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# Non BPS topological defect associated with two coupled real field

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## Abstract

We investigate a stability equation involving two-component eigenfunctions which is associated with a potential model in terms of two coupled real scalar fields, which presents non BPS topological defect.

**Key-words:** Couple fields; Kink; Topological defect.

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## I. INTRODUCTION

We consider the non Bogomol'nyi [1] and Prasad-Sommerfield [2] (non BPS) classical soliton (defect) solutions of two coupled real scalar fields. They are static, nonsingular classically stable solutions of the field equations with finite localized energy [3–6]. Here we consider an approach to a particular case of such two-coupled-field systems, independent of the time [4,7], so that they are non BPS soliton solution.

Such a static classical configurations in 1+1 dimensions are called kinks, in 2+1 dimensions are named vortices and in 3+1 dimensions are called domain walls which are bidimensional structures.

Recently the case of BPS topological defect associated with two coupled real scalar fields to a system containing up to sixth-order powers in the fields has been investigated via supersymmetry in quantum mechanics [8]. In this work, into the asymptotic region, we find the zero mode eigenfunction associated with the stability equation of a relativistic system containing up to fourth-order powers in the fields without supersymmetry.

Recently, one-loop quantum corrections to soliton energies and central charges in the supersymmetric $\phi^4$ and sine-Gordon models in (1+1)-dimensions have been investigated [9]. The reconstruction of 2-dimensional scalar field potential models has been considered and quantum corrections to the solitonic sectors of both potentials are pointed out [10].

## II. TOPOLOGICAL DEFECTS FOR TWO COUPLED SCALAR FIELDS

The topological defects considered in this work are called kinks. Our potential model of two coupled real scalar fields in 1+1 dimensions presents $Z(2)$ symmetry and the classical soliton solutions.

The Lagrangian density for such a nonlinear system in the natural system of units ($c = \hbar = 1$), is given by

$$\mathcal{L}\left(\phi, \chi, \partial_\mu\phi, \partial_\mu\chi\right) = \frac{1}{2}\left(\partial_\mu\phi\right)^2 + \frac{1}{2}\left(\partial_\mu\chi\right)^2 - V(\phi, \chi), \tag{1}$$

where $\partial_\mu = \frac{\partial}{\partial x^\mu}$, $x^\mu = (t, x)$ with $\mu = 0, 1$, $x_\nu = \eta_{\nu\mu}x^\mu$; $\phi = \phi(x, t)$, $\chi = \chi(x, t)$ are real scalar fields, and $\eta^{\mu\nu}$ is the metric tensor given by

$$\eta^{\mu\nu} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

Here the potential $V = V(\phi, \chi)$ is given by

$$V(\phi,\chi)=\frac{\lambda}{4}\left(\phi^2-\frac{m^2}{\lambda}\right)^2+\frac{\lambda}{4}\left(\chi^2-\frac{\mu}{\lambda}\right)^2+\gamma\chi^2\phi^2, \tag{3}$$

where $\lambda>0, m>0, \mu>0$. Observe that this potential has a discrete symmetry as $\phi\to-\phi$ and $\chi\to-\chi$ so that we have a necessary condition that it must have at least two zeroes in order that solitons can exist. However, this condition is not sufficient.

The general classical configurations obey the following equations of motion:

$$\frac{\partial^2}{\partial t^2}\phi-\frac{\partial^2}{\partial x^2}\phi+\frac{\partial}{\partial\phi}V=0, \qquad \frac{\partial^2}{\partial t^2}\chi-\frac{\partial^2}{\partial x^2}\chi+\frac{\partial}{\partial\chi}V=0, \tag{4}$$

which, for static soliton solutions, become a system of nonlinear differential equations given by

$$\begin{aligned}\phi''&=\frac{\partial}{\partial\phi}V=\lambda\phi\left(\phi^2-m^2+2\gamma\chi^2\right)\\ \chi''&=\frac{\partial}{\partial\chi}V=\lambda\chi\left(\chi^2-\mu^2+2\gamma\phi^2\right),\end{aligned} \tag{5}$$

where primes represent differentiations with respect to the space variable.

There is two vacua given by

$$M_i=\left(\pm\frac{m}{\sqrt{\lambda}},0\right) \quad N_i=\left(0,\pm\frac{m}{\sqrt{\lambda}}\right). \tag{6}$$

However, $M_i$ or $N_i$ is a false vacuum. Choosing the vacuum state so that $\mu^2<\nu^2$, in this case $M_i$ becomes a true vacuum given by

$$\begin{aligned}M_1&=\left(\frac{m}{\sqrt{\lambda}},0\right), \quad x\to\infty\\ M_2&==\left(-\frac{m}{\sqrt{\lambda}},0\right), \quad x\to-\infty.\end{aligned} \tag{7}$$

We see that these solitons are non BPS because the Bogomol'nyi form of the energy is not consisting of a sum of squares and the surface term, i.e.

$$E_B\neq\int dx\frac{\partial}{\partial x}\Gamma[\phi(x),\chi(x)]. \tag{8}$$

The conserved topological current ($\partial_\mu j^\mu=0$) can not be written in terms of the continuously twice differentiable function $\Gamma(\phi,\chi)$, viz.,

$$j^\mu\neq\epsilon^{\mu\nu}\partial_\nu\Gamma(\phi,\chi), \quad \epsilon^{00}=\epsilon^{11}=0, \quad \epsilon^{10}=-\epsilon^{01}=-1. \tag{9}$$

Therefore, does not exist a superpotential $\Gamma(\phi,\chi)$ and, so, the topological charge of such a system is not equivalent to the minimum value of the energy. Next we consider the particular case in that $\gamma=\lambda^2$. The case with $\gamma\neq\lambda^2$ will be consider elsewhere [17].

Now let us analyze the connection between an approach for $\phi^4$ model according Rajaraman's method so that we choose an ellipse orbit in the plane $(\phi, \chi)$, viz.,

$$\chi^2(x) = \beta^2 \left(1 - \frac{\phi^2(x)}{\alpha^2}\right) \tag{10}$$

where $\alpha$ and $\beta$ are real constants. Observe that in this trajectory as $x \to \infty, \phi \to \phi_{vacuum} = \alpha$ because $\chi = 0$. From (10) and (5) and doing $\phi = \alpha\psi$, we get the following Riccati equation

$$\psi''(x) = 2(\mu^2 - m^2)[\psi^3(x) - \psi(x)], \tag{11}$$

which provides a particular solution given by

$$\psi(x) = \tanh[(\mu^2 - m^2)x]. \tag{12}$$

This Riccati equation is a particular case of field equations in scalar potential models in 1+1 dimensions [11]. In similar way we also have a Riccati equation associated to field $\chi$ which has a solution that satisfies the above orbit given in Eq. (10). Therefore, we find the following static classical configurations

$$\phi(x) = \alpha \tanh[(\mu^2 - m^2)x], \quad \alpha = \frac{m}{\sqrt{\lambda}}$$
$$\chi(x) = \beta sech[(\mu^2 - m^2)x], \quad \beta = \sqrt{2(3m^2 - \mu^2)}. \tag{13}$$

The classical stability of the kinks in this non-linear system [7,8] is analyzed by considering small perturbations around $\phi(x)$ and $\chi(x)$:

$$\phi(x,t) = \phi(x) + \eta(x,t) \tag{14}$$

and

$$\chi(x,t) = \chi(x) + \rho(x,t). \tag{15}$$

Next let us to expand the fluctuations $\eta(x,t)$ and $\xi(x,t)$ in terms of the normal modes:

$$\eta(x,t) = \sum_n \epsilon_n \eta_n(x) e^{i\omega_n t} \tag{16}$$

and

$$\rho(x,t) = \sum_n c_n \rho_n(x) e^{i\tilde{\omega}_n t}. \tag{17}$$

Thus, if $\tilde{\omega}_n = \omega_n$, the equations of motion for the two fields become from (4) a Schrödinger-like equation for two-component wave functions $\Psi_n$. If $\tilde{\omega}_n \neq \omega_n$ we obtain

$$\mathcal{H}\Psi_n = \tilde{\Psi}_n, \quad n = 0, 1, 2, ..., \tag{18}$$

where the fluctuation operator can be written as

$$\mathcal{H} = \left( \begin{array}{cc} -\frac{d^2}{dx^2} + V_{11} & V_{12} \\ V_{21} & -\frac{d^2}{dx^2} + V_{22} \end{array} \right)_{|\phi=\phi(x),\chi=\chi(x)} \tag{19}$$

with

$$\begin{aligned} V_{11} &= m^2\{3tanh^2[(\mu^2 - m^2)x] - 1\} \\ V_{21} &= V_{12} = 4\lambda^3\alpha\beta\tanh[(\mu^2 - m^2)x]sech[(\mu^2 - m^2)x] \\ V_{22} &= (-\mu^2 + 2m^2\lambda)\tanh^2[(\mu^2 - m^2)x] \end{aligned} \tag{20}$$

and

$$\tilde{\Psi}_n = \left( \begin{array}{c} \omega_n^2\eta_n(x) \\ \tilde{\omega}_n^2\rho_n(x) \end{array} \right). \tag{21}$$

When the two-component normal modes in (18) satisfy ${\omega_n}^2 \geq 0$ and $\tilde{\omega}_n^2 \geq 0$ we have ensured the stability of the defects.

We can calculate the two-component eigenfunction associated to the zero mode via classical analysis of second order differential equation. However, here we present only the results to a particular case. Making an extension for the case of only one single real scalar field we can realize, *a priori*, the asymptotic ($x \to \infty$) behaviour so that we obtain the following zero mode eigenfunction ($\tilde{\omega} = \omega = 0$)

$$\Psi_0(x) = C_1 \left( \begin{array}{c} e^{-bx} \\ cos(\kappa x + \theta) \end{array} \right), \tag{22}$$

where $\theta$ is a constant. As $m^2 < \mu^2$ implies $\kappa^2 = 2\frac{m^2}{\lambda} - \mu^2 < 0$, for $\lambda \geq 2$. Besides, our results for kinks are ensured when $\mu$ and $\nu$ are in the following interval: $\frac{1}{3} \leq \frac{m^2}{\mu^2} < 1$.

## III. CONCLUSIONS

The connection between the non-relativistic quantum mechanics with two-component wave functions and the stability equations associated with defect (soliton) solutions for a model of two coupled real scalar fields in 1+1 dimensions has been presented. In this work, we have considered an application for a potential associated with the $\phi^4$ model in 1+1 dimensions which, was solved by the trial orbit method treated in the Ref. [4].

We see that, if $\Psi_-^{(0)} = \Psi_0(x)$ is a normalizable two-component eigenstate of the bosonic sector, one cannot write $\Psi_+^{(0)}$ of the fermionic sector in terms of $\Psi_-^{(0)}$ in a similar manner to ordinary supersymmetric quantum mechanics. This fact can be checked in the example treated here. A detailed analysis of our work will be published elsewhere.

Static classical field configurations have been recently exploited into the context of defect that live inside topological defects [12], and triple junctions via $N = 1$ supersymmetry theories [13] and without supersymmetry providing networks of domain wall [14], and using the vinicity of BPS bound states [15]. Recently the BPS saturated objects with axial geometry (wall junctions, vortices), in generalized Wess-Zumino models have been investigated [16].

We are investigating the issue of the stability, via arguments based on supersymmetric quantum mechanics, for the BPS and non-BPS states from two domain walls in a potential model in both situations, without supersymmetry and in the case of a minimal model ($N = 1$-SUSY),[17].

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## REFERENCES


[1] E. B. Bogomol'nyi, *Sov. J. Nucl. Phys.* **24**, 489 (1976).

[2] M. K. Prasad, C. H. Sommerfied, *Phys. Rev. Lett.* **35**, 760 (1975).

[3] R. Jackiw, *Rev. Mod. Phys.* **49**, 681 (1977).

[4] R. Rajaraman, *Phys. Rev. Lett.*, **42**, 200 (1979); *Solitons and Instantons*, (North-Holland, Amsterdam, 1982).

[5] S. Coleman, *Aspects of Symmetry*, (Cambridge University, London, 1985), pp. 190 - 191; S. Coleman, *Classical Lumps and their Quantum Descendants*, 1975 Erice Lectures published in *New Phenomena in Sub-Nuclear Physics*, Ed. A. Zichichi, Plenum Press, New York (1977).

[6] A. P. Balachandran, G. Marmo, B. S. Skagerstam and A. Stern, *Classical Topology and Quantum States*, (World Scientific, Singapore, 1991), pp. 104-106; E. J. Weinberg, *Annu. Rev. Nucl. Part. Sci.* **42**, 177 (1992).

[7] L. J. Boya and J. Casahorran, *Phys. Rev.* **A39**, 4298 (1989).

[8] R. de Lima Rodrigues, P. B. da Silva Filho and A. N. Vaidya, *Phys. Rev.* **D58** (1998) 125023.

[9] N. Grahan and R. L. Jaffe, *Nucl. Phys.* **B544**, 432 (1999); preprint hep-th/9808140, 7 April 2000.

[10] G. H. Flores and N. F. Svaiter, preprint Notas de Física, CBPF-NF-014/01; hep-th/01017043.

[11] P. B. Burt, *Phys. Rev. Lett.* **32**, 1080 (1974).

[12] D. Bazeia and F. A. Brito, *Phys. Rev.* **62D**, 101701 (R)(2000).

[13] P. M. Saffin, *Phys. Rev. Lett.* **83** 4249 (1999); S. M. Carroll, S. Hellerman and M. Trodden, *Phys. Rev.* **61D**, 065001 (2000).

[14] D. Bazeia and F. Brito, *Phys. Rev. Lett.* **84**, 1094 (2000).

[15] C. A. G. Almeida, D. Bazeia and L. Losano, *J. Phys. A: Math. Gen.* **34**, 3351 (2001).

[16] M. Shifman and T. ter Veldhuis, *Phys. Rev.* **62D**, 065004 (2000).

[17] G. de Sousa Dias, E. L. Graça and R. de Lima Rodrigues, *Stability equation involving two-component eigenmodes associated to two domain walls*, in a forthcoming paper.