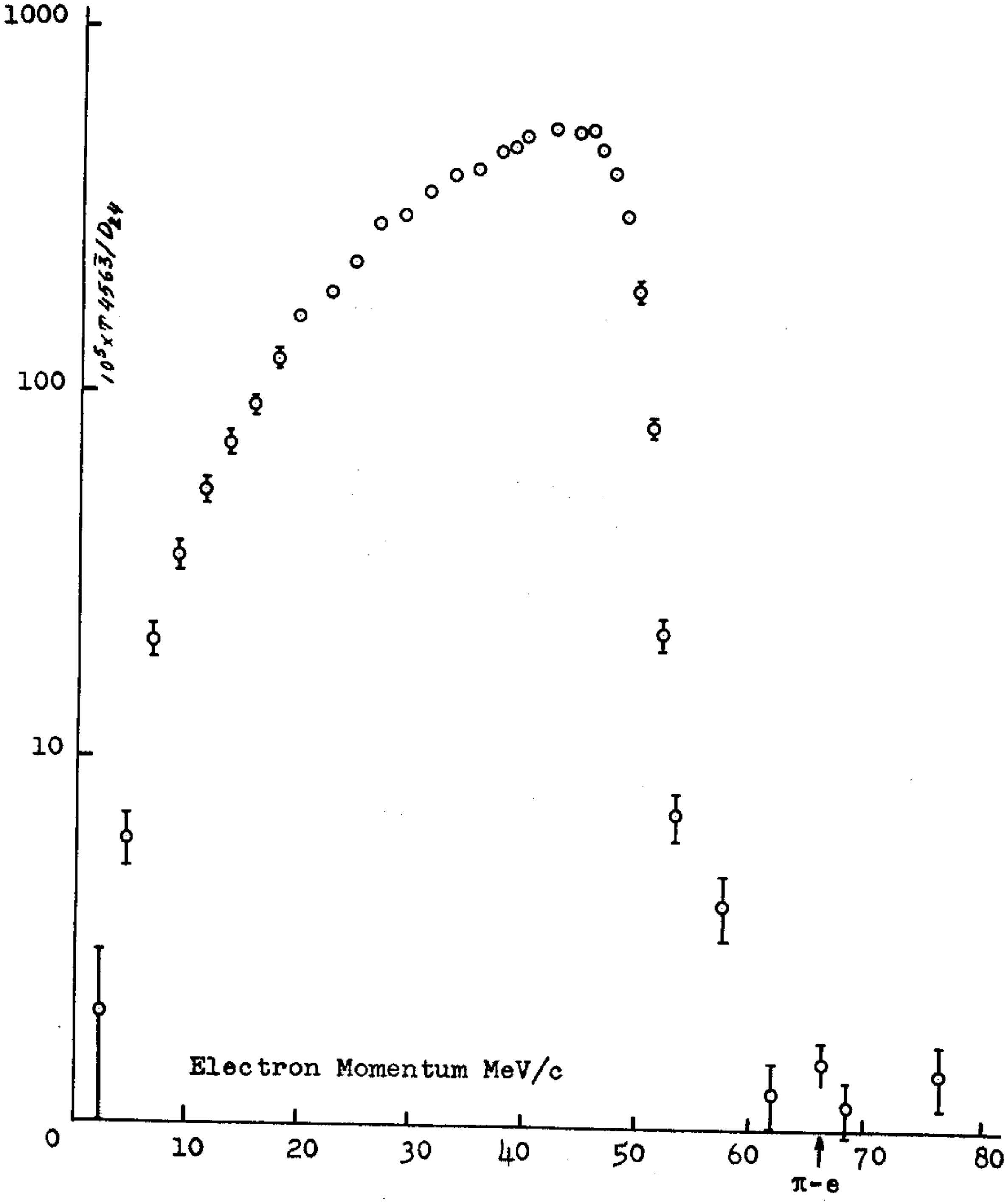


Fig. 7

Electron spectrum obtained by observing the triples T_{456} per 10^5 D_{24}. To reduce accidental coincidences in the μ electron spectrum, anti-coincidence in no.3 was added for the counts below 55 MrV/c.

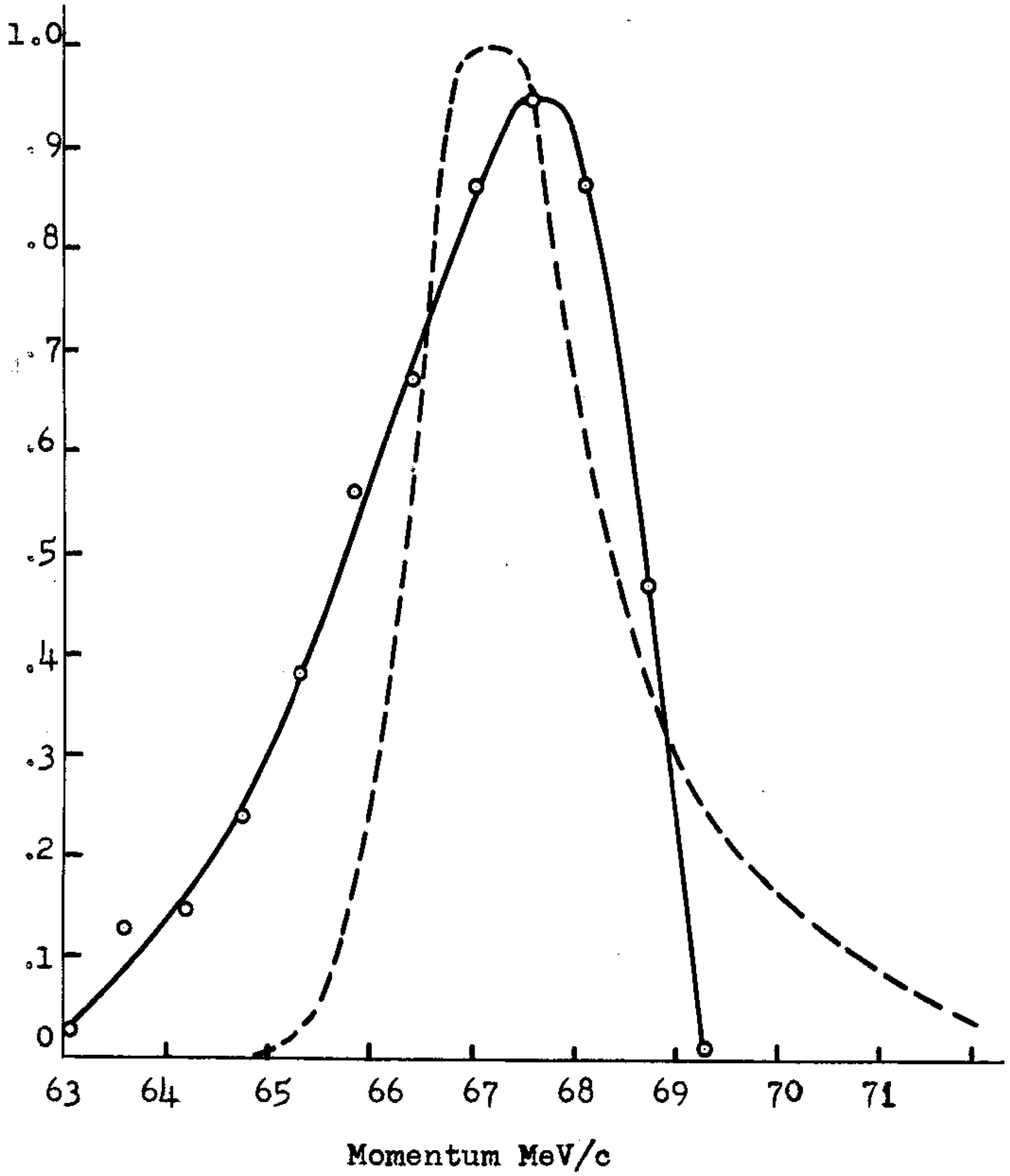
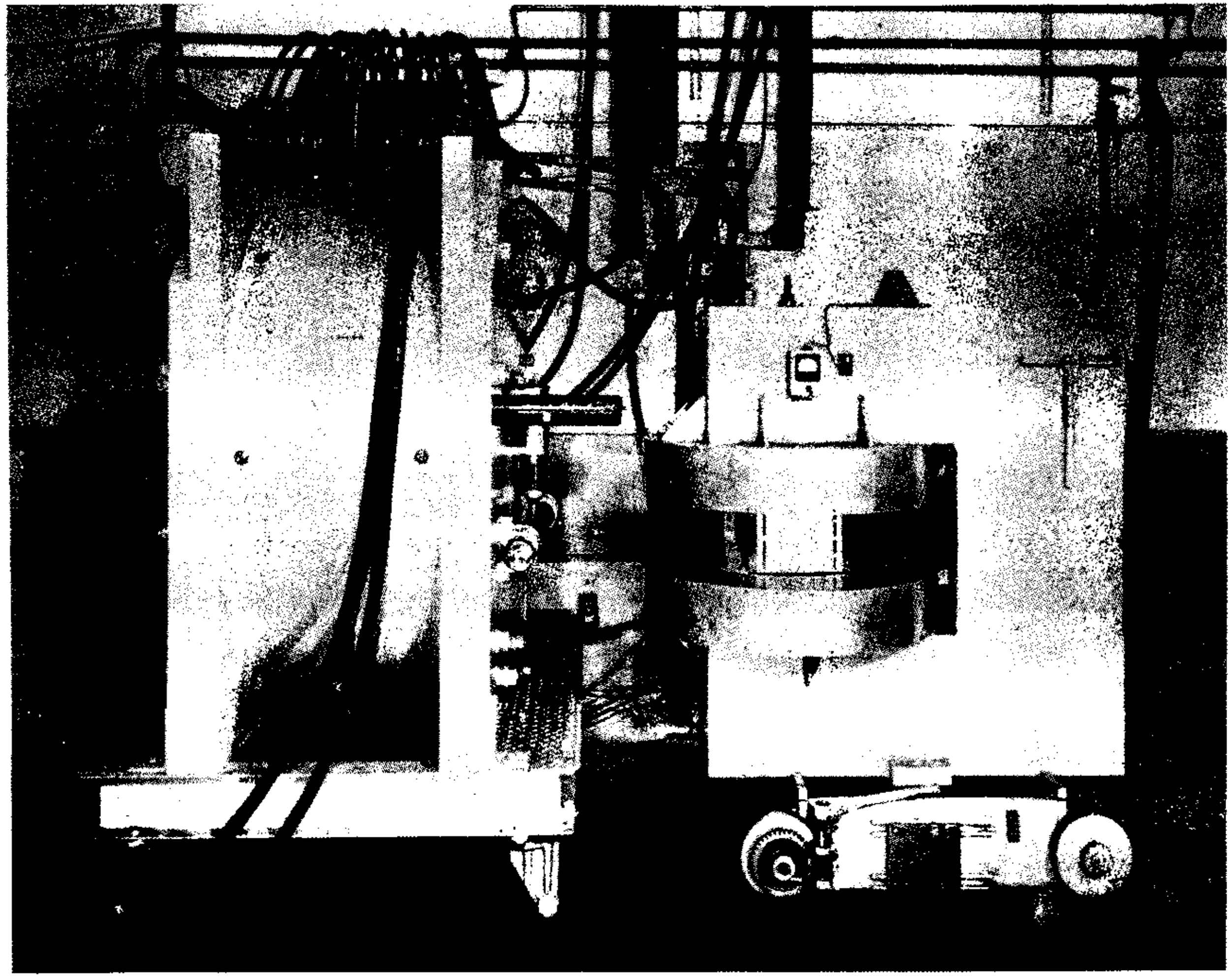


Fig. 8a

Momentum distribution expected for the π electrons due to energy loss in the source (full curve). The points were obtained from the pulse height distribution in no. 4 due to μ-electrons observed at 450 amperes. For μ-electrons, the enitial energy was to be taken at 69.8 MeV. The dotted curve is the momentum response of the spectrometer for a current setting of 760 amperes.



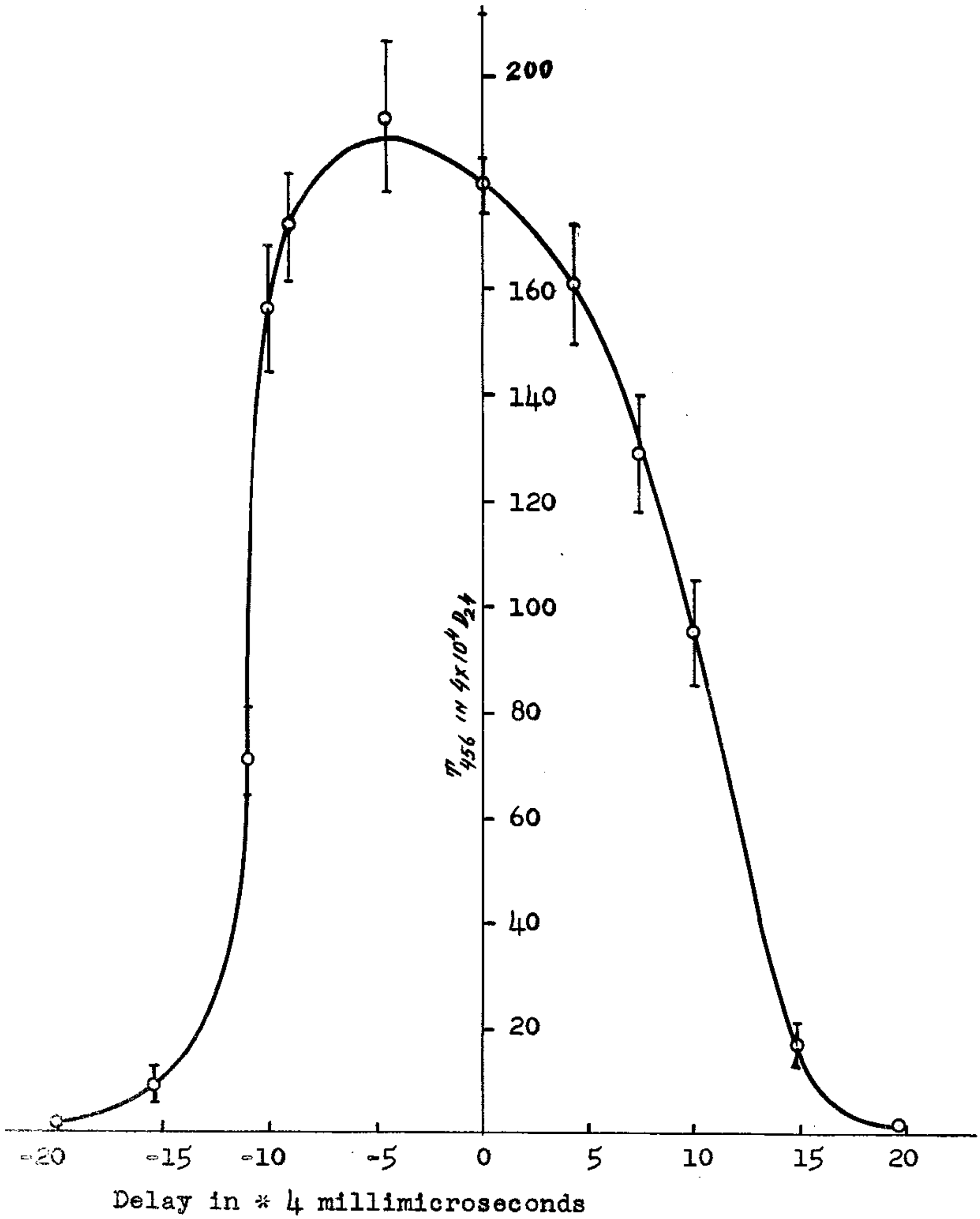


Fig. 9

Delay curve obtained by delaying counter no. 4
with respect to 5, 6 in channel C.

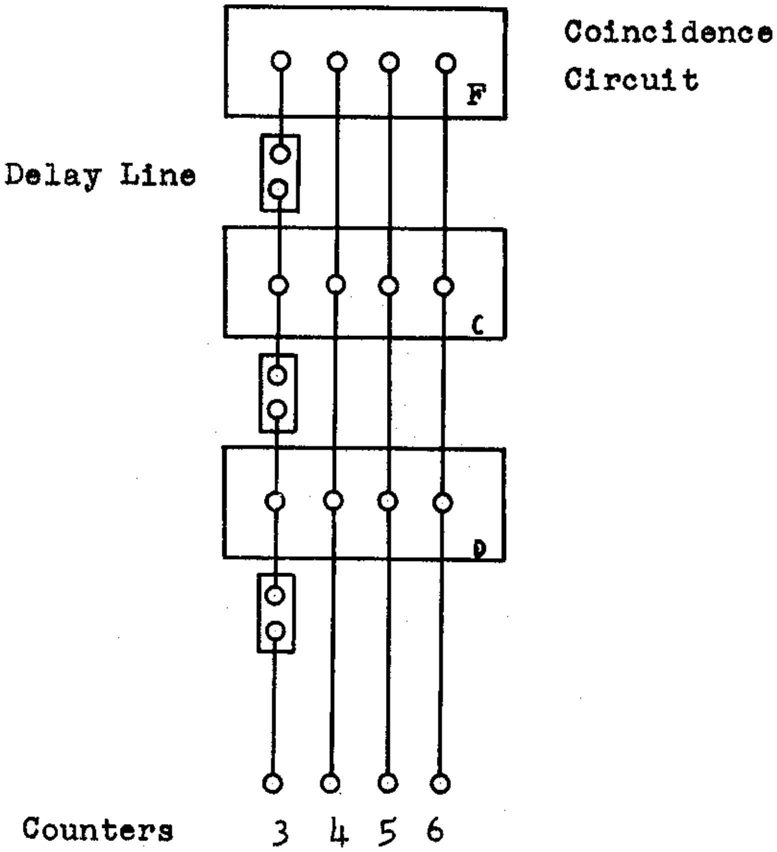


Fig. 10. Block diagram of the quadruple coincidence circuits showing the manner of delaying the pulse from no. 3.