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by

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SUPERCONVERGENCE RELATIONS IN $\pi - A_1$ SCATTERING *

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Strong-interaction "superconvergent" sum rules have recently been proposed invoking the high-energy behaviour of certain invariant amplitudes [1, 2]. The saturation of these superconvergent relations by a finite number of intermediate states leads, for small value of momentum transfer, to self-consistent relations among the coupling constants and masses of the involved particles as, for example, demonstrated in the case of $\rho - \pi$ scattering [1,3]. In this paper we discuss similar sum rules for the axial vector meson A_1 and π scattering, restricting ourselves only to forward $(t = 0)$ case.

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The $\pi - A_1$ scattering amplitude can be written in terms of invariant amplitudes A, B, C and D as

$$T(s,t) = (\varepsilon_1 \cdot P)(\varepsilon_2 \cdot P)\ A(s,t) + \frac{1}{2}\Big[(\varepsilon_1 \cdot P)(\varepsilon_2 \cdot Q) + (\varepsilon_1 \cdot Q)(\varepsilon_2 \cdot P)\Big] B(s,t) +$$

$$+ (\varepsilon_1 \cdot Q)(\varepsilon_2 \cdot Q)\ C(s,t) + (\varepsilon_1 \cdot \varepsilon_2)\ D(s,t)\ , \qquad (1)$$

where $P = \frac{1}{2}(p_1 + p_2)$; $Q = \frac{1}{2}(q_1 + q_2)$, $p_{1,2}$ are the momenta of the initial and final pion and $q_{1,2}$ and $\varepsilon_{1,2}$ are the momenta and polarizations of the A_1-meson. Reggepole theory predicts the high-energy behaviour $A^{(I)}(s,t) \sim s^{\alpha_I - 2}$, $B^{(I)}(s,t) \sim s^{\alpha_I - 1}$, $C(s,t) \sim s^{\alpha_I}$ and $D^{(I)}(s,t) \sim s^{\alpha_I}$, where I is the isospin in the t-channel. The amplitudes A and B also satisfy the crossing symmetry relations $A^{(1)*}(v,t) = - A^{(1)}(-v,t)$, $A^{*(2)}(v,t) = A^{(2)}(-v,t)$, $B^{(2)}(v,t) = - B^{(2)}(-v,t)$, where $v = (P.Q)/m_A$. From the experimental result $0 \leqslant \alpha^{(1)}(0) < 1$ and assuming[4] $\alpha^{(2)}(0) < 0$ we are led to the following nontrivial, super-convergent dispersion relations for $t \sim 0$:

$$\int_{-\infty}^{\infty} \mathrm{Im}\ A^{(1)}(v,t)\ dv = 0\ , \qquad (2)$$

$$\int_{-\infty}^{\infty} v\ \mathrm{Im}\ A^{(2)}(v,t)\ dv = 0 \qquad (3)$$

and[5]

$$\int_{-\infty}^{\infty} \mathrm{Im}\ B^{(2)}(v,t)\ dv = 0\ , \qquad (4)$$

Next we discuss the coupling constant sum rules obtained by approximately saturating these relations by the contributions of a number of s-channel resonances, say, ρ, B(1210), D(1285), $\eta_\nu(1050)$, $f^o(1250)$, $f^{o'}(1500)$, $\rho'(1650)$ etc.

For the two independent $A_1\rho\pi$ couplings, we take them as

$$g_L \left(p_\mu - \frac{(p.q)q_\mu}{q^2}\right)\left(q_\lambda - \frac{(p.q)p_\lambda}{p^2}\right)\varepsilon^\lambda \varepsilon'^\mu$$

for the longitudinal coupling and

$$\left(g_T/m^2_{A_1}\right)\varepsilon^{\lambda\alpha\beta\nu}\,\varepsilon_{\mu\alpha'\beta'\nu}\,p_\alpha\, q_\beta\, p^{\alpha'}\, q^{\beta'}\varepsilon_\lambda\, \varepsilon'^\mu$$

for the transverse coupling. The $BA_1\pi$ coupling is written as $g_B\, \varepsilon_{\mu\nu\lambda\beta}\; p^\mu q^\nu \varepsilon^\lambda \varepsilon'^\beta$. Here p(q) and $\varepsilon(\varepsilon')$ are the momentum and polarization of A_1(ρ or B). The coupling $f_o A_1\pi$ is written as $g_f\; h^{\mu\nu}\,\varepsilon_\mu\, p_\nu$. Other couplings are similarly defined. The sum rules obtained are given in the Appendix.

The sum rules derived, for t = 0, from relations (2) and (3) lead to the interesting result that in order to obtain nonvanishing values for the couplings we must include at least one of the known higher resonances $f^{o'}(1500)$, $\rho'(1650)$. However, in order to obtain nonnegative solutions for the squares of the coupling constants, compatible with the observed small partial decay rates for $A_1\pi$ mode of the resonances under consideration, we must include both $f^{o'}$ and ρ' for the saturation of the above sum rules.

Even then a solution consistent with the decay widths of the particles involved for the $A_1\pi$ mode is not possible as is verified, say, by solving for g_D^2. Here the only positive term comes from the transverse coupling of the ρ' meson. A non-negative solution for g_D^2 requires too large a decay width (about fifty times the total width) for ρ' into $A_1\pi$ channel.

We, thus, conclude that the presently known resonances cannot saturate the above sum rules.

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APPENDIX

The three sum rules for $t = 0$, are

$$- g_L^2 m_{A_1}^2 \left(1 - \left[\frac{m_{A_1} + \nu_\rho}{m_\rho}\right]^2\right) - g_T^2(\nu_\rho^2 - m_\pi^2) + g_B^2\, m_{A_1}^2 +$$

$$+ g_f^2 \left[\frac{1}{6} - \left(\frac{1}{2}\frac{m_\pi^2}{m_f^2} + \frac{1}{3}\frac{m_\pi^2 + m_{A_1}\nu_f}{m_f^2}\right) + \frac{2}{3}\left(\frac{m_\pi^2 + m_{A_1}\nu_f}{m_f^2}\right)^2\right] +$$

$$+ g_D^2\, m_{A_1}^2 + g_{\eta_\nu}^2 + f' \text{ term} + \rho' \text{ term} = 0 \;, \qquad (A.1)$$

$$g_L^2\, m_{A_1}^2 \left(1 - \left[\frac{m_{A_1} + \nu_\rho}{m_\rho}\right]^2\right) \nu_\rho + g_T^2(\nu_\rho^2 - m_\pi^2)\,\nu_\rho - g_B^2\, m_{A_1}^2\, \nu_B +$$

$$+ g_f^2 \left[\frac{1}{6} - \left(\frac{1}{2}\frac{m_\pi^2}{m_f^2} + \frac{1}{3}\frac{m_\pi^2 + m_{A_1}\nu_f}{m_f^2}\right) + \frac{2}{3}\left(\frac{m_\pi^2 + m_{A_1}\nu_f}{m_f^2}\right)^2\right]\nu_f +$$

$$+ g_D^2\, m_{A_1}^2\, \nu_D + g_{\eta_\nu}^2\, \nu_{\eta_\nu} + f' \text{ term} + \rho' \text{ term} = 0 \;, \qquad (A.2)$$

$$g_L^2\, m_{A_1}^2\left(1-\left[\frac{m_{A_1}+\nu_\rho}{m_\rho}\right]^2\right)-g_T^2(\nu_\rho^2-m_\pi^2)\left(1+\frac{2\nu_\rho}{m_{A_1}}\right)+g_B^2(m_{A_1}^2+$$

$$+\,2m_{A_1}\nu_B)+g_f^2\left[-\frac{5}{6}+\left(-\frac{1}{2}\frac{m_\pi^2}{m_f^2}+\frac{2}{3}\frac{m_\pi^2+m_{A_1}\nu_f}{m_f^2}\right)+\frac{2}{3}\left(\frac{m_\pi^2+m_{A_1}\nu_f}{m_f^2}\right)^2\right]$$

$$-\,g_D^2(m_{A_1}^2+2m_{A_1}\,\nu_D)+g_{\eta\nu}^2\;+f'\text{ term}+\rho'\text{ term}=0\,, \qquad (A.3)$$

where

$$\nu_x=\frac{m_x^2-m_{A_1}^2-m_\pi^2}{2m_{A_1}}\,.$$

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