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An anthology of non-local QFT and QFT on noncommutative spacetime

Dedicated to Detlev Buchholz on the occasion of his 60th birthday

Bert Schroer
presently: CBPF, Rua Dr. Xavier Sigaud 150
22290-180 Rio de Janeiro, Brazil
Prof. em., Institut für Theoretische Physik, FU-Berlin
email: schroer@cbpf.br

Abstract

Ever since the appearance of renormalization theory there have been several differently motivated attempts at non-localized (in the sense of not generated by point-like fields) relativistic particle theories, the most recent one being at QFT on non-commutative Minkowski spacetime. The often conceptually uncritical and historically forgetful contemporary approach to these problems calls for a critical review of contemporary ideas in the light of previous results on this subject.

Key-words: Non-local QFT; Non-commutative spacetimes.

1 History of attempts at non-local fields

To attribute in-depth investigations of non-local QFT to the last decade, as it is done in most contemporary articles on this subject, is historically incorrect; there was already a flurry of interest (quite a strong one considering the number of particle physicist at that time) in non-local aspects of QFT leading to significant results as far back as the late 50s[1]. This interest in non-local aspects originated in the wake of renormalization of QED and of the LSZ scattering theory. The main physical motivating idea was to get to more and "better" (in the sense of less singular) interactions by using L-invariant structure functions in the interaction part of the Lagrangian. In addition there was the desire to understand whether (pre-QCD) meson-nucleon structure, as it became experimentally accessible in nucleon formfactors, could be consistently used in a Lagrangian formalism; in other words if what nowadays would be considered as "effective interactions" could be of a more fundamental significance within a non-local Lagrangian setting.

The extensive work of Kristensen and Møller [1] as well as an important contribution by Claude Bloch [2] and Hayashi [3] with prior remarks by Pauli and Rayski led to lively discussions and also called the critical attention of leading field theorists of the post renormalization era as Kallen and Lehmann [4]. The issue of non-locality attracted many Japanese researchers and became even the main topics of a 1954 conference in Japan [5].

Since relativistically invariant non-local interactions even on a formal level do not fit well with the canonical equal time formalism (and neither with the associated functional integral approach), most investigation were carried out in the Yang-Feldman setting which is independent of the canonical formulation and most closely related to both the Wightman theory and the LSZ framework of scattering theory.

The main problem in this setting, which relates the interacting Heisenberg fields with on-shell in- and out- fields, is the proof that if the incoming on-shell field is assumed to have the standard free field commutation relation, this property is inherited by the on-shell outgoing field so that the S-matrix is unitary. This is a problem which is in principle decidable in perturbation theory. The original structural argument in favor of this property in Bloch's work was not correct[2]; indeed this was noticed somewhat later [3] by explicit calculation up to fourth order of the commutator of the outgoing field within the Kristensen-Møller setting (which uses a L-invariant formfactor instead of a pointlike interaction vertex).

Although this was the result of a particular non-local model, it was believed that this negative answer is generic to non-local interactions. A very clearly written re-investigation confirming this negative result within a more modern context appeared two decades later [6]. Since the standard derivation of the unitary S-matrix in QFT, as first shown by Haag and Ruelle [7], requires the validity of the spacelike cluster property for the correlation functions of the Heisenberg fields, an explicit proof that the on-shell outgoing field does not fulfill the free field commutation relation also shows at the same time that the cluster property is violated. The latter is part of what one summarily calls "macro-causality"[3]. Consider a division of the field coordinates in a correlation function into two clusters which are separated in the sense that the field localization points of one cluster are left to those of the other. Increasing the cluster distance to spacelike infinity, the vacuum expectation factorizes in those of the two clusters and the connected part goes to zero with the well-known Yukawa fall-off which is given by the smallest invariant mass of the intermediate state above the vacuum. However if the clusters are overlapping, one first has to disentangle them to the previous non-overlapping position which requires the application of spacelike commutation relations.

In non-local theories the commutator decreases in some way, and there exists a remarkable theorem of Borchers and Pohlmeyer [8] which for Bose fields amounts to the following statement (in a massive theory fulfilling Wightman's axioms except locality)

Theorem 1 *A spacelike fall-off of the commutator of the (Heisenberg) fields in the relative distance which is faster than Yukawa's exponential decrease leads back to a micro-causal theory.*

Here faster than Yukawa means that the decrease is dominated by $e^{-\lambda r^{1+\varepsilon}}$ with $\lambda, \varepsilon > 0$ and r the relative spatial distance. So in a somewhat intuitive sense a genuine non-local theory must have a fall-off

[1] This is of course meant relative to the much smaller number of particle physicists at that time.

[2] This can easily be seen by realizing that e.g. the very existence of creation/annihilation operators fulfilling the Zamododchikov-Faddeev relations provides a counterexample.

[3] Strictly speaking this is "macrolocality" whereas macrocausality is more related to the absence of timelike precursors.

coming from the commutator which is *milder* than that coming from the lowest mass state; in other words the commutator fall-off has to dominate the spacelike decrease coming from intermediate states above the vacuum.

This indicates that the exponential clustering in genuine non-local QFTs is not any more determined by the kinematical properties of the intermediate state spectrum[4], but rather suffers modifications which depend on the kind of interaction. In a way these results suggest that, contrary to what one may have expected intuitively and contrary to the deceivingly lighthearted world of cutoffs and regulators, *causality is a very rugged property* and that a notion of "a little bit non-local" or "a little bit acausal" is not much more sensical than "a little bit pregnant".

In fact nowadays it is generally excepted among experts that among all physical principles which underlie standard QFT, Einstein causality for local observables is the most sturdy property from a conceptual viewpoint; no matter how many words have been spoken and how many papers had been written on cut-offs, regularizations and other ad hoc modifications, nobody has any idea (beyond a wishful incantation) what such manipulations really mean in terms of operators in a Hilbert space (if they mean anything at all). Hence it comes as no surprise that most attempts of introducing deviations from micro-causality actually amount to violating macro-causality in the wake; but macro-causality is the absolute borderline between physics and the realm of poltergeists.

Among the more prominent attempts there was the proposal by Lee and Wick to modify the Feynman rules by pairs of complex poles together with their complex conjugates. After formulating this idea in a field theoretic setting in such a way that the S-matrix within the Yang-Feldman setting came out unitary, Marques and Swieca [9] showed that the proposal led to physically untenable timelike power "precursors"[5].

This raised the question whether there exist consistent relativistic unitary and macro-causal particle theories at all. This question was positively answered by Coester's idea [10] of construction of "direct particle interactions" which was first formulated on 3-particle systems and then generalized in collaboration with Polyzou [23] to multiparticle theories. As a pure relativistic particle theory without vacuum polarization, it has no natural second quantization setting, but on the other hand it fulfills all properties which are expressible in terms of particles without explicitly inferring fields. In particular these theories fulfill the very nontrivial cluster separability properties of the associated Poincaré invariant unitary S-matrix; as a result their existence contradicts a dictum (which is ascribed to S. Weinberg) saying that a Poincaré invariant unitary S-matrix with these properties is characteristic for a (local) QFT.

Although these Coester-Polyzou theories of direct particle interactions have acquired some popularity in phenomenological (but conceptually well-founded) treatment of medium energy meson-nucleon interaction calculations (where the limitation in energy prevent the appearance of real vacuum polarization clouds [23]), our interest in them, as far as this paper is concerned, remains strictly limited to the desire to broaden our conceptual scope of the meaning of macrocausal and non-local behavior even if the objects which achieve that are strictly speaking not QFTs since they allow no natural second quantization.

With all the above mentioned negative results about nonlocal QFT coming from different sources, the ideas about Lorentz invariant non-locality was laid to rest up to the beginnings of the 90's when quite different well-founded physical considerations led to quantum field theory on non-commutative Minkowski spacetime which turned out to have a more subtle relation to the old notion of non-locality. The starting point was an observation by Doplicher, Fredenhagen and Roberts [14] that a quasiclassical interpretation of Einstein's general relativity equation coupled with the requirement that mere localization measurements without the action of additional dynamical processes should not generate black hole horizons. Such a requirement was not entirely new; one finds a mentioning of localization measurements and black holes within a very restrictive and physically less motivated setting as far back as 1964 [15]. However the formulation of uncertainty relations allowing for localization regions of any shape, derived from a few physically plausible assumptions, really starts with the DFR work. In recent publications one also finds references to a much older 1947 paper by Snyder [16] which is sometimes quoted in a wrong context since Snyder's motivating idea was to mimic lattice regularization in a covariant way which led to noncommuting spatial position operators for technical reasons. Without the clarification of the idea of renormalization there was anyhow no chance in 1947 to say something relevant on quantum field theoretic

[4]The pre-exponential strength factors in local QFT do of course depend on details of the interaction.

[5]Only in the case of external interactions only the precursors have a more amenable exponential behavior.

aspects of gravitation.

The DFR way of arguing in favor of uncertainty relations for position operators resembles vaguely the Bohr-Rosenfeld invocation of uncertainty relation for the electromagnetic field based on a quasiclassical interpretation of electromagnetism coupled to Schroedinger quantum matter.

Requirements of simplicity then led DFR to a certain kind of non-commuting relations between coordinate operators even in the absence of curvature i.e. to a kind of non-commutative Minkowski spacetime. Although Wigner's particle picture in the DFR setting is fully incorporated, the Fourier transforms of the canonical momentum space creation/annihilation operators involves operators q_μ fulfilling commutation relations which are described in terms of an antisymmetric matrix Q with variable (center-valued) entries. This particular realization of the uncertainty relations has of course no direct bearing on "quantum gravity" since it lacks the property of general covariance and background independence. The situation is appropriately described as a QFT in non-commutative Minkowski spacetime, often shortly referred to as "non-commutative QFT". The novel feature is that there is an additional Hilbert space of wave functions which describes the localization aspects of the q-operators.

Picking a family of localizing wave functions, e.g. the family which belongs to minimal localization (which cannot be pointlike in all 4 components as a result of the uncertainty relations [14]) and forgetting that these wave functions unlike test functions suffer an active change under L-transformations, this setting may appears like a modern form of the pre-Einsteinian "ether" but since these are only special wave functions in an altogether Poincaré covariant representation space this historical analogy is misleading. Only if we were to work in an irreducible representation of the coordinate operators in which their commutators obey necessarily non-L-invariant numerical valued commutations relations and the Poincaré ceases to have an automorphic action on the algebra, this would indeed amount to a return of a kind of (quantum) ether. One also should refrain from interpreting the $q's$ as some kind of quantum mechanical observables (the terminology "uncertainty relations" in [14] may invite such an interpretation) as e.g. a quantum mechanical Newton-Wigner [17] localization operator[6]; rather what is being quantized here is Minkowski spacetime itself. Although the DFR scheme does not seem to lead back to objects which are covariant in the standard pointlike sense of tensorial/spinorial fields, the new fields are covariant in a generalized sense which incorporates the spectrum Σ of Q. The conceptual problems posed by such an unusual situation of a QFT on noncommutative Minkowski spacetime are best discussed in the setting of free fields.

There is also a lack of standard covariance, although a less radical one, in the already mentioned "direct particle interaction" models since the requirement of additivity in particle interactions and the cluster requirement cannot be met simultaneously in one formula for the interacting Lorentz generators. This dependence of the description on additional data (on localization wave functions in the case of the DFR model, the choice of the scattering equivalence in the C-P setting) has to be expected because the kind of localization and associated cluster properties depend on this choice. But only if this drops out asymptotically, i.e. if the S-matrix comes out to be a good old-fashioned transition matrix between incoming and outgoing Wigner multiparticle states can the theory be potentially useful in particle physics. This raises the interesting question whether the DFR ideas also can be formulated in such a way that a scattering matrix can be abstracted from the asymptotic behavior.

Starting from some observations about the role of noncommutative tori in string theory, there has been a different line of thought leading to noncommutative and explicitly non-L-invariant theories[62][63]. Such attempts are expected to carry all the conceptual fragility of string theory, which employs a lot of advanced mathematical techniques but tends to be atrophic on the side of physical concept and physical interpretations. It is impossible to critically review the content of all these string theory-motivated papers from our viewpoint which, although not intended to be encyclopedic, has a strong commitment to history and faithfulness to the principles underlying particle physics.

In some of these papers the spacetime symmertry is described by $P(2)\otimes E(2)$ tensor product of a two-dimensional Poincaré- with the two-dimensional Euclidean- group $E(2)$ which happens to be a subgroup of $P(4)$. The required locality in the 2 dimensional QFT is associated with the $P(2)$ factor, whereas the Euclidean degrees of freedom are just "spectators" which do not contribute anything to localization. As

[6]Although the asymptotically covariant Newton-Wigner localization does not play a direct role in the formulation of QFT on non-commutative Minkowski spacetime, it is crucial (as for any relativistic particle theory) in the extraction of the asymptotic behavior which leads to the scattering operator.

a consequence of this setting and the well-known fact that d=1+1 causality imposes a Lorentz invariant shape on the associated positive energy momentum spectrum, the 4-dimensional Wigner particle concept gets lost and therefore such a model can have hardly be expected to have any relevance with respect to 4-dimensional particle physics. As a 2-dimensional local theory it of course fulfills the associated TCP theorem as proven by Jost as well as the Bargmann-Hall-Wightman unicity theorem [40]. Combined with the reflection invariance of the euclidean spectator theory this then leads to the TCP operation. But since the theory does not have a particle mass shell in the sense of 4-dimensions, the TCP remains physically empty.

Apparently the authors in [18] were somehow aware of these shortcomings. They propose a theory which maintains the usual particle concept associated with the mass hyperboloids, but breaks L-invariance through the application of a kind of Wigner-Moyal star-functor to the Wightman functions of a local Poincaré-invariant QFT (see equ. II.3 in [18], the mathematical difficulties in attributing a precise meaning to such a functor are not discussed there). They show that if one lets this functor also change the commutator to a kind of spacelike star-commutativity, the weak locality [40] assures that the old TCP operator and hence the S-matrix remain unchanged under the action of the star functor (which is hardly a desired situation). The star-functor associated with a fixed set of numerical-valued q-commutators does not commute with the Lorentz boosts and therefore "breaks" L-invariance, but without harming the Wigner mass shell support of particles[7]. Unfortunately the value $\sigma_{0i} = 0$ (which corresponds to the absence of space-time non-commutation in [18] and other papers quoted therein) does not occur in the DFR Q-spectrum which, in view of the fact that the Q-spectrum results from the simplest implementation of physically well-motivated uncertainty relations, makes it less appealing.

Most of the string-theory based papers on "noncommutative QFT" lack an intrinsic physical definition under what circumstances a mathematical subgroup can be called a "broken symmetry" of the ambient group. Apart from the well-defined notion of spontaneous symmetry breaking (this happens to internal symmetry groups in the Goldstone setting and to the Poincaré group for a QFT in a thermal KMS state where the boosts fail to be implementable), I am not aware of such a meaning. The acceptance of such a terminology would therefore indeed amount to a return of the pre-Einsteinian (non-dynamical) ether in the setting of quantum physics.

With the particular proposal in [18] one has in addition the uneasy feeling that one should not be able to generate a new physical reality by doing nothing else than applying a star-functor to the standard Wighman correlation functions[8]. This last objection could be removed by defining the theory via a new perturbation approach which takes into account the change of ordinary products to star-products in the perturbative interaction density instead of doing this on the Wightman functions of an already constructed standard QFT. In this case one would loose the TCP-property which was however was never to be expected anyhow to arise as a structural consequence of a non-local theory. The most one can hope for is that (as in the aforementioned case of relativistic direct particle interactions) one can add this discrete symmetry to restrict the analytic form of interactions.

The only way to counteract the return to a (quantum) "ether" however is to find an extension in which "broken symmetries" become part of a larger Poincaré-invariant setting. This is precisely what the DFR theory [19][20][21] achieves; it does so by building a QFT over a non-commutative version of fully Poincaré-invariant Minkowski spacetime. In a more colorful artistic terminology one may say that the static ether is rendered dynamic, but insinuating analogs by semantic wrappings can be no substitute for a yet incompletely understood conceptual situation.

Interpreting the word "non-local" in its widest sense as referring to *any theory whose content cannot be expressed in terms of pointlike fields*[9], the recently discovered string localized fields also feature under the heading "non-local" in this article. By string-localized fields we mean fields $A(x,e)$ as they arose in connection with constructing the field theory behind the massless infinite spin (rather infinite helicity

[7] It should be clear that our critical remarks towards proposals which try to implement spacetime noncommutativity at the prize of the return of a kind of (quantum) ether are not directed against their mathematical content (which appears to be correct).

[8] In this context it is helpful to remind the reader that the Heisenberg (commutator-) ring is non-isomorphic to the Poisson ring, i.e. the transition is "artistic" and not functorial (remember Nelson's famous saying: second quantization is a functor but first quantization remains a mystery).

[9] In the terminology of AQFT any theory whose net of algebras cannot be generated by additivity from arbitrarily small double cone localized algebras.

tower) Wigner representation of the Poincaré-group [11]. Here x denotes a point in Minkowski space and e is a "fluctuating spacelike direction" i.e. a localization point of a quantum field in 3-dimensional de Sitter space. Such strings are Einstein causal if their spacelike half-lines $x + R_+ e$ are relatively spacelike (not just their endpoints) and they also exhibit the timelike causal influence- and shadow- properties. There are good reasons to believe that there exist interactions of string-localized fields which maintain the string localization in each order. There also exist semiinfinite string-localized massive free fields. Even though they generate the same Wigner one-particle spaces as their pointlike counterparts, there are good reasons to believe that they widen the concept of particle interactions while maintaining appropriately adjusted causal localization properties [42]. One very encouraging property of massive string fields is that different from pointlike free fields, their short distance behavior does not increase with the spin [43].

Finally it is worthwhile to point out that nonlocal objects play a crucial *intermediate role* in the non-perturbative construction of local theories. A well-known example is supplied by the vacuum-polarization-free-generators (PFGs) of d=1+1 factorizing models whose Fourier transforms are momentum space creation/annihilation operators which fulfill the Zamolodchikov-Faddeev algebra relations and are therefore nonlocal. They turn out to be wedge localized and in fact generate the wedge algebra. They do not have the form of a smeared pointlike localized field with the wedge being the support of the smearing function, rather they are linked in an inexorable way to the global wedge region [13]. They loose their usefulness if one passes to sharper localized subwedge algebras by intersecting wedge algebras whose operators necessarily have vacuum polarization. These ideas about PFGs of wedge algebras are important because they show that one of the motivating pillars for studying nonlocal theories namely the idea that local theories are inexorably beset by ultraviolet problems is not true. What causes these problems is not the intrinsic local structure of QFT but rather the use of "pointlike field coordinatizations" and their singular correlations in the standard calculational approach. Whereas the use of problem-creating singular coordinates in geometry can be avoided, the "coordinates" of standard QFT obtained via Lagrangian quantization in the form of fields are inherently singular.

This is of course the main reason why the algebraic approach, which is based on nets of (bounded operator) algebras, was proposed instead of the standard pointlike field formulation. There are myriads of (composite) fields which all generate the same net of algebras. The spacetime indexed net of algebras corresponds to local equivalence classes of fields (Borchers classes) and not to individual pointlike fields. The application of these ideas to factorizing models clearly shows that the whole ultraviolet short distance issue evaporates if one succeeds to find a constructive procedure which avoids the use of necessarily singular pointlike field coordinatizations; in other words in those cases the ultraviolet aspects are limitations of the Lagrangian quantization approach and not an intrinsic aspect of the theory.

In our presentation we avoid anything which could smack like an axiomatic formulation. We think that the scarcity of controllable examples and the poor mathematical and conceptual status presently does not warrant an axiomatic approach to non-locality and non-commutativity.

The content is organized in the following way.

In the next section we present the relativistic direct particle interaction setting which fulfills cluster factorization and the time like aspects of macro-causality (absence of precursors). In section 3 semiinfinite string-localized fields are introduced, whereas section 4 is dedicated to show the usefulness of non-local (in fact wedge-localized) operators in the nonperturbative construction of local theories. Finally section 5 contains some details about QFT on noncommutative Minkowski spacetime and its encoding into nonlocal QFT on standard Minkowski spacetime. Some open problems are mentioned in the last section.

2 Direct particle interactions and macro-causality

It is known since the early days of particle physics that a interacting relativistic 2-particle system of two massive particles (for simplicity of equal mass) is simply described by going into the c.m. system and modifying the mass operator

$$\begin{aligned} M &= 2\sqrt{\vec{p}^2 + m^2} + v \\ H &= \sqrt{\vec{P}^2 + M^2} \end{aligned} \tag{1}$$

The interaction v may be taken as a function of the relative coordinate which is conjugate to the relative momentum p in the c.m. system; but since the scheme does not lead to local differential equations, there is nothing to be gained to insist in such a locality property. One may the follow Bakamjian and Thomas (BT) [22] and choose the P-generators in such a way the interaction does not appear[10] in the formulas for the c.m. momentum $\vec{P} = \vec{P}_0$, the total angular momentum $\vec{J} = \vec{J}_0$, as well as in the position conjugate to the c.m. momentum $\vec{X} = \vec{X}_0$, whereas the boost generators depend implicitly on the interaction through

$$\vec{K} = \frac{1}{2}(HX_0 + X_0H) - \vec{j} \times \vec{P}(M + H)^{-1} \tag{2}$$

where the Wigner canonical spin $\vec{j} = \vec{J}_0 - \vec{X}_0 \times \vec{P}$ commutes with $\vec{X}_0$ and $\vec{P}$ and results from applying the P boost transformation on the Pauli-Lubanski vector W

$$\begin{aligned} L_P W &= \left(1, 0, 0, M\vec{j}\right) \\ W_\mu &= \frac{1}{2}\sum_{\nu\rho\sigma} J^{\rho\sigma} P^\nu \varepsilon_{\nu\rho\sigma\mu} \end{aligned} \tag{3}$$

It is now easy to check that the commutation relations of the Poincaré generators are a result of the above definitions and the canonical commutation relations of the single particle canonical variable which furnish a complete (irreducible) set of operators in terms of which any operator in the Hilbert space may be written. Furthermore for short ranged interactions v the sequence of unitaries $e^{iHt}e^{-iH_0t}$ converges strongly towards the isometric Møller operators from which in turn one may compute the S-matrix

$$\begin{aligned} \Omega_\pm(H, H_0) &= s - \lim_{t\to\pm\infty} e^{iHt}e^{-iH_0t} \\ S &= \Omega_+^*\Omega_- \end{aligned} \tag{4}$$

The unitarity of S in the subspace orthogonal to the possible bound states is a consequence of the identity of ranges of $\Omega\pm$. Since the Hamiltonian is frame-dependent, and a relativistic particle theory is only physically acceptable if the scattering operators are frame-independent, we must have

$$\Omega_\pm(H, H_0) = \Omega_\pm(M, M_0) \tag{5}$$

which indeed follows from the same kind of short range assumptions which already assured the validity of the asymptotic convergence [23]. The same short range is also responsible for the cluster property, namely the statement that the infinite translation of one particle to spatial infinity results in the S-matrix converging against the identity (the product of the single particle S-matrices) $\Omega_\pm \to 1$.

The BT form of the generators can be achieved for arbitrary number of particles. As will be seen the advantage of this form of the representation is that in passing from n-1 to n-particles the interactions simply add and one ends up with Poincaré group generators for an interacting n-particle system. But whereas this iterative construction in the nonrelativistic setting complies with cluster separability, this is not the case in the relativistic context.

This problem shows up for the first time in the presence of 3 particles [10]. The BT iteration from 2 to 3 particles gives the 3-particle mass operator

$$\begin{aligned} M &= M_0 + V_{12} + V_{13} + V_{23} + V_{123} \\ V_{ij} &= M(ij, k) - M_0 \\ M(12, 3) &= \sqrt{\vec{p}_{12,3}^2 + M_{12}^2} + \sqrt{\vec{p}_{12,3}^2 + m^2} \end{aligned} \tag{6}$$

Here $M(12, 3)$ denotes the 3 particle invariant mass in case the third particle is a "spectator" which does not interact with 1 and 2. The momentum in the last line is the relative momentum between the (12)-cluster and particle 3 in the joint c.m. system i.e. $\vec{p}_{12,3} = L_P^{-1}(\vec{p}_{12} - \vec{p}_3), \vec{p}_{12} = \vec{p}_1 + \vec{p}_2$. As in the

[10] The group theory alone does not select a particular way of introducing interactions; in fact the C-P scheme uses this flexibility in the multiparticle sectors for the implementation of the all important cluster properties.

nonrelativitic case one can always add a totally connected contribution. This generally changes the S-matrix of the full system while keeping the lower particle S-matrices. But contrary to the nonrelativistic case, the unitary representation of the Poincaré group constructed with the BT generators via M do not fulfill the cluster separability requirement. The latter demands that if e.g. the interaction between two clusters is removed, the unitary representation factorizes into the product of the unitaries of the two clusters. Applied to the case of three particles with the third particle being a spectator one expects that shifting the third particle to infinity will result in a factorization $U_{12,3}(\Lambda) \to U_{12}(\Lambda) \otimes U_3(\Lambda)$. But what really happens [24] in the limit is $U_{12,3}(\Lambda) \to U_1(\Lambda) \otimes U_2(\Lambda) \otimes U_3(\Lambda)$. The reason for this violation of the cluster separability property (as a simple calculation using the transformation formula from c.m. variables to the original $\vec{p}_i, i = 1, 2, 3$ shows) is that the translation in the original system (instead of the c.m. system) does remove the third particle to infinity but it also drives the two-particle mass operator (with which it does not commute) towards its free value (for an explicit calculation of this limit see [24]).

The BT construction is very well suited to manufacture a Poincaré covariant 3-particle interaction which is additive (6) in the respective c.m. interaction terms, but the resulting system will not be cluster-separable in the sense of the previously two-particle system.

Fortunately there always exist unitary equivalences which transform BT systems into cluster-separable systems without affecting the S-matrix. Such transformations are called *scattering equivalences*. The phenomenon behind this observation is vaguely reminiscent of the insensitivity of the S-matrix against local changes in the interpolating field-coordinatizations which in field theoretic terminology means changing the pointlike field by passing to another (composite) field in the same Borchers class, or in the setting of AQFT by picking another operator from a local operator algebra[11]. The notion of scattering equivalences is conveniently described in terms of a subalgebra of *asymptotically constant* operators C defined by

$$lim_{t\to\pm\infty} C^{\#} e^{itH_0}\psi = 0 \tag{7}$$

where $C^{\#}$ stands for both C and C^*. These operators which vanish on dissipating free wave packets in configuration space form a *-algebra which extends naturally to a C^*-algebra $\mathcal{C}$. A scattering equivalence is a unitary member V of $\mathcal{C}$ which is asymptotically equal to the identity

$$lim_{t\to\pm\infty} (V^{\#} - 1) e^{itH_0}\psi = 0 \tag{8}$$

The relation to scattering theory comes about through the change of the Møller operators according to $\Omega_\pm(VHV^*, VH_0) = V\Omega_\pm(H, H_0)$ which leaves the S-matrix unchanged. Scattering equivalences do however change the interacting representations of the Poincaré group $U(\Lambda, a) \to VU(\Lambda, a)V^*$

It has been shown by Sokolov [25] that these scattering equivalences can be utilized for achieving cluster separability of the (interacting) representation of the Lorentz group while maintaining the S-matrix. For example if we were to take the BT representation for the full 3-particle mass operator (6), then there exists a unitary B which does the following

$$H_{cl} = BH_{BT}B^* \tag{9}$$

where H_{cl} denotes the Hamiltonian associated with the clustering representation. In general these B's have a complicated (non-algebraic) functional analytic dependence on the interaction data in H_{BT}, since scattering theory enters in their calculation in an essential way. The simplest case for an explicit construction is the above mentioned case of 3-particles with one spectator which already served as a demonstration of the failure of clustering in the BT setting [24].

For a consistent formulation, including bound states involving more than two interacting particles, one needs the theory of rearrangement collisions which uses in addition to the Hilbert space of the interacting system $\mathcal{H}$ an auxiliary Hilbert space $\mathcal{H}_f$ describing the free moving stable fragments (elementary and bound particles). Since the Hamiltonian of the free dissipating fragments and the actual Heisenberg Hamiltonian operate in different Hilbert spaces [23] we need an isometric map Φ which relates the two in such a way that the Møller operators

$$s - lim_{t\to\pm} e^{itH}\Phi e^{-itH_f} = \Omega_\pm(H, \Phi, H_f) \tag{10}$$

[11] The class of fields which interpolate the same S-matrix is much larger (it comprises all "almost local" fields [7]) but in a local theory there is no demand for such generality.

are now isometries with the same ranges between the two spaces and $S = \Omega_+(H,\Phi,H_f)^*\Omega_-(H,\Phi,H_f)$ is a unitary operator in $\mathcal{H}_f$. In case of absence of bound states (or for two particles after projecting onto the subspace of scattering states) one may choose $\Phi = 1$ and the formalism reduces to that in one Hilbert space. Although there is a lot of freedom in the choice of Φ, some knowledge about bound state wave function is necessary.

The proof that a clustering Poincaré invariant and unitary S-matix exists (for a given 2-particle interaction) is inductive [23]. The induction starts with two particles as above where the BT additivity and the cluster property hold simultaneously. As stated above, the cluster property does not hold for three particles with one particle in a spectator position. The induction assumption is

- For each proper subsystem $\mathcal{C}_i$ of an N-body system there is a representation $U_{cl,\mathcal{C}_i}(\Lambda,a)$ which clusters and is scattering equivalent to a BT representation $U_{cl,\mathcal{C}_i}(\Lambda,a) = B_{\mathcal{C}_i}U_{BT,\mathcal{C}_i}(\Lambda,a)B^*_{\mathcal{C}_i}$.
- Let $\mathcal{P} = \cup_i \mathcal{C}_i$ be a partitioning of the N-body system into proper subsystems then $U_{\mathcal{P}}(\Lambda,a) = \prod_i U_{cl,\mathcal{C}_i}(\Lambda,a)$ a representation of the Poincaré group associated with the cluster partition $\mathcal{P}$.

Theorem 2 *(Coester-Polyzou, [23][26]) $U_{\mathcal{P}}(\Lambda,a)$ is scattering equivalent to a BT system $U_{BT,\mathcal{P}}(\Lambda,a)$ corresponding to an additive mass operator $M_{BT,\mathcal{P}}$*

The last step considers in writing down the BT mass operator for the full N-body system in terms of an additive formula which is the N-particle analog of (6). Another scattering equivalence B then transforms this N-body BT system into a system which clusters for *any* partitioning

$$s - lim_{\|a_i\|\to\infty}(U_{cl}(\Lambda,a) - \prod_i U_{cl,\mathcal{C}_i}(\Lambda,a))e^{i\sum_i P_{\mathcal{C}_i}a_i} = 0 \qquad (11)$$

where the last factor denotes the translation which shifts the clusters of the chosen clustering to infinity ($P_{\mathcal{C}_i}$ is the momentum of the ith cluster). The induction starts at N=2 where all the properties are obviously fulfilled.

The direct particle interaction setting of Coester and Polyzou incorporates all aspects of particle physics which can be formulated without local fields. Micro-causality, locality, the crossing property of the S-matrix are not belonging to those properties whereas Poincaré-invariance and the cluster separability of the scattering operator are naturally incorporated. In addition there are a number of properties which are compatible but cannot be derived in this setting: antiparticles, TCP symmetry, spin and statistics[12]. The direct particle interaction theory has been included in this anthology of non-local particle theories because the properties on which it is founded define a catalogue of properties referring to Wigner particles which are indispensable in any non-local model.

As a curious side remark it is worthwhile to mention that Dirac's hole theory model of particles/antiparticles interacting with a quantized electromagnetic field, which started out as a relativistic particle theory and later (as a result of the interactions between the particles in the negative energy sea) was looked upon as a QFT of particles, finally turned out to be inconsistent both in the particle- as well as in the quantum field- sense. This is is surprizing since it led to many correct low order perturbative results (see Heitler's book) and its shortcomings only showed up on the level of renormalization which only works in the charge-symmetric formulation. The reason is that (apart from some d=1+1 integrable models) the processes of filling the Dirac "sea" and "switching on the interaction" do not commute.

The C-P direct particle interaction theory is the only known consistent relativistic particle framework which avoids the use of quantum fields and maintains consistency on the level of particles. It is nonlocal because the only notion of localization which (as in nonrelativistic theory) can be expressed in terms of localizing projectors (the Newton-Wigner localization) is only asymptotically L-covariant and local. But this is enough to obtain a L-invariant and clustering S-matrix. In fact Wigner particles only enter QFT through the LSZ asymptotic and therefore the limitation of localization not being describable in terms of localized covariant projectors operators does not cause harm. It rather points to the fact that causal

[12] According to Kuckert [27] a spin statistics connection can be derived in the setting of nonrelativistic QM which possibly lacks a second quantization formulation. This suggests the interesting question whether arguments like this can be carried over to the Coester-Polyzou direct relativistic particle interaction setting.

propagation is to be discussed in terms of expectation values which clearly show that the faster than light propagation (which would be formally allowed if one were to use Newton-Wigner kind of projectors) do not occur. The finite propagation (as distinguished from asymptotic scattering) requires a different localization which is inherent in the formalism of QFT but also has a well-defined intrinsic meaning on particle states (related to the modular localization in section 4). It does not seem to play a role in the Coester-Polyzou setting which is basically a scattering theory of relativistic particles which implements Heisenberg's relativistic S-matrix program.

An antagonism as pronounced as expressed in the title " Reeh-Schlieder defeats Newton-Wigner" [28] does not really exist in my opinion. Instead of emphasizing the lack of covariance and causal localizability of localization concepts build on the existence of projectors onto subspaces [29] and in particular of the N-W particle localization, it is more helpful for particle physics to emphasize that they are *asymptotically covariant and causal* and indeed indispensable for the derivation of scattering theory were only the asymptotic localization properties are relevant and the probability interpretation associated to projections is needed. The statement that their use in propagation over finite times creates contradictions [30] to Fermi's two-atom Gedankenexperiment (designed to illustrate the validity of the maximal velocity principle in the relativistic quantum realm) is quite irrelevant; the causal propagation aspects are to be described by the modular localization which leads to projectionless expectation values [31] in complete harmony with causal propagation and Fermi's attempt to demonstrate this in the quantum setting.

It is also important to note that scattering theory and asymptotic completeness lead to the description of the Hilbert space of the Wightman field theory in terms of a Fock space of multi-particle states. Whereas field coordinatizations are highly arbitrary (even more so than coordinatizations in geometry), particles belong to the intrinsic and unique content of particles physics. Pure QFTs, as those one encounters in generic curved spacetimes, lack this powerful particle aspect.

A similar loss of particle structure is encountered if one passes from a ground state to a thermal situation by placing the system into a heat bath; instead of Wigner particles one confronts an ensemble of dissipating quanta which in genuine interacting situations do not comply with scattering theory, nor relate to a Fock space structure (for attempts to obtain a substitute for the lost scattering properties see [32]). This situation also prevails if the thermal aspect is generated by localization via a causal horizon as in the Hawking-Unruh effect. There is no LSZ scattering behavior and no Fock space structure associated with Unruh excitations[13]. The hallmark of an Hawking-Unruh situation as compared to a generic heat bath caused thermality is that there exists a "territorial" extension [33][34] which is accompanied by an extension of the type III_1thermal von Neumann algebra with a KMS state at the Hawking-Unruh temperature to a standard type I ground state algebra with a Wigner particle structure and an associated scattering theory which permits to describe the Hilbert space of the theory in terms of a Fock space of incoming particles; this extension is only possible at that particular temperature. Note that our use of the notion of (Wigner) particles, as those objects in terms of which all field states above the ground state can be asymptotically resolved, is more restrictive that in [35][36] where the thermal quanta are also called particles even if they do not lead to a Fock space strucure.

3 String-localized fields, stability under perturbations

The simplest illustration of string-localization results from the attempt to associate a localized quantum field to the famous class of Wigner's zero mass representation of the Poincaré group in which the stability group (the "little group") of a lightlike vector has a faithful representation. Instead of one helicity as in the conformally invariant cases of photons and massless neutrinos, this (non conformally covariant) zero mass representation contains a tower of all integer- or halfinteger-valued helicity degrees of freedom[14] and we will therefore refer to this class of irreducible massless representations which depends on one continuous parameter κ (a kind of euclidean mass) as the Wigner infinite *helicity tower representation* [11].

[13]The quantization of an interaction-free theory on Rindler spacetime of course has the Fock space structure of the free quanta, but without the validity of the LSZ scattering theory in a Rindler world this does not extend to the interacting case.

[14]In order to remove a common misunderstanding it needs to be stressed that string-localized fields arising from zero mass infinite helicity tower Wigner representations have nothing in common with zero string tension objects of string theory.

It had been known for some time that there can be no pointlike covariant field which applied once to the vacuum leads to a "one-field subspace" containing such a representation [37]. In more recent times the application of the spatial version of modular theory applied to positive energy representation (via forming intersections of wedge-localized subspaces in positive energy Wigner representations) has led to a theorem that they are always localizable in arbitrary thin spacelike cones [38]. Since the cores of such cones are semiinfinite linear spacelike strings (in analogy to points being the cores of compact double cone), it is natural to look for a string-localized field as being the best (tightest localized) field theoretic description of Wigner's zero mass helicity tower representations. This object indeed exists and in the following we will describe this construction in some detail.

The irreducible zero mass helicity-tower representation of the orthochronous proper Poincaré group $\mathcal{P}_+^\uparrow$ is induced (as all Wigner representations) from unitary irreducible representations of its stabilizer subgroup of a fixed light-like vector (the "little group") . This stabilizer group is isomorphic to the two-dimensional Euclidean group $E(2)$, consisting of rotations R_ϑ by an angle $\vartheta \in \mathbb{R}$ mod 2π and translations by $c \in \mathbb{R}^2$. Let $\varphi_1 \cdot \varphi_2 = \int \delta(|k|^2 - \kappa^2)\overline{\varphi_1(k)}\varphi_2(k)$ (where the bar denotes complex conjugation) be the scalar product on the Hilbert space H_κ of functions on the plane, restricted to the circle of radius κ. An irreducible unitary action of $E(2)$ on H_κ, with the Pauli-Lubanski invariant parameter κ (a kind of Euclidean mass labelling nonequivalent representations), is given by the formula

$$\left(D_\kappa(R_\vartheta, c)\varphi\right)(k) = e^{ick}\varphi(R_\vartheta^{-1}k) \tag{12}$$

where $(R_\vartheta, c) \in E(2)$ are the Euclidean rotation and translation. The representation can be linearized by Fourier transformation with respect to k.

Let $\psi(p)$ be an H_κ-valued wave function of $p \in \mathbb{R}^4$, square integrable with respect to the Lorentz invariant measure $d\mu(p) = \theta(p^0)\delta(p^2)$ on the mantle ∂V^+ of the forward light cone V^+. The unitary Wigner transformation law for such a wave function reads

$$U(a, \Lambda)\psi(p, k) = e^{ipa}D_\kappa(R(\Lambda, p))\psi(\Lambda^{-1}p) \tag{13}$$

where $R(\Lambda, p) = B_p^{-1}\Lambda B_{\Lambda^{-1}p} \in E(2)$ denotes the Wigner "rotation" with B_p an appropriately chosen family of Lorentz "boosts" that transform the standard vector $p = (1, 0, 0, 1)$ to $p \in \partial V^+$.

The Fourier series decomposition of $k \in S_\kappa^1$ leads to the discrete integer-valued (or hafinteger-valued) helicities. Different from the zero mass finite spin representations, the occurrence of the opposite helicity in the infinite helicity tower makes a doubling of the representation (in order to achieve TCP invariance) unnecessary. A valuable hint as to how to investigate the localization aspects of this representation comes from placing it into the setting of tensor product representations of the form

$$U^0(\Lambda, a) \otimes U^1(\Lambda) \tag{14}$$

acting on the tensor product $H^0 \otimes H^1$ of the representation space of a spinless massive representation of the Poincaré group tensored with a unitary representation of the homogeneous Lorentz group. The latter is precisely the setting for the Bros-Morschella localization on 3-dimensional de Sitter space which can be made explicit with the help of the Hörmander-Fourier transformation [39]. The de Sitter space localization arising from the second factor implies a spacelike directional localization in the d=1+3 Minkowski spacetime which together with the pointlike localization coming from the first factor amounts to a localization along semiinfinite spacelike strings. For the analytic details of the modular localization subspaces and the associated intertwiners the reader is referred to the literature [11][42].

The associated string-localized field operators are defined on the Fock-space over the irreducible representation space and turn out to be of the form

$$\begin{aligned}
&\Phi^\alpha(x, e) = \int_{\partial V^+} d\mu(p)\left\{e^{ipx}u^\alpha(p, e)\cdot a(p) + e^{-ipx}\overline{u^\alpha(p, e)}\cdot a^*(p)\right\} \qquad (15)\\
&D_\kappa(R(\Lambda, p))u^\alpha(\Lambda^{-1}p, e) = u^\alpha(p, \Lambda e)\\
&u^\alpha(p, e) \equiv e^{-i\pi\alpha/2}\int d^2z e^{ikz}\left(B_p\xi(z)\cdot e\right)^\alpha\\
&with\ \xi(z) = (\frac{1}{2}\left(|z|^2 + 1\right), z_1, -z_2, \frac{1}{2}\left(|z|^2 - 1\right))
\end{aligned}$$

where the intertwiner u^α, which depend on a complex parameter α[15] (and on p via the boost B_p) are determined by the intertwining property in the second line and certain analyticity requirements (with a complex parameter α to be explained). The dot between the pre-factors $u^\alpha(p,e)$ and the creation and annihilation operators $a*(p), a(p)$ (that depend also on $k \in \mathbb{R}^2$, suppressed by the notation) stands for integration over k with respect to the measure $\delta(|k|^2 - \kappa^2)d^2k$. The k-dependence of u^α, a and a^* has thus been transferred to the dependence of the field $\Phi(x,e)$ on the space-like direction e. The field $\Phi(x,e)$ is an operator-valued distribution in x and e which describes a quantum field which fluctuates in 4-dimensional Minkowski space as well in 3-dimensional de Sitter space [39]. The following properties of Φ justify the attribute "string-localized":

- If $x + \mathbb{R}^+ e$ and $x' + \mathbb{R}^+ e'$ are space-like separated then

$$\left[\Phi^\alpha(x,e), \Phi^{\alpha'}(x',e')\right] = 0 \tag{16}$$

  while the commutator is nonzero for some e, e' if the endpoints only are space-like separated

- The covariant transformation law is consistent with this localization:

$$U(a,\Lambda)\Phi^\alpha(x,e)U(a,\Lambda)^{-1} = \Phi^\alpha(\Lambda x + a, \Lambda e) \tag{17}$$

- After smearing with tests functions in x and e (in certain α-ranges such e-smearing is unnecessary), where it is sufficient to let x and e vary in an arbitrary small region, the field operators generate a dense set in Fock space when applied to the vacuum vector $|0\rangle$. This is the appropriately string-adapted version of the Reeh-Schlieder [40] property which plays a crucial role in the mathematical physics literature on algebraic QFT.

In the standard finite spin case the second statement is a result of the above intertwining properties of $u^a(p,e)$. This intertwiner function must have a certain complexity since according to the first above property of Φ it has to amalgamate Minkowski and De Sitter spacetime in such a way that the commutation properties are not simply those of a tensor product between two-point function in both spaces[16] (which would have too strong commutation properties). For an explicit representation of $u^a(p,e)$ in terms of the Hörmander-Fourier transformation on de Sitter space we refer to [11].

Although it does not seem to be possible to express the string-localization intertwiner u^α and the resulting c-number commutator in terms of known functions, its existence and analytic properties can be proven from the representation. It turns out that the continuous parameter α, apart from the integer nonnegative values 0,1,.., can be chosen at will and the different relatively local string fields in the same Fock space correspond to the linear part of the local equivalence (Borchers) class. They are analogs of the discrete family of fields associated to the standard (m,s) Wigner representation by taking $u(p,s)$ and $v(p,s)$ intertwiners of increasing length $l \geq 2s+1$[41]. The important question of whether the operator algebra generated by these string-localized fields contains compactly localizable subalgebras (existence of pointlike localized composite fields) is presently under investigation [42].

The same construction with alpha-dependent intertwiners $u^\alpha(p,e)$ applied to the massive (m,s) representation also leads to covariant semiinfinite string-localized fields; their application to the vacuum generates the same one-particle Wigner Hilbert space as the e-independent standard textbook [41] intertwiners $u(p)$ which belong to pointlike fields. However this does not mean that they are useless. Whereas the case of interacting pointlike localized fields has been studied since the very beginning of field theory and led to the result of renormalized perturbation theory in which the pointlike localization is maintained in every order, the investigation of interactions of string-localized fields has barely begun. The most intriguing property of these massive free string-localized fields is that, contrary to standard pointlike free fields, their short-distance behavior does not get worse with increasing spin [43]. This, in addition to the

[15] Although the Bros-Moschella setting fixes the parameter α to $\mathrm{Re}\,\alpha = -1$, the intertwiners which convert the Euclidean degrees of freedom into internal fluctuating string degrees of freedom exist (in the distribution theoretical sense) for all α; in fact the continuous parameter α characterizes the linear part of the string Borchers class.

[16] In particular it cannot be reduced to standard type Jordan-Pauli type of zero mass commutators.

expected stability of string localization under suitably defined perturbations, makes string-localized fields $A(x,e)$ interesting objects for enlarging the realm of interactions while still maintaining an extended form of locality/causality.

Another type of string-localized field arises from the massive Wigner representation in d=1+2 for (abelian) generic real values of the spin (different from (half)integer). Such generic "anyonic" values activate the rich covering structure of the 3-dimensional Lorentz group. The covering structure beyond the double covering requires to define the real localization subspaces more carefully by attaching a spacelike direction e such that $W + e \subset W$ [44]. The directions e are again points in a de Sitter space, but since this space is now a two-dimensional de Sitter space, it has an infinite covering. In this way the pairs $\tilde{W} \equiv (W,e)$ and finally also the sharpened pointlike data (x,e) can be used to model a substitute for a covering space via spacelike strings which matches the Bargmann covering structure of the Lorentz group[17]. The geometric role of the spacelike directions is to have a substrate on which the center of the covering can act nontrivially. In this way the string nature of the resulting objects is already preempted by the covering structure of the modular wedge formalism which requires the definition of a reference wedge from which the "winding number" is counted. One easily shows that unlike the (half)integer spin case there is now a complex phase factor between the symplectic complement of $H_r(\tilde{W})$, $\tilde{W} := (W,e)$ and its geometric complement i.e. $H_r(\tilde{W})' = Phase \cdot H_r(\tilde{W}')$ [44]. Since the symplectic complement is the one-particle projection of the von Neumann commutant and the geometric complement is given by the square root of the 2π spin rotation, the phase is obviously related to the spin-statistics phase of abelian braid group commutation relations (anyons). The presence of string localization can be analytically confirmed by showing that the complex phase requires the triviality of compactly localized subspaces. The smallest intersections of wedges which fulfill $\mathcal{C} + e \subset \mathcal{C}$ are spacelike cones with arbitrary opening angles.

This anyonic (more generally plectonic) string is very different from the previous ones in that it owes its stringlike nature not to the presence of an internal structure of the little group; fields on coverings carry no more degrees of freedom than Bosons or Fermions. Whereas anyonic strings resemble what one expects from Mandelstam string (alias "Jordan string" in QED [46]) and more general strings in a gauge theoretical setting[18], the strings with the internal degrees of freedom in the form of an *infinite helicity tower* are in some sense more like the objects of string theory (see however the caveats below).

The anyonic strings are for various reasons (e.g. potential applications in solid state physics) of more immediate physical interests, but their nontrivial aspects as a result of their inherent off-shell field nature (the vacuum polarization is necessary to sustain the braid group commutation relations) requires more research.

String-localization is a radically different property from the properties of quantized Nambu-Goto strings on which string theory is based. There exist two different quantizations of the N-G string. The more intrinsic approach is due to Pohlmeyer *[48]* and consists in extracting a complete set of invariants which together with the generators of the Poincaré group form a closed Poisson algebra. This task has been almost completed and in this way the Nambu-Goto system was seen to be an integrable system in the classical sense. The quantization of this algebra i.e. the search for an algebra of operators which have commutation relations which mimic the Poisson structure as much as possible is still an open problem[19]. The Pohlmeyer strings are not string-localized, but their algebra of invariants share with the string-localized fields the absence of a mysterious and unphysical preference (resulting from the non-intrinsic canonical quantization) of d=10,26 spacetime dimensions, rather they exist as Poincaré invariant theory in each dimension ≥ 3.

The other approach is that which led to string theory proper; it consists in canonically quantizing before eliminating the constraints and afterwards taking care of the constraints in the spirit of the BPS

[17]The activation of covering groups by anomalous (anyonic) spin or anomalous operator dimensions (in case of conformal QFT) inexorably leads to interactions governed by braid group commutation structure (time-like braids in the conformal case [45]) There can be little doubt that QFTs with these group-theoretic induced interactions will be the first ones for which a complete mathematical control will be achieved.

[18]The rigorous LQP strings coming from the classification of admissible superselection charges in the presence of a mass gap were actually thought of by their protagonists [47] as a rigorous model-independent version of strings in massive gauge theories with confinement.

[19]Assuming that a specific conjecture about the classical algebra of conserved charges is true, there actually exists a concrete realization of the quantum algebra of Pohlmeyer charges [49].

cohomological approach. The localization aspect was recently investigated and was found that they are neither string-localized nor do they fulfill the causal dependency property of a relativistic causal pointlike localization (for an on-line discussion of these aspects see [50]).

Since one classical Lagrangian in its quantization should not lead to two different QT, this raises the question which is the right one. In my view it is the Pohlmeyer approach because among other things it does not lead to a weird distinction of spacetime dimensions which on the rather simple-minded conceptual level of finding the quantized version of a constraint Lagrangian system should not happen if one does it correctly. On the other hand the desire to phrase the construction and content of the dual model S-matrix in an auxiliary Lagrangian setting requires the Polyakov method of dealing with the Nambu-Goto Lagrangian irrespective of whether this is the correct method for quantizing the N-G Lagrangian or not. It is interesting to note that Pohlmeyer's invariant N-G charges have recently attracted the interest of researchers of the so-called loop variable approach to quantum gravity [51].

The exploration of the standard formulation of string theory and its various extensions has led to novel mathematical conjectures (which were backed up by later proofs), so from a mathematical viewpoint it is a useful endavour, even if it turns out to be a failed physical theory.

4 Wedge-localized PFGs

Non-local auxiliary operators play a pivotal role in nonperturbative (and non-Lagrangian) constructions of QFT models. Their purpose is to generate operator algebras which are localized in extended (and without loss of generality) causally closed regions. The best studied case is that of wedge localized generators without vacuum polarization, which leads among other things to the d=1+1 Zamolodchikov-Faddeev algebras of factorizing models.

Let us briefly look at this fascinating new aspects of local QFTs by starting with the meaning of vacuum-**p**olarization-**f**ree **g**enerators (shortly called PFGs) [59][13]. With a causally closed simply connected region $\mathcal{O}$ in Minkowski spacetime (typically a wedge $\mathcal{W}$, a spacelike cone $\mathcal{C}$ or a compact double cone $\mathcal{D}$) one associates a PFG (unbounded) operator F which is affiliated to the operator algebra $\mathcal{A}(\mathcal{O})$ (denoted by $F\eta\mathcal{A}(\mathcal{O})$) and creates a vector $F\Omega$ which contains only a one-particle component but no vacuum polarization parts (i.e. no continuous multiparticle contributions).

The operator algebras of a free field theory clearly possess PFGs for every spacetime region. However wedge-localized PFGs even exist in those interacting theories which have have one-particle states. This is a general consequence of modular theory [13]. But in most cases these PFG are not useful because their translates do not admit Fourier transforms and their extremely bad domain properties prevent their successive applications as in Wightman field theory. The useful PFG are the so-called "tempered" PFG and they generally do not exist in the presence of interactions even if we allow the region to be as large as a wedge in order to have the best compromise between one particles states and one-field states. If we include the attribute of temperateness in the PFG terminology then there are (with one exception which will be explained below) no interacting theories with wedge localized PFG. If a PFG exist for a subwedge region as e.g. a spacelike cone C (the largest subwedge region), then it follows that the PFG F is in fact a smeared free field (with support of the test function in that region). The analytic part of the proof is very similar to the proof of the Jost-Schroer theorem [40][52], the complication due to the fact that the operator F has a prescribed localization property but no simple (tensorial) covariance behavior is easy to handle. So the non-existence of subwedge PFG is the precise *intrinsic local characterization for the presence of interactions* which turns out to be equivalent (under the assumption of validity of the crossing property of formfactors) to $S_{scat} = 1$. PFGs in this stronger sense of temperateness only exist in case of interactions without real (on-shell) creation and annihilation in scattering processes [13]. But purely elastic S_{scat} are only possible in d=1+1 and it is believed that this possibility is exhausted by the family of factorizing models. Schematically, limiting the notation to the simplest case of a scalar scattering matrix in a factorizing theory involving one particle only (the Sinh-Gordon model) we have in terms of the following notation [59]

$$A(x) = \int (Z(\theta)e^{ip(\theta)x(\chi)} + h.c.)d\theta \tag{18}$$

$$p(\theta) = m(ch\theta, sh\theta),\ x(\chi) = r(sh\theta, ch\theta)$$

where the second line defines the mass shell rapidity parametrization as well that of the right wedge. The $Z^{\#}s$ are not the standard creation/annihilation operators but they rather fulfill the Zamolodchikov-Faddeev algebra relations:

$$
\begin{aligned}
Z(\theta)Z^*(\theta') &= S(\theta-\theta')Z^*(\theta')Z(\theta) + \delta(\theta-\theta') \\
Z(\theta)Z(\theta') &= S(\theta'-\theta)Z(\theta')Z(\theta)
\end{aligned} \tag{19}
$$

and $A(x)$ can therefore not be a local field. Some application of modular theory of operator algebras for the wedge region shows that

$$
\begin{aligned}
A(f) &= \int A(x)f(x)d^2x, \; suppf \subset W \\
\mathcal{A}(W) &= alg\{A(f), \; suppf \subset W\}
\end{aligned} \tag{20}
$$

generate the wedge algebra. In contradistinction to pointlike localized fields the sharpening of the support of f to a subregion inside W does not help since the localization region of resulting operator does not follow the classical picture of the support of f. The only way to restrict the localization to D is to define algebras with the more restricted localization by forming the intersections of wedge algebras which contain D i.e.

$$
\mathcal{A}(D) = \cap_{W\supset D}\mathcal{A}(W) \tag{21}
$$

which for the case of d=1+1 at hand can be shown to reduce to

$$
\begin{aligned}
\mathcal{A}(D) &= \mathcal{A}(W) \cap \mathcal{A}(W_a)' \\
D &= W \cap W_a'
\end{aligned} \tag{22}
$$

the condensed notation requires some explanation. W_a is the result of applying a right spacelike translation by a to W and the upper dash on regions denotes their spacelike complement which in the case of a wedge is just the opposite left wedge. The dash on operator algebras denotes the von Neumann commutant in the ambient Hilbert space whereas on regions it stands for the causal complement.

The computation of intersections of von Neumann algebras is a complicated problem for which no general methods exist. However if the generators of wedge algebras have a simple structure as in the present case, one can solve the characterizing relations for the desired $\mathcal{A}(D)$ affiliated operators A as those which obey the relation

$$
A \subset A(W) \; s.t. \left[A, U(a)A(f)U(a)^{-1}\right] = 0 \tag{23}
$$

In the spirit of the old LSZ formalism one can then make an Ansatz in form of a power series in $Z(\theta)$ and $Z^*(\theta) \equiv Z(\theta - i\pi)$ (corresponding to the power series in the incoming free field in LSZ theory)

$$
A = \sum \frac{1}{n!}\int_C \ldots \int_C a_n(\theta_1, \ldots\theta_n) : Z(\theta_1)\ldots Z(\theta_n)
$$

Each integration path C extends over the upper and lower part of the rim of the $(0, -i\pi)$ strip in the complex θ-plane. The strip-analyticity of the coefficient functions a_n expresses the wedge-localization of A. It is easy to see that these coefficients are identical to the vacuum polarization form factors of A

$$
\langle \Omega \left| A \right| p_n, ..p_1 \rangle^{in} = a_n(\theta_1, \ldots\theta_n)
$$

whereas the crossing of some of the particles into the left hand bra state leads to the connected part of the formfactors

$$
{}^{out}\langle p_1, ..p_l \left| A \right| p_n, ..p_{l+1}\rangle^{in}_{conn} = a_n(\theta_1 + i\pi, \ldots\theta_l + i\pi, \theta_{l+1}, ..\theta_n)
$$

The algebraic identity (23) involving the in Z linear generator $U(a)A(f)U(a)^{-1}$ relates the a_n coefficients whose n differs by 2. This is nothing else than the famous "kinematical pole condition" first introduced as one of the construction recipes by Smirnov [53]. The solutions together with the Payley-Wiener meromorphic characterization of the size of D defines a space of formfactors or $A's$ in the sense of bilinear forms. If one wants A to have simple covariance properties one should think of a basis of pointlike fields (the generalization of Wick monomials to the realm of interaction); in this context does not create short distance problems since formfactors by definition do not contain short distance fluctuations. Hence once it is known that the intersected algebras $\mathcal{A}(D)$ are nontrivial, then the present approach serves as a calculational tool for the formfactors of these operators. Direct calculations of correlation functions with the aim of showing the existence via a GNS kind of reconstruction theorem failed because one has not been able to control the sum over intermediate particle states which formally links correlation functions of products of operators with formfactors of these operators.

In this situation it is interesting to note that there has been a recent proposal by Buchholz and Lechner [54] to use powerful modular methods of operator algebras ("modular nuclearity") instead if handling infinite intermediate state sums using formfactors. If this modular nuclearity property can be checked for the wedge algebras, then the nontriviality of the intersection is guarantied and one would have succeeded to proof the existence of a large class of nontrivial models whose fate would otherwise fall under the spell of the unresolved short distance problem.

The idea in this form depends on the existence of nontrivial wedge-localized PFG (only possible in absence of inelastic scattering processes) and can not be generalized to non-factorizing models or to higher dimensions. Since the presentation of some highly speculative ideas how to get around these problems would lead to far away from the main subject of this paper we refer the reader to a forthcoming publication [55].

The present modular localization approach to PFGs should be combined with the work of Brunetti Guido and Longo [38] who constructed the family of wedge-localized real subspaces of positiv energy Wigner representation spaces by combining purely group theoretical ideas with aspects of modular theory. Although the subwedge (spacelike cones, double cones) localized subspaces have no geometric modular characterization, they can easily be constructed by intersecting wedge localized spaces. The work of these authors culminated in a theorem that positive energy representations of the Poincaré group always have nontrivial spacelike cone localized subspaces for arbitrary small opening angles of the spacelike cone and that the wedge like localization can be re-obtained by additivity from spacelike cone localized subspaces. In case of (half)integer spin representations one can show that the compact spacelike cone intersections are nontrivial by constructing a dense set of double cone localized wave functions. However there were two class of representations which did not allow compact localization (i.e. the double cone localization spaces were trivial), the d=1+2 representations with "anyonic" spin (s$\neq$ (half)integer) and the famous Wigner family of zero mass infinite helicity representations. Both cases lead to nontrivial spacelike cone localized subspaces and were presented in the previous section. The associated wedge algebra for the anyonic case turns out to have a structure which is analogous to a Z-F algebra [56]; however the d=1+2 dimensionality of the problem renders the computation of intersected algebra $\mathcal{A}(D)$ more difficult [57]. The fascination of this problem relates to the conjecture that this is the only case for which the Wigner representation theory combined with modular localization contains enough information to determine the vacuum polaritation clouds and the generating string-localized fields of "free" anyons (for an explanation of this terminology see .)

The modular approach to the d=1+1 factorizing models is very similar in that the principle structure is always associated with wedge-localized algebras and the subwedge-localized objects are formed by intersections. The main difference is that in all the situations of positive energy Wigner representations (with the exception of the case of d=1+2 anyons) there also exist subwedge-localized PFGs and the algebras are obtained from a spatial modular theory in a functorial way (the CCR or CAR functor). This had led me already in a very early stage of my investigations to view the whole constructive program based on modular theory as an extension of the functorial relation between spatially and algebraically localized objects to the realm of interactions.

Here I am using two different looking but nevertheless equivalent totally intrinsic definitions of the meaning of "interacting". On the one hand one can take the absence of PFGs associated to subwedge-localized algebras as the local definition of absence of interactions (and their presence in the opposite situation). On the other hand there exists the global definition in terms of the triviality of the S-matrix i.e. S=1. The modular aspects of wedge-localized algebras permits to identify these two definitions.

5 Non-local aspects of QFT in noncommutative Minkowski spacetime

About a decade ago Doplicher, Fredenhagen and Roberts [14] started to investigate the feasibility of a QFT on noncommutative Minkowski space. Their original motivation was to study implications of new uncertainty relations which arose from limitations on measurements in small volumes if one requires that no black hole horizons should be created by such measurements alone. More specifically one combines the result of the Heisenberg uncertainty relation, which state that in order to achieve an uncertainty $\Delta x_0,..\Delta x_3$ one needs an energy transfer $E \sim \frac{\hbar c}{a}, a = min\{\Delta x_\mu\}$, with the Einstein equations, which relate energy with mass and couple mass in turn to the gravitational field leading possibly (as exemplified by the Schwarzschild solution) to black hole horizons which could wreck the measurement process. The requirement that such a destruction of the measurement process via formation of black hole horizons is forbidden leads to the two uncertainty relations

$$\Delta x_0(\Delta x_1 + \Delta x_2 + \Delta x_3) \geq \lambda_P^2 \tag{24}$$
$$\Delta x_1\Delta x_2 + \Delta x_1\Delta x_3 + \Delta x_2\Delta x_3 \geq \lambda_P^2$$

DFR then realized that, in case one ignores curvature, these uncertainty relations can be saturated in terms of a noncommutative Minkowski spacetime i.e. an algebra affiliated with hermitian localization operators which fulfill (in a suitable normalization)

$$[q_\mu, q_\nu] = i\lambda_P^2 Q_{\mu\nu},\, [q_\lambda, Q_{\mu\nu}] = 0 \tag{25}$$
$$Q_{\mu\nu}Q^{\mu\nu} = 0,\, (\frac{1}{16}Q_{\mu\nu}\tilde{Q}^{\mu\nu})^2 = 1$$

where the matrix Q is central-valued and transforms under L-transformation like an antisymmetric tensor and $\tilde{Q}$ the associated pseudo tensor. The "Planck length" λ_P is a parameter which originates from the DFR uncertainty relations and whose intended role is that for large distances the spacetime is to return to Minkowski spacetime. A irreducible representation of the $q's$ would require numerical values σ of Q which would destroy L-invariance. A L-transformed $q' = \Lambda q$ leads to another irreducible representation with another commutator matrix σ' which results from L-transforming the antisymmetric tensor $\sigma_{\mu\nu}$. The minimal $*$-algebra on which the Poincaré transformations can act as automorphisms is therefore a direct integral over Λq with the Haar measure of the L-group. Since the Lorentz group only enters the algebraic structure via its transitive action on the $\sigma_{\mu\nu}$ matrices, the direct integral actually involves a measure on Σ which is induced by the Haar measure of the L-group. It is comforting to know that this formal *algebra on which the Poincaré group acts as an automorphism can be elevated to a mathematically respectable C^*-algebra whose regular representation is unique (up to quasi equivalence) [14]. The spectral values which the central elements Q take consists of all values $\sigma \in \Sigma$ with the above restriction for the matrix entries; explicitly this set Σ turns out to be isomorphic to the 4-dimensional tangent space of a sphere extended by a reflection $\Sigma \simeq TS^2 \times \{-1, 1\}$.

There exist Poincaré covariant representations of the algebra affiliated with these commutation relations and the full spectrum Σ (which simply consists of the space of numerical matrices fulfilling the above algebraic conditions 25) [14] of the nontrivial center; this situation is summarily referred to as the non-commutative Minkowski spacetime M_{qu}. The representation of the translations (incorporation of the momentum space) requires to tensor the Hilbert space H which carries the q-representation with its conjugate $H_{rep} = H \times \bar{H}$. Different from the positive energy representations featuring in particle physics the translational part is not related to support properties in momentum space (in particular no positive energy restriction).

What is meant by a field on such a noncommutative Minkowski spacetime can be best explained in case of a free field[20]. Formally it is obtained by substituting the spacetime variable by the above operator

[20] This restriction is not only taken for pedagogical reasons; it is presently not quite clear whether the standard implementation of interactions in terms of polynomials of (non-commutative) field products is appropriate.

q

$$A(q) = \int (e^{-ipq}a(p) + h.c.)\frac{d^3p}{2p_0} \tag{26}$$
$$H = H_{rep} \otimes H_{Fock}$$

where the second line denotes the Hilbert space of the free model and the first factor is the previously explained covariant representation space. The Poincaré transformations do not only act on the particle variables (including the transformations of their "non-commutative positions"), but they also act on the central operators Q and transform those parts of their spectrum together the localizing wave functions. In this way we clearly keep the Wigner particle picture as one of the pillars of any relativistic particle theory. If we want to return to a Fock space description, we have to take the (partial) expectation value in a state ω on the localization space M_{qu} which leads to expressions of the form [14][20] e.g.

$$\omega \to \omega(A(q+x_1)...A(q+x_n)) = \tag{27a}$$
$$= \int f_\omega(x_1, x_2, ...x_n; \xi_1, \xi_2, ...\xi_n)A(\xi_1)A(\xi_2)....A(\xi_n)d\xi_1 d\xi_2...d\xi_n$$

which leaves us with smeared non-local usual operator in Fock space (written in terms of a nonlocal monomial of local free fields) where the non-locality comes from collecting the product of q-exponentials in one exponential only[21] on which the state ω over the q-algebra may be evaluated. If one starts e.g. with an ω corresponding to "minimal localization" [14], the spectral values of Q are restricted to a compact submanifold Σ_1 which is left invariant by the rotation and translation subgroups whereas a L-boost will change its position within Σ. Since L-invariant states on M_{qu} do not exist, the field theoretic expressions on H_{Fock} are even before test-function smearing already delocalized and non-covariant (in the sense that they are not fitting the tensor/spinor calculus of Lagrangian quantization). The commutator of two such fields evaluated on Σ_1 associated minimal states ω exhibits a Gaussian decrease in spacelike direction. This is faster than the Yukawa decrease allowed by the Borchers-Pohlmeyer theorem mentioned in the introduction but one has to keep in mind that a free field theory on M_{qu} is quite different from a Wightman theory on ordinary Minkowski spacetime M.

Although there is no problem with free noncommutative QFT, the interacting situation is still in a very precarious state. The problem with interactions is similar to those problems on which the old attempts (mentioned in the introduction) failed. The indispensable property which any particle physics theory (whether P-covariant and non-local in the old sense or P-covariant and non-local in the new non-commutative sense) has to deliver is a P-invariant S-matrix which describes scattering between Wigner multi-particle in and out states. Before one tries to formulate this requirement in terms of new interacting fields on M_{qu} in a LSZ setting of asymptotic convergence, one may try to make mathematical experiments with a Gell-Man-Low like Ansatz for an S-matrix or similar attempts with formulas close to the standard ones. The results in this direction have been only partially successful. On the one hand it was possible to overcome previous difficulties with unitarity and ultraviolet divergences [19]. These perturbative calculations can even be arranged in such a way that Feynman graphs supplemented with other rules continue to be useful [21]. On the other hand it is presently not known of how to obtain an S-matrix in the aforementioned sense i.e. one which has forgotten its memory on the M_{qu}-localization ω.

Note that in the C-P direct particle interaction theory the non-locality also prevented the existence of covariant tensor objects corresponding to Heisenberg fields[22]. But thanks to the scattering equivalences this did not spoil the existence of a clustering unitary S-matrix. Conceptually the DFR model is presently in a worse shape since there it is not clear whether interacting correlation functions in $\omega \times \omega_{vac}$ states will still fulfill cluster fall-off properties which is a prerequisite for the existence of a clustering S-matrix.

On the other hand one should perhaps not put too much faith in the particular DFR model of fields on M_{qu} but investigate more profoundly other implementations of their physically motivated commutation relations. Perhaps there is also a message to be drawn from the successful modular localization setting

[21] The non-commutativity of the q-exponentials converts the ordinary product of field operators into a "twisted" product. The twist factors are center-valued and it is only their evaluation in a state ω on the M_{qu} which leads to $\mathbb{C}$-valued weighted integrals over the $\vec{Q}$-spectrum.

[22] The theory is not capable to produce formfactors i.e. particle matrix elements of covariant currents etc.

[38]. The representation of the P-group (including reflections) on $H_{rep} \times H_{Wig}$ (where H_{Wig} stands for the Wigner irreducible particle representation) has also a wedge-associated Tomita operator and therefore is subject to the setting of modular localization (although with less restrictive properties since the representation in the first factor is not of positive energy [38]) which seems to allow for a wider starting point than that defining the DFR model.

6 Concluding remarks, outlook

In this work I presented (according to the best of my knowledge) all not outright unreasonable attempts to formulate theories in particle physics which either go beyond the frontiers of relativistic causality (spacelike locality and timelike causality) but are still maintaining Wigner's particle concept, or proposals which aim at local theories but use non-local operators in intermediate steps in an essential way.

Since the word "non-local" is as revealing as a "non-elephant", one needs to specify in what sense they non-local theories. This is most easy done in case of the wedge-localized objects of section 4. In that case one uses the simplicity of generators of algebras of large but causally nontrivial (i.e. situations in which the causal completion is still smaller than the total Minkowski spacetime) localization regions because their vacuum polarization properties of operators which are allowed to spread over large regions are simpler than those of small regions. More precisely the absence of any operator which applied to the vacuum creates a one-particle state without any other admixture for causally complete subwedge regions is characteristic for absence of interactions ($S_{scat} = 1$).

On the other hand for the wedge region such operators exist in d=1+1 if the interactions do not lead to particle creation through scattering [13]. There are arguments [58] that these d=1+1 theories are exhausted by factorizing models for which it is indeed easy to write down wedge algebra generators [59][60] in terms of creation/annihilation operators which fulfill the Zamolodchikov-Faddeev algebra. Their smearing with test functions never produces operators which have a localization better than that of a wedge. Rather operators localized in smaller regions (e.g. double cones) have to be constructed by the very nontrivial process of intersectiong algebras which leads inevitably to operators which generate one-paticle states accompanied by vacuum polarization clouds. In this approach the coordinatizations in terms of covariant (and necessarily singular) pointlike fields only emerges at the end. These objects are not unique but they are generators of algebras of arbitrary localizations.

An approach based on interseting algebras and avoiding correlations between pointlike fields in favour of formfactors will never enter into the muddy waters of ultraviolet divergences, although its field-coordinatizations at the end are as singular as required in a Wightman setting (namely inverse power short distance behavior).

There are interesting but still very speculative ideas of how to generalize this to non-factorizing and higher dimensional QFTs with a particle structure. It is clear that as soon as one is able to formulate an (iterative) calculational scheme in terms of formfactors of operators in particle states instead of correlation functions of point- (or string-) like fields the ultraviolet problem will have disappeared and together with it all those motivations for non-local theories which are based on ultraviolet finiteness. The idea that there may be such a formulation of relativistic particle physics was already the basis of the ill-fated bootstrap program of Chew, the new message is to incorporate the S-matrix into the family of formfactors (as the formfactor of the identity operator) and to use this as part of QFT and not against it.

On the other hand the "direct particle interaction" approach of Coester and Polyzou has its point of departure in the extreme opposite setting of relativistic quantum mechanics i.e. a framework without any vacuum polarization structure. It is not possible to formulate this approach in one frame only; in inducing from n to n+1 particles one has to play between two kinds of relativistic descriptions: one in which the cluster property holds and another one (the BT description) in which the interactions add up in a way one is accustomed to in nonrelativistic QM (but with the k-particle interaction operator with $k \leq n-1$ in the n-particle system being different from the k-particle interaction in the n-1 particle system). This change from one to the other description keeps the same Hilbert space and the same S-matrix; the unitary operators which implement these transitions belong to the class of "scattering equivalences" since they leave the S-matrix unchanged (they are the analogs of the more restrictive changes of the field coordinatizations in QFT [40]). This direct particle approach can be generalized to models with nonelastic processes (e.g. the Lee model [23]) since the positive energy representation theory of the Poincaré group is not subject to the mass superselection rule of Galilei-group representation theory..

The interest in contrasting the aforementioned algebraic approach to QFT via wedge algebra generators with the Coester Polyzou particle setting is that one may get a deeper understanding of vacuum polarization as the most characteristic property of local QFT. Since the days when Heisenberg discovered that (even without interactions) near the surface of spatial localization regions of charges there are always (infinitely strong for sharp surfaces) vacuum polarization clouds and ever since Furry's and Oppenheimer's observation that interactions implemented by Lagrangian perturbations inevitably lead to field-states (states obtained from the vacuum by the one-time application of a field) whose one-particle contribution are inexorably accompanied by vacuum polarization clouds, it became desirable to have a more intrinsic quantitative nonperturbative understanding of these phenomena. Important effects as the on-shell crossing property and the localization-induced thermal effects (Hawking-Unruh radiation from causal Rindler horizons, area proportionality of localization induced entropy etc.) depend on a better understanding of vacuum polarization. The shape of polarization clouds associated with operators localized in the same localization region is expected to be related to the modular theory. The mathematics needed for the control of Coester-Polyzou direct particle theories is well-known and their conceptual fabric is quite well understood. The framework is very flexible because it breaks the general superselection rules which are a consequence of locality and the ensuing vacuum polarization as well as the already mentioned nonrelativistic mass superselection rule associated with Galilei invariance[23] i.e. it can describe creation of particles. One could envisage a whole family of intermediate models between microcausal QFT and macrocausal C-P type of particle models (e.g. relativistic quantum mechanical nucleons with a photon or meson field) which because of the lack of full vacuum polarization are expected to be more mathematically manageable than pure QFT (such ideas are implicit in [61]).

The non-local aspect of string-localization in section 3 has a clear historical motivation. Far from being a mere invention, it was there ever since Wigner wrote his famous article on the classification of irreducible representations of the P-group. But whereas for all the finite halfinteger spin representations there was a description in terms of local Euler-Lagrange setting following the rules of Jordan's field quantization, the Wigner zero mass infinite helicity representation turned out to be outside this historically cherished framework. This is the reason why the associated string-localized operator theory was discovered only recently; one needed a good understanding of modular localization before one could solve this problem. The open problem is whether in addition to filling a historical loophole string localization can lead to new physics. The hope that this could be the case is founded on the observation that there exist string-localized massive free fields $A(x,e)$ with nice properties which, as the zero mass infinite spin fields, live simultaneously on Minkowski spacetime (labeled by x) as well as on one dimensional lower de Sitter manifold of spacelike directions (labeled by e). One useful property which emerges from a study of their two-point function is that their short distance worsening with increasing spin and that for parallel directions it is milder than for perpendicular $e's$[24]. Another important property is that the LSZ asymptote of interacting string-localized fields does not loose its "stringy" localized nature i.e. the timelike LSZ maintains the dependence on the de Sitter space localization coordinate e. These are ideal prerequisites for studying perturbations which maintain the stringlike-localized nature in every order. Assuming that these ideas can be backed up by calculations, one would expect to obtain a new realm for formulating interactions.

In the past the quest for extending QFT into the non-local regime was often motivated by the desire to improve the ultraviolet behavior associated with the renormalizability aspects of Lagrangian quantization. But with accumulating evidence that these limitations may be related to the use of singular field objects rather than originating from inherent frontiers set by the causality/locality properties, the motivation is shifting. Although nearly nothing is known about the quantum aspects of gravity, most theoreticians would agree that the global notion of Einstein causality (which is still maintained in QFT on curved spacetime) has no place in a future "quantum gravity". The enormous difficulties encountered in attempts to go beyond causal QFT while maintaining physical interpretability may actually indicate

[23]The fragmentation of a bound target into a fixed number of constituents is not subject to the conservation of mass (if the inertial and Galileian masses are identified).

[24]I owe this observation to J. Mund, private communication.

that a consistent non-local theory (beyond string-localized fields) which contains the present setting as a limiting case will be quantum gravity per excellence.

The aim of this anthology would be fulfilled if among newcomers to QFT it is able to generate some admiration and enthusiasm about the depth of the ideas of the past protagonists and a desire to overcome the present (basically sociological caused, certainly not related to a decrease in intelligence) crisis through a return of a more conceptual-based approach to particle physics.

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