ISSN 0029 - 3865

CBPF-NF-011/84

THE FERMIONIC DETERMINANT FOR MASSLESS QCD_2

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*Work supported by CNPq

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Abstract.

We calculate exactly the fermionic functional determinant for massless SU(N) bidimensional QCD

Key-words: Field Theory; Functional Integral

In the last years, analysis of Quantized Field Theories in two-dimensional space-time have been valuable in order to understand physical properties of more realistic Field Theories[1]. Recently, several attempts to solve the bidimensional SU(2) Quantum Chromodynamics ($SU(N)-QCD_2$) have been made [2-5]

Alvarez[4] in a recent work calculated the fermion functional determinant in a reduced model obtained by replacing the Dirac equation with a suitable modification. We show in this short note that Alvarez's method works in the general case with the full Dirac operator by choosing appropriately the decoupling gauge[5] introduced by Roskies.

We start our analysis by considering the generating functional for Euclidean $SU(2)-QCD_2$ with massless fermions

$$Z = \int D\bar{\Psi}\ D\Psi\ DA\ \exp\left\{ -\int (\bar{\Psi} D\Psi + \frac{1}{4} F^2_{\mu\nu} + \text{gauge fixing term}) d^2x \right\} \tag{1}$$

where $\not{D} = \gamma_\mu D^\mu = i\not{\partial} + \not{A}.t$ and $\{t_a\}$ are the generators of the colour group SU(2)

In order to compute the fermionic determinant, $\ell n \det \not{D}$, we follow Roskies work[5] and choose the decoupling gauge.

$$i\gamma^\mu A_\mu.t = \gamma^\mu(\partial_\mu U)U^{-1} \tag{2}$$

where U is given by

$$U = e^{-\gamma_5\ \xi.t} \tag{3}$$

In this gauge the operator $\not{D}$ can be written as

$$\not{D} = i\gamma^{\mu} U \partial_{\mu} U^{-1} \tag{4}$$

We can introduce now a one - parameter family of operators:

$$\not{D}^{(\tau)} = i\gamma^{\mu} \, U_{(\tau)} \partial_{\mu} U^{-1}{}_{(\tau)} \tag{5}$$

where $U_{(\tau)} = e^{-\tau\gamma_5 \, \xi . t}$ with $\tau \varepsilon \, [0,1]$. Defining vector field $A_{\mu}^{(\tau)}$ and $V_{\mu}^{(\tau)}$ by the relation

$$U_{(\tau)} \, \partial_{\mu} \, U^{-1}_{(\tau)} = V_{\mu}^{(\tau)} + A_{\mu}^{(\tau)} = G_{\mu}^{(\tau)} \tag{6}$$

with $A_{\mu}^{(\tau)}$ and $V_{\mu}^{(\tau)}$ given in Ref. 6 and we remark that $A_{\mu}^{(\tau)}$ has no direct relation to the gauge field defined in (2). We can express now the operator $[D^{(\tau)}]^2$ in the following convenient form [4]

$$\left[\not{D}^{(\tau)} \right]^2 = - (\partial_{\mu} + W_{\mu}^{(\tau)})^2 + F^{(\tau)} \tag{7}$$

with

$$W_{\mu}^{(\tau)} = V_{\mu}^{(\tau)} + i\varepsilon_{\mu\nu} \, \gamma_5 \, A_{\nu}^{(\tau)}$$

$$F^{(\tau)} = - \partial_{\mu} A_{\mu}^{(\tau)} - \left[V_{\mu}^{(\tau)}, A_{\mu}^{(\tau)} \right] + i\varepsilon_{\mu\nu} \, \gamma_5 \left[A_{\mu}^{(\tau)}, A_{\nu}^{(\tau)} \right] \tag{8}$$

and in deriving (7) and (8) we have used the integrability conditions [4] of (6).

Regulating now the determinant, $\ell n \det[\not{D}^{(\tau)}]^2$, by the proper time method and using the property satisfied by (5), namely:

$$\frac{d}{d\tau} D^{(\tau)} = \gamma_5 \xi . t \not{D}^{(\tau)} + \not{D}^{(\tau)} \gamma_5 \xi . t \qquad (9)$$

we get the following differential equation

$$\frac{d}{d\tau} \mathrm{Tr}\, \ell n[\not{D}^{(\tau)}]^2 = 4 \lim_{\varepsilon\to 0} \int d^2x \, \mathrm{Tr}_{C x \gamma} <x|\gamma_5 \xi . t \exp[-\varepsilon(\not{D}^{(\tau)})^2|x> \qquad (10)$$

where $\mathrm{Tr}_{C x\gamma}$ denotes the trace over the colour and Dirac indices. The diagonal part of $\exp[-\varepsilon(D^{(\tau)})^2]$ has well-known asymptotic expansion[7]. For the operator in (7) we have:

$$<x|\exp[-\varepsilon(\not{D}^{(\tau)})^2]|x> \underset{\varepsilon\to 0}{\to} \frac{1}{4\pi\varepsilon} [1+ \varepsilon F^{(\tau)} + 0(\varepsilon^2)] \qquad (11)$$

Substituting (11) in (10) we get:

$$\mathrm{Tr}\, \ell n\left[\not{D}^{(\alpha)}\right]^2 = C(\infty) + \frac{1}{\pi}\int d^2x \int_0^\alpha d\tau \, \mathrm{Tr} \left\{(\partial_\mu A_\mu{}^{(\tau)} + \left[V_\mu{}^{(\tau)}, A_\mu{}^{(\tau)}\right])\right.$$

$$\left. \xi . t \, \gamma_5 \right\} - \frac{i}{\pi}\int d^2x \int_0^\alpha d\tau \, \varepsilon_{\mu\nu} \mathrm{Tr} \left\{\left[A_\mu{}^{(\tau)}, A_\nu^{(\tau)}\right] \xi . t\right\} \qquad (12)$$

Using the integrability conditions [4] and the properties of $\not{D}^{(\tau)}$ we can show with a straightforward algebra that

$$\mathrm{Tr}\, \ell n \left[\not{D}^{(\tau)}\right]^2 = C(\infty) - \frac{1}{4\pi} \int d^2x \, \mathrm{Tr}\left[\not{B}^{(\tau)} \not{B}^{(\tau)}\right] +$$

$$- \frac{1}{2\pi} \int d^2x \int_0^\tau d\alpha \, \mathrm{Tr}\left[\gamma_5 \not{B}^{(\alpha)} \xi . t \, \not{B}^{(\alpha)}\right] \qquad (13)$$

with $G_\mu^{(\tau)}$ defined by formula (6). The infinite constant in (12) can be absorbed by the zero-point energy renormalization, the second term is the non-abelian extension of the Schwinger mechanism and the last term can be shown [3] to correspond to the two dimensional analogue of the Wess-Zumino functional [8] recently considered by Witten [9] in a four-dimensional chiral model. It is important to note that the expression (13) coincides with the result obtained by Gamboa Saravi, Schaposnik and Solomin [3] where the fermionic determinant is calculated by performing a non-abelian chiral change of variables. The method discribed in this note is an alternative way to calculate the Euclidean ferimionic functional determinant for two-dimensional SU(N) massless QCD.

Acknowledgements

The authors are indebted to Prof. F.A. Schaposnik for calling their attention to the papers of reference 3-5, and to Profs. C.G. Bollini, J.J. Giambiagi, J.A. Mignaco and K.D. Rothe for some discussions.

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