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THE FORM FACTORS IN  $K_{\mu 3}$  AND  $K_{e 3}$  DECAY

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ABSTRACT

An investigation of the form factors in  $K\mu_3$  and  $Ke_3$  decay amplitudes, on the assumption of local production of the fermion pair, is carried out by means of dispersion relations techniques. The approximation is made of taking explicit account of  $K\pi$ -scattering in the unitarity condition and all the other contributions are incorporated into a single arbitrary parameter. The validity of this approximation is discussed. The results are expressed in terms of S and P phase shifts for  $K\pi$ -scattering.

1. Introduction

A systematic investigation of decay processes has been lately carried out within the framework of a universal Fermi interaction

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with V-A coupling.<sup>1</sup> However, with regard to  $K_{\mu 3}$  and  $K_{e3}$  one has not yet come to a definite conclusion. The theoretical analysis predicts that for each given type of coupling the dependence on the lepton energy is well determined, whereas the effects of strong interactions appear as form factors which depend only on the pion energy.<sup>2</sup> There has been some argument to the effect that these form factors are rather slowly varying functions of the pion energy over the physical region for the decay processes. It is the purpose of this work to investigate this question in some more detail making use of dispersion theory.

## 2. Determination of the Form Factors From Dispersion Relations.

We consider the processes:

$$K_{\mu 3} \rightarrow \pi + \mu + \nu \quad (\text{I})$$

$$K_{e3} \rightarrow \pi + e + \nu \quad (\text{II})$$

in which the K-meson has momentum  $p_K$  the outgoing particles have momenta  $p_\pi, p$  and  $q$  respectively. The matrix element for this decay process on the assumption of local production of the fermion pair may be written in the form:

$$(4p_{\pi 0} p_{K 0} p_0 q_0)^{1/2} \langle \pi \mu \nu | K \rangle = (2\pi)^4 \delta(p_\pi + p + q - p_K) (4p_{\pi 0} p_{K 0})^{1/2} \quad (\text{I})$$

1. R.P.Feynman and M.Gell-Man., *Phys.Rev.* 109,193 (1958).

E.C.G.Sudarshan and R.E.Marshak., *Phys.Rev.* 109,1860 (1958).

2. S.Furuichi et al., *Progr. Teor.Phys.*(Japan) 17, 89 (1957).

A .Pais and S.B.Treiman, *Phys.Rev.* 105, 1616 (1957)

S .W.Mac Dowell, *Nuovo cimento* 6, 1445 (1957).

$$\langle \pi | J_\alpha(0) | K \rangle \bar{u}(p) \gamma_5 (1 - \gamma_5) v(q) \quad (1)$$

We have, in addition, restricted to V or A the type of coupling and used the two component theory of the neutrino. Taking the  $\Lambda$  parity, by convention, to be the same as that of the nucleons it follows from conservation of parity in strong interactions, that the vector or pseudo-vector character of the baryon current is associated with negative or positive parity of K-mesons.

The strong interaction effects are contained in the factor  $\langle \pi | J_\alpha(0) | K \rangle$  which in the K-meson rest system, depends only on the pion energy.

From Lorentz invariance it follows that:

$$(4p_{\pi 0} p_{K 0})^{1/2} \langle \pi | J_\alpha^I(0) | K \rangle = \frac{1}{2} (p_K + p_\pi)_\alpha f_+^I + \frac{1}{2} (p_K - p_\pi)_\alpha f_-^I \quad (2)$$

where the index  $I = 1/2, 3/2$  indicates that  $J_\alpha^I(0)$  transforms like an isospinor of the respective rank, and the  $f^I$ 's are functions of

$$\omega = -p_\pi p_K / m_K m_\pi \quad (3)$$

We assume that each covariant amplitude has a representation of the form:

$$f(\omega) = \frac{1}{\pi} \int_1^\infty \frac{\rho(\omega')}{\omega' - \omega} d\omega' \quad (4)$$

where

$$\frac{1}{2} (p_K + p_\pi)_\alpha \rho_+(\omega) + \frac{1}{2} (p_K - p_\pi)_\alpha \rho_-(\omega) = \sqrt{2p_{K0}} \sum_n \delta(p_n + p_\pi - p_K) \cdot$$

$$\langle 0 | J_\alpha(0) | n \rangle \langle n | j_\pi(0) | k \rangle \quad (5)$$

The lower limit  $\omega_0 = 1$  in (4) comes from the threshold energy of the least massive state  $|n\rangle = |K\pi\rangle$ .

As on similar problems<sup>3</sup> it is practically impossible to take into account the contributions from intermediate states with more than two particles. Therefore we shall neglect in the sum all but the terms which come from  $|n\rangle = |K\pi\rangle$  and  $|n\rangle = |N\bar{Y}\rangle^*$  in the hope that the contribution from other states is relatively small. On the other hand the matrix elements involved in (5) vanish except for those intermediate states with total angular momentum  $j = 0, 1$ . The equations for  $f(\omega)$  become decoupled when expressed in terms of transition amplitudes in states of given angular momentum, in the center of mass of the  $(K, \pi)$  system. The connection between  $f_{\pm}$  and  $f_j$  is:

$$f_0 = f_- + \frac{m_K^2 - m_\pi^2}{W^2} f_+ ; \quad f_1 = f_+ \quad (6)$$

and

$$f_- = f_0 - \frac{m_K^2 - m_\pi^2}{W^2} f_1 \quad (7)$$

We obtain likewise for  $f_0$  and  $f_1$  a representation of the form (4) except that  $f_0$  has a pole at  $W^2 = 0$  ( $\omega = \omega_0$ ) with residue  $(m_K^2 - m_\pi^2)f_1(\omega_0)$ .

The relation between  $W^2$  and  $\omega$  is:

$$W^2 = m_K^2 + m_\pi^2 + 2m_K m_\pi \omega \quad (8)$$

In the two particles approximation the weight functions are

3. M. L. Goldberger and S.B. Treiman, Phys.Rev. 110, 1178 (1958); 111, 335, (1958).  
P. Federbush, M.L. Goldberger and S.B. Treiman, Phys.Rev. 112, 642 (1958).

\* Throughout this paper  $Y$  stands for  $\Lambda$  or  $\Sigma$ .

given by:

$$\rho_0(\omega) = k f_0(\omega) \phi_0(\omega)^* \theta(W^2 - (m_K + m_\pi)^2) + k^0 F_0(\omega) \psi_0(\omega)^* \theta(W^2 - (M_Y + M)^2) \quad (9)$$

$$\rho_1(\omega) = k f_1(\omega) \phi_1(\omega)^* \theta(W^2 - (m_K + m_\pi)^2) + k^0 F_1(\omega) \psi_1(\omega)^* \theta(W^2 - (M_Y + M)^2) \quad (10)$$

where  $\theta$  is the step function,  $k$  and  $k^0$  are the momenta in the center of mass of the systems  $(K, \pi)$  and  $(N, \bar{Y})$  respectively related to the total energy  $W$  by:

$$k^2 = (W^2 - (m_K + m_\pi)^2) (W^2 - (m_K - m_\pi)^2) / 4W^2$$

$$k^0{}^2 = (W^2 - (M_Y + M)^2) (W^2 - (M_Y - M)^2) / 4W^2$$

$\phi_{0,1}$  and  $\psi_{0,1}$  are the (S,P) amplitudes for transitions.

$$K + \pi \longrightarrow K + \pi$$

$$N + \bar{Y} \longrightarrow K + \pi$$

$F_{0,1}(\omega)$  is the scalar (vector) part of  $\langle 0 | J_\alpha(0) | N\bar{Y} \rangle$ .

Within the framework of a primary Fermi interaction the last term in (9,10) is the source of the decay process while the first term gives an induced contribution to the decay amplitudes due to  $K-\pi$  interaction. In order to get some insight into the details of the decay mechanism let us investigate the general structure of the term with two baryons intermediate state.

We can write:

$$\left( \frac{p_{N_0} p_{\bar{Y}_0}}{M M_Y} \right)^{1/2} \langle 0 | J_\alpha(0) | N\bar{Y} \rangle = \bar{u}(-p_{\bar{Y}}) \gamma_\alpha (G_V - \gamma_5 G_A) u(p_N) \quad (11)$$

and

$$\left( \frac{2p_{K_0} p_{N_0} p_{\bar{Y}_0}}{M M_Y} \right)^{1/2} \langle N\bar{Y} | j_\pi(0) | K \rangle = \bar{u}(p_N) O(A + Q B) u(-p_{\bar{Y}}) \quad (12)$$

where  $O = 1$  for pseudoscalar K-mesons and  $O = \gamma_5$  for scalar K-mesons\*, and  $Q = \frac{1}{2} (p_K + p_\pi)$ . Therefore:

$$\rho_0^{(2)} = \frac{2k^0}{W^2} G \int_{-1}^1 \left[ (p_{N_0} M_Y + p_{\bar{Y}_0} M) A + k k^0 Z (-p_{N_0} + p_{\bar{Y}_0}) B \right] 2\pi dZ \quad (13)$$

$$\rho_0^{(2)} = \frac{k^0}{kW} G \int_{-1}^1 \left[ k^0 Z (M_Y \pm M) A + k(2k^0{}^2 Z^2 + \frac{1}{2} (W^2 - M_Y^2 - M^2) \pm M_Y M) B \right] 2\pi dZ \quad (14)$$

and the upper (lower) signs stand for pseudoscalar (scalar) K-mesons. Since, in perturbation theory, for fixed Z, A and B behave like  $1/W^2$  for  $W^2 \rightarrow \infty$ , it follows that  $\rho_0 \sim 1/W^2$  and  $\rho_1 \sim \text{const.}$  It then follows that at least one subtraction is necessary in the spectral representation of  $f_1$  but  $f_0$  may still have none.

On the other hand for small values of  $k^0$  one has the following behaviour of  $\rho_0^{(2)}$  and  $\rho_1^{(2)}$ :

For pseudoscalar K-mesons:  $\rho_0^{(2)} \sim k^0{}^3 (M_Y - M)$ ;  $\rho_1^{(2)} \sim k^0 M M_Y$

For scalar K-mesons:  $\rho_0^{(2)} \sim k^0 M M_Y$ ;  $\rho_1^{(2)} \sim k^0{}^3$

\* We are assuming that  $\Lambda$  and  $\Sigma$  have the same parity. If they have opposite parities then for  $Y = \Sigma$  one should take  $O = \gamma_5$  for pseudoscalar and  $O = 1$  for scalar K-mesons.

Therefore on kinematic grounds only one might expect a small contribution to  $f_0$  from  $\rho_0^{(2)}$  as compared to the contribution of  $\rho_1^{(2)}$  to  $f_1$  while the opposite holds for scalar K-mesons.

Assuming that K-mesons are indeed pseudoscalar, we are then justified in neglecting the contribution from  $\rho_0^{(2)}$ . Moreover we can cast the contribution from  $\rho_1^{(2)}$  into the arbitrary constant which appears as the result of one subtraction in the spectral representation for  $f_1$ . This arbitrary constant represents the global result of all dynamical effects which cannot be taken into account individually.

We thus arrive at the following representations for  $f_0$  and  $f_1$ :

$$f_0(\omega) = \frac{m_K^2 - m_\pi^2}{\omega^2} f_1(\omega_0) + \frac{1}{\pi} \int_1^\infty \frac{\rho_0^{(1)}(\omega')}{\omega' - \omega} d\omega' \quad (15)$$

$$f_1(\omega) = f_1(\omega_0) + \frac{1}{\pi} \int_1^\infty \frac{\omega'^2}{\omega'^2} \frac{\rho_1^{(1)}(\omega')}{\omega' - \omega} d\omega' \quad (16)$$

These equations are of the Omnès type,<sup>4</sup> since  $\rho^{(1)}(\omega)$  is given by the product of  $f(\omega)$  by the amplitude for K- $\pi$  scattering and kinematical factors. Therefore one can write down explicit solutions for  $f_0$  and  $f_1$  in terms of the S and P wave phase shifts for K- $\pi$  scattering, defined by:

$$\operatorname{tg} \delta_0 = \frac{k \operatorname{Re} \rho_0}{1 - k \operatorname{Im} \rho_0} \quad ; \quad \operatorname{tg} \delta_1 = \frac{k \operatorname{Re} \rho_1}{1 - k \operatorname{Im} \rho_1} \quad (17)$$

The solutions are:

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4. R. Omnès, Nuovo cimento 8, 316 (1958).



$$f_0(\omega) = \frac{m_K^2 - m_\pi^2}{\omega^2} f_1(\omega_0) \exp. \left( - \frac{1}{\pi} \int_1^\infty \frac{\omega'^2}{\omega'^2} \frac{\delta_0(\omega')}{\omega' - \omega} d\omega' \right) \quad (18)$$

$$f_1(\omega) = f_1(\omega_0) \exp. \left( - \frac{1}{\pi} \int_1^\infty \frac{\omega'^2}{\omega'^2} \frac{\delta_1(\omega')}{\omega' - \omega} d\omega' \right) = f_+(\omega) \quad (19)$$

which substituted into (7) gives:

$$f_-(\omega) = \frac{m_K^2 - m_\pi^2}{\omega^2} f_1(\omega_0) \left\{ \exp. \left( - \frac{1}{\pi} \int_1^\infty \frac{\omega'^2}{\omega'^2} \frac{\delta_0(\omega')}{\omega' - \omega} d\omega' \right) - \exp. \left( - \frac{1}{\pi} \int_1^\infty \frac{\omega'^2}{\omega'^2} \frac{\delta_1(\omega')}{\omega' - \omega} d\omega' \right) \right. \quad (20)$$

The physical content of the approximations made is that the direct contribution from the primary weak interaction is taken into account in the determination of  $f_+(\omega)$  but  $f_-(\omega)$  is obtained with the neglect of those direct effects and therefore entirely induced by the  $K=\pi$  interaction.

It is this circumstance that led us to a result which depends only on one arbitrary parameter  $f_1(\omega_0)$  and is such that the ratio of the two amplitudes  $f_-/f_+$  is independent of this parameter. If, however, the experiments do not confirm this remarkable feature of our results, one concludes that a subtraction is indeed necessary in (15) and that the direct contributions to  $f_-$  are not negligible.

### 3. Evaluation of the Form Factors

The physical range of values of  $\omega$  in the decay process (I) is from  $\omega_1 = -1$  at the lower end of the pion spectrum to  $\omega_2 = -(m_K^2 + m_\pi^2 - m_\mu^2)/2m_K m_\pi$  at the upper end.

We shall now estimate the variation of  $f_+(\omega)$  in this range.

We find:

$$\frac{1}{f_1} \frac{df_1}{d\omega} = \frac{\delta_1(\bar{\omega})}{\pi} \frac{1}{1-\omega} \quad (21)$$

where  $\bar{\omega} > 1$  depends on  $\omega$ . The variation of  $f_+(\omega)$  between the two ends of the spectrum is then approximately given by:

$$\frac{\Delta f_+}{f_+(-1)} \approx \frac{\delta_1(\bar{\omega})}{\pi} \frac{\Delta \omega}{2} = \frac{\delta_1(\bar{\omega})}{\pi} \frac{(m_K - m_\pi)^2 - m_\mu^2}{4m_K m_\pi} = 0.44 \frac{\delta_1(\bar{\omega})}{\pi} \quad (22)$$

If there is a P-wave resonance at low enough energy a large variation of  $f_+(\omega)$  is expected. If, however, the P-wave is small the variation is negligible.

We turn now to the analysis of the behaviour of  $f_-(\omega)$ . In order to simplify the discussion we assume  $\delta_1(\omega)$  negligible and  $\delta_0(\omega)$  given by:

$$\text{tg } \delta_0 = a (W^2 - (m_K + m_\pi)^2)^{1/2} \quad (23)$$

The integrations can then be performed yielding the following results:

1)  $a > 0$  (repulsive S-wave):

$$f_- = \frac{(m_K^2 - m_\pi^2) a}{1 + a(m_K + m_\pi)} \frac{1}{W^2} \left[ \left( (m_K + m_\pi)^2 - W^2 \right)^{1/2} - m_K - m_\pi \right] < 0 \quad (24)$$

The ratio

$$\frac{f_-(\omega_2)}{f_-(\omega_1)} \approx \frac{1}{2} \left( 1 + \sqrt{\frac{2m_K m_\pi}{m_K^2 + m_\pi^2}} \right) = 0.85 \quad (25)$$

is independent of the scattering length.

2)  $a < 0$  (attractive S-wave):

$$f_- = \frac{(m_K^2 - m_\pi^2) a}{1 - a \left( (m_K + m_\pi)^2 - W^2 \right)^{1/2}} \frac{1}{W^2} \left[ \left( (m_K + m_\pi)^2 - W^2 \right)^{1/2} - m_K - m_\pi \right] > 0 \quad (26)$$

As  $a$  varies from 0 to  $-\infty$  the ratio  $f_-(\omega_2)/f_-(\omega_1)$  decreases from 0.85 to 0.6.

So far we have discussed the behaviour of the individual amplitudes ignoring isotopic spin dependence. In general  $f_+$  and  $f_-$  are linear combinations of amplitudes with isospin indices  $I = 1/2$  and  $I = 3/2$  which in turn depend on the  $k-\bar{\pi}$  scattering amplitudes in the corresponding isostates. If however, we adhere to the selection rule  $I = 1/2$  (which, for instance, arises from a Fermi interaction involving  $\Lambda P$  as the baryon pair) then there will be only one amplitude, namely that with  $I = 1/2$ .

#### 4. Final Remarks

The analysis we have made is also valid if the weak interaction proceeds via an intermediate vector field of mass  $m$ . The only difference is that the transition amplitude (1) must then be multi-

plied by the boson propagator  $(W^2 - m^2)^{-1}$  .

The  $K_{e3}$  decay is naturally more appropriate for an investigation of the form factor  $f_+(\omega)$  since  $f_-(\omega)$  will not contribute in this case. On the other hand information concerning  $f_-(\omega)$  can only be obtained from data on  $K_{\mu 3}$  decay and due to interference with  $f_+(\omega)$  more extensive and accurate experiments are required. In both cases however, one needs the overall determination of the kinematics of the events observed experimentally.

The results we have obtained and the assumptions underlying them can then be checked when these data become available.

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