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PAIR-CREATION CROSS SECTION OF SPIN ONE-HALF PARTICLES POSSESSING AN ANOMALOUS MAGNETIC MOMENT* +

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INTRODUCTION

Very recently the pair production of μ-mesons by gamma rays was measured at Stanford University[1]. The experimental cross section is used in the present paper to estimate the magnetic moment of the μ-meson.

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Since the μ-meson does not seem to possess any strong nuclear interaction, and since its spin[2] is most likely 1/2, its magnetic moment is expected to be close to the value $e\hbar/2mc$, predicted by the Dirac equation. The deviation from the Dirac value is expressed in terms of a dimensionless parameter λ, such that the value for the total magnetic moment is given by $(1 + \lambda)e\hbar/2mc$. Fowler[3] reviewed the various sources of experimental information concerning the μ-meson, with the conclusion that $\lambda \ll 1$. This result is based mainly on the high-energy bremsstrahlung processes which give rise to cosmic-ray burst. However, the method of calculation used in Fowler's analysis has questionable validity[4] at these high energies. The pair-creation experiment mentioned above, being done near threshold, therefore provides welcome additional information on the μ-meson.

The assumptions used in this calculation are the following: The μ-meson is treated as a spin - 1/2 particle and its anomalous magnetic moment, taken as being small but different from zero, is described phenomenologically by means of a Pauli term

$$\lambda(e\hbar/2mc)(1/2)F_{\mu\nu}\gamma_\mu\gamma_\nu \tag{1}$$

The effect of this additional interaction upon the pair-creation cross section is calculated in first Born approximation using the Feynman rules[5] for writing the matrix element. The physical processes and interactions which give rise to λ are thereby ignored. Nevertheless, the present calculation is expected to be valid for energies which are sufficiently large so that the interactions of the μ-mesons in the final state can be neglected, but small enough so that no additonal particles will be created. The value of λ, used in the type of phenomenological procedure chacterized by the term (1), can be taken as a measure of the deviation from the behavior of the "normal" spin 1/2 particle, and the limits for λ obtained below indicate that the μ-meson is indeed close to being a "heavy electron".

In Sec. I the main steps for the calculation of the differential cross section for pair production as a function of λ are described. In Sec. II the result is integrated over the variables of one of the pair particles in order to obtain the experimentally observed cross section. The finite size of the bombarded nucleus is taken into account by means of a nuclear form factor in a manner similar to a procedure previously presented[6]. The details of the calculation and complete theoretical formulas, which are too lengthy to be given here, are contained in the author's dissertation[7].

I. OUTLINE OF THE CALCULATION

The addition of the term (1) to the Dirac equation yields, for the total interaction Hamiltonian, the expression

$$H_{int} = ie\gamma_{\mu}A_{\mu} - i\lambda(e\hbar/2mc)(1/2)F_{\mu\nu}\gamma_{\mu}\gamma_{\nu} \qquad (2)$$

where $F_{\mu\nu}$ are the components of the electromagnetic field tensor given by $F_{\mu\nu} = \partial A_{\nu}/\partial x_{\mu} - \partial A_{\mu}/\partial x_{\nu}$, A_{μ} are the electromagnetic potentials and γ_{μ} are the Dirac matrices. In the momentum representation, the effect of the added term of Eq. (2) can be accounted for by the substitution

$$\gamma_\mu \rightarrow \gamma_\mu + i(\lambda/4)(\gamma_\mu \not{p} - \not{p}\gamma_\mu) \tag{3}$$

for the Dirac matrices associated with the interaction vertices. Here $\not{p}$ stands for $p_\mu \gamma_\mu$, where p_μ are the momentum components transferred to the particle from the electromagnetic field at the interaction vertex.

For the case of pair creation the Feynman diagrams contain two interactions: one with the electromagnetic field of the incident photon and the other with the electrostatic field of the recoiling nucleus. In the matrix element for this process the replacement (3) is therefore used twice, with p standing for either the recoil momentum q or the photon momentum k. Since the initial and final states are described by the plane-wave spinors $u(-p_+)$ and $u(p_-)$, where p_+ and p_- are respectively the momenta of the positively and negatively charged pair particles, the pair-creation matrix element can be simplified by the use of the following relations:

$$(i\not{p}_+ - 1)u(-p_+) = 0$$

$$\bar{u}(p_-)(i\not{p}_- + 1) = 0$$

$$k + \not{q} = \not{p}_+ + \not{p}_-$$

$$\not{a}\not{b} + \not{b}\not{a} = 2(ab) = 2a_\mu b_\mu, \tag{4}$$

where a and b are any two sample vectors.

The spur which represents the square of the absolute value of the matrix element summed over the spin states of the pair particles is a very lengthly algebraic expression in the momenta and polarization vectors involved in the process. To insure the correctness of the final result, the aforementioned simplification of the matrix element was carried out by two different methods and the two spur calculations were followed through independently. One of these methods makes use of a convenient grouping of the final terms, and the other combines terms of equal powers of λ.

The result for the differential pair-creation cross section can be expressed in the form

$$\frac{d\phi}{d\Omega_+ \, d\Omega_-} = \frac{Z^2}{2\pi}\,\frac{r^2}{137}\,\frac{4p_+ \, p_-}{k}\,\frac{1}{q^4}\,\frac{R}{2\pi}\cdot\left|\,F(q)\right|^2$$

$$R = R_{1/2} + \lambda R_1 + \lambda^2 R_2 + \lambda^3 R_3 + \lambda^4 R_4 \qquad (5)$$

where p_+ and E_+ are respectively the absolute value of the momentum and the energy of the positively charged particle emitted into the solid angle $d\Omega_+$; the subscript (-) indicates the corresponding quantities for the negatively charged particle. The number of protons contained in the recoiling nucleus is denoted by Z, $r^2/137 = (e^2/mc)^2/137 = 1.35 \times 10^{-32} cm^2$, m = 207 electron masses, k and q are the absolute values respectively of the photon and recoil momenta, and F is the nuclear form factor[6]. The expressions for R are given in Eq. (2.2) of Ref. 7. The $R_{1/2}$ term, describing normal spin-1/2 pair creation, is equal to $4k^2$ times the expression written in curly brackets in Eq. (6) of Ref. 8. The coefficient R_1 is equal to

$$R_1 = 2(R_0 + R_{1/2}) \tag{6}$$

where R_0 is the expression for R in the case of spin-zero pair creation[9]. The other terms are of less simple interpretation.

The high-energy limit of the expressions is derived and an estimate of the total cross section is obtained in Ref. 7. Since in the case of electrons, λ incorporated the effect of some of the radiative corrections to pair creation, the energy dependence of our results is of some interest. Because of the relation (6), it can be seen that the coefficient of the first power of λ depends logarithmically on the photon energy k. The coefficient of λ^2 depends quadratically on k. The remaining coefficients have not been examined in any detail, but they seem to depend even more strongly on the photon energy. The higher the power of λ, the more the large angles, and therefore also the high recoil momenta, contribute to the cross section. As a conclusion one is led to expect that the radiative corrections to electron pair creation of order $(e^2/\hbar c)^2$ cannot be neglected at cosmic-ray energies and might be responsible for the anomalous trident production which has been observed recently[10].

II. COMPARISON WITH THE EXPERIMENT

In the experiment of Ref. (1) a target of aluminum is bombarded by a 600-Mev bremsstrahlung photon beam, and emitted mesons (with 180-Mev kinetic energy) are observed at 12°

and 23° with respect to the incident beam. The two cross sections $\sigma(\theta)$ (per effective photon) thus obtained are subtracted and divided by the corresponding difference calculated[6] for the case of normal spin-1/2 particles. The following experimental value for the quantity $\Delta(\sigma_{exp}/\sigma_{1/2})$ is obtained:

$$\Delta\left(\frac{\sigma_{exp}}{\sigma_{1/2}}\right) = \frac{\sigma_{exp}(12^{\circ}) - \sigma_{exp}(23^{\circ})}{\sigma_{1/2}(12^{\circ}) - \sigma_{1/2}(23^{\circ})} = 1.6 \pm 0.86 \qquad (7)$$

In order to compare this result with the present calculation, the differential cross section, Eq. (5), is integrated over the variables of one of the pair particles by a method previously presented[6]. A nuclear form factor, corresponding to a uniformly charged nucleus of radius $1.2 \times 10^{-13} A^{1/3}$ cm, where A is the atomic number, was used. The result is given by

$$\sigma_{\lambda} = \frac{d^2 \phi}{d\Omega_{+}\, dE_{+}} = \frac{Z^2}{2\pi}\frac{r^2}{137} R'$$

$$R' = \int (4p_{+}p_{-}/k)(R/2\pi)(F^2/q^4)\, d\Omega' \qquad (8)$$

where the expressions for R' are given by Eq. (3.4) of Ref. 7. Instead of averaging σ_{λ} over the photon spectrum, the following method, which is adequate for the present experimental accuracy, is used. A sample value for the photon energy of 550 Mev is chosen, and the contributions to R' from the terms in λ^{0}, λ and λ^2 are calculated for the two angles.

The result is the following:

$$R'(12^{o}) = 0.71 + 2 \times 1.23\lambda + 13.68\,\lambda^{2}$$
$$R'(23^{o}) = 0.24 + 2 \times 0.313\lambda + 4.17\,\lambda^{2} \qquad (9)$$

Their difference is divided by $R'_{1/2}(12^{o}) - R'_{1/2}(23^{o})$ and the result, denoted by $\Delta(\sigma_{\lambda}/\sigma_{1/2})$, is equal to

$$\Delta(\sigma_{\lambda}/\sigma_{1/2}) = 1 + 3.9\lambda + 20\,\lambda^{2}. \qquad (10)$$

Figure 1 shows the quantity (10) plotted as a function of λ. The higher powers of λ, for $\lambda < 1/2$, give a contribution to (10) of less than 10 per cent and can therefore be neglected. The experimental value (7) is inserted in the form of dotted lines in Fig. 1 and gives for the limits of λ the result

$$-0.4 < \lambda < 0.2 \qquad (11)$$

It should be noted that the procedure just described does not take into account the possibility of exciting the nucleus into one of several final states. This effect increases the cross section by a certain amount, the upper limit of which can be estimated[6] by replacing the static form factor squared F^2 by $F'^2 = F^2 + (1 - F^2)/Z$. If R' is calculated using F'^2 instead of F^2, the difference $R'_{1/2}(12^{o}) - R'_{1/2}(23^{o})$ increases only by 6 per cent, and the inelastic effect on the expression (7) can be neglected for the present purpose.

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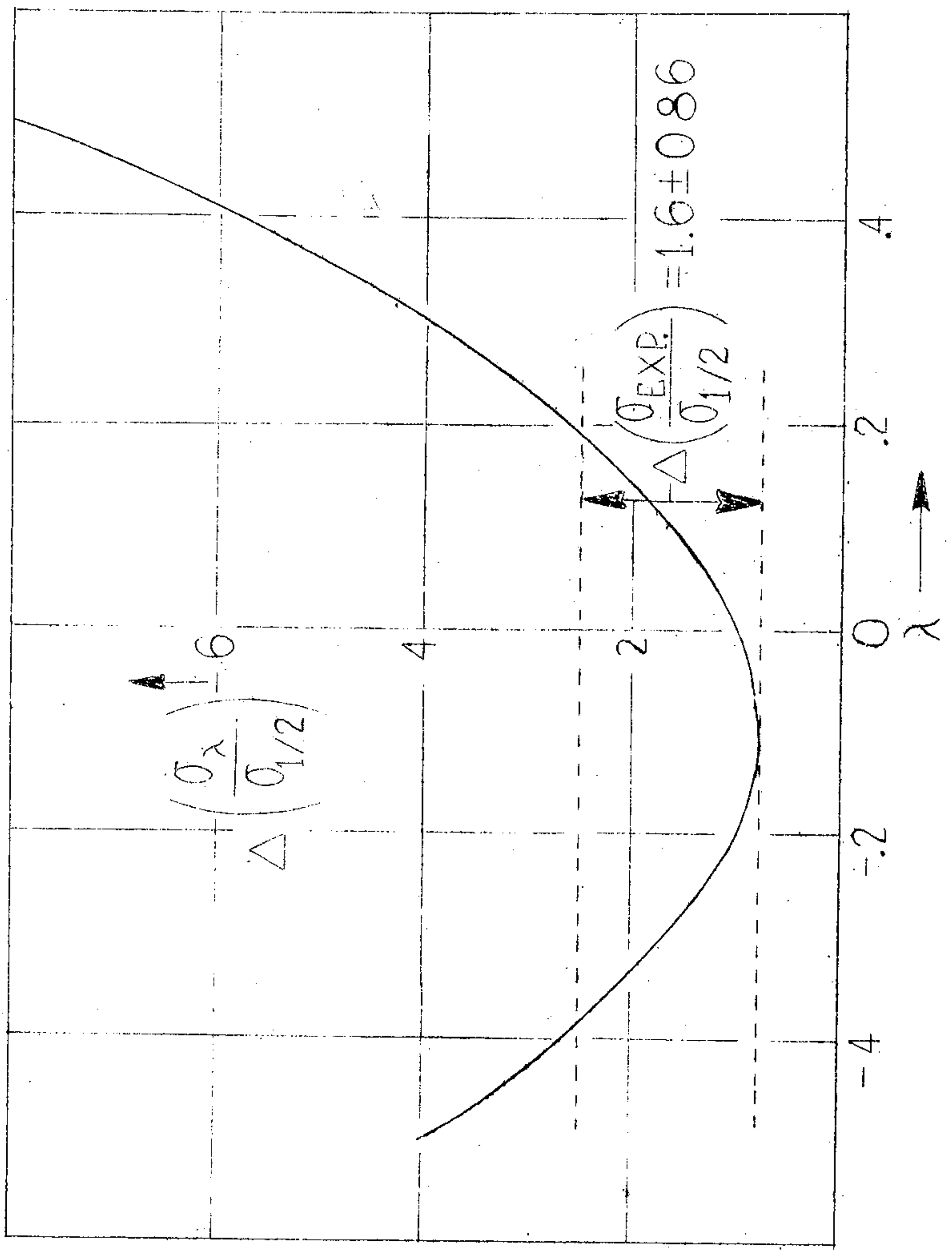

Figure 1