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THE GAUGE EFFECT: AN APPLICATION TO THE ELECTRON SELF ENERGY

by

R.M. Doria*

Centro Brasileiro de Pesquisas Físicas - CBPF/CNPq
Rua Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

*Also, ICTP, Trieste, Italy

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R.M. Doria

Centro Brasileiro de Pesquisas Físicas - Rio de Janeiro - Brasil

and

International Centre for Theoretical Physics - Trieste, Italy

Abstract

The influence of the gauge rotation in perturbation theory is investigated. As an example, the fermion propagator is studied in a gauge invariant form. The calculation is parametrized for a straight line. A gauge independent renormalization constant is obtained.

I. Introduction

The gauge principle is being considered as a source for generating interactions. At the moment, our best promise of unification lies here. However the characterization of the limitations of this principle has not been fully developed yet. This work will be an effort to understand the consequences of the gauge transformation

$$\psi(x) \rightarrow e^{i\lambda(x)} \psi(x) \tag{1}$$

The presence of the above relation in the calculations of physical entities through perturbation theory is going to be investigated. The difference between these new numbers generated by the field rotation (1) and the old numbers calculated when the gauge concept was not considered will be referred to as the Gauge Effect.

Thus, inspite of the present success of Gauge Theories, two kinds of criticisms can be levelled towards it. They are the Bohm Aharanov Effect and the Gauge Effect value for some process being analised. We intend to explore the second aspect in QED for the Electron Self Energy.

The dynamics in field theory is given through a Lagrangian. Usually the Green functions are defined, in the interaction picture, in the following way [1]

$$G(x_1, x_2, \ldots x_n) = \langle 0 | T\ \phi(x_1) \ldots \phi(x_n)\ S | 0 \rangle_{conn} \tag{2}$$

where

$$S = u(-\infty,\infty) = T\ e^{i\int d^4x \mathcal{L}_{int}(\phi)}$$

The aspect to be analysed here is that although the gauge principle (1) unchange the Lagrangian it does not preserve the Green function (2). In the literature an appropriate Gauge Invariant Green function is

$$G_{GI}(x,y) \equiv G_{NEW}(x,y) = \langle 0 | T \left[\psi(x)\ e^{ie\int_{x,c}^{y} A_\mu(x')dx'^\mu}\ \bar{\psi}(y)\ e^{ie\int \mathcal{L}_{int} dx^4} \right] | 0 \rangle \tag{3}$$

that is being explored under differents considerations [2]. It is invariant under the gauge transformation

$$\begin{aligned} \psi'(x) &= e^{i\lambda(x)}\psi(x) \\ \psi'(y) &= e^{i\lambda(y)}\psi(y) \\ A'_\mu(x') &= A_\mu(x') - \frac{1}{e}\partial_\mu\lambda(x) \end{aligned} \tag{4}$$

The equation that correlates (2) and (3) is

$$\int_x^y A_\mu(x')\ dx'^\mu = \lambda(x) - \lambda(y) \tag{5}$$

We understand (4) as a relation that represents electron and photon fields interdependence. They are connected through the parameter $\lambda(x)$ that is unknown. Thus (5) represents a type of connection between A_μ and λ. The conventional interpretation for propagator (2) is that an electron is destroyed in x and created in y. In (3) the presence of the term $e^{ie\int_{x,c}^{y} A_\mu dx'^\mu}$ creates a type of cable between x and y. In field theory the action at a distance is usually defined as a limit sum over infinitesimal paths. Therefore a study of infinitesimal distances is suf ficient. In (2) and (3) we will thus consider $x \to y$. It is then natural to take the path C as being the straight line even though the phase factor is path dependent

$$\int_{x,c}^{y} dx'^{\mu} A_{\mu}(x') = \int_{0}^{1} du\, A_{\mu}(ux)\, x^{\mu} \tag{6}$$

Our objective will be to calculate the renormalization constant Z_2 from (3). Intuitively from

$$G_{GI}^{R} = Z_2^{NEW}\, G_{GI} \tag{7}$$

we expect that as G_{GI} is gauge invariant (by construction) and also G_{GI}^{R} (because it is connected with experimental results), then Z_2^{NEW} will also be gauge invariant. Considering the influence of the renormalization constant parameters on the experimental results a measure of the gauge effect will depend on the difference between Z_2^{NEW} in (6) and the usual Z_2 from (2). In order to calculate it we are going through the following steps: in part I, an expansion formula is established up to second order; in part II and III, an expression for the new pieces of the propagator is developed. Afterwards we calculate through a method developed in Appendix A the corresponding renormalization constants. In part VI the cancellation of the gauge dependence is shown followed by the conclusion.

II. The Gauge Invariant Propagator up to Second order

Expanding (3) gives

$$G_{NEW}(x,y) = \langle 0|T[\psi(x)\,\overline{\psi}(y)]|0\rangle +$$
$$+ \sum_{n=1}^{\infty} \frac{(ie)^n}{n!} \int d^4x_1 \ldots d^4x_n \langle 0|T[\psi(x)\mathcal{L}_{int}(x_1)\ldots\mathcal{L}_{int}(x_n)\overline{\psi}(y)|0\rangle$$
$$+ \sum_{m=1}^{\infty} \frac{(ie)^m}{m!} \int_x^y d\,x_1'^{\mu}\, dx_2'^{\nu}\ldots \langle 0|T[\psi(x)A_\mu(ux_1')A_\nu(\nu x_2')\ldots\overline{\psi}(y)]|0\rangle$$
$$+ \sum_{n,m=1}^{\infty} \frac{(ie)^n(ie)^m}{n!\,m!} \iint_x^y d^4x_1\, dx_1'^{\mu}\ldots d^4x_n\, dx_n'^{\nu} \langle 0|T[\psi(x)A_\mu(ux_1')\mathcal{L}_{int}(x_1)\ldots$$
$$\ldots A_\nu(x_n')\mathcal{L}_{int}(x_n)\overline{\psi}(y)|0\rangle \qquad (8)$$

Considering up to second order

$$G^{(2)}_{NEW}(x,y) = \frac{(ie)^2}{2!} \int d^4x_1 d^4x_2 \langle 0|T[\psi(x)\mathcal{L}_{int}(x_1)\mathcal{L}_{int}(x_2)\,\overline{\psi}(y)]|0\rangle +$$
$$+ \frac{(ie)^2}{2!} \int_x^y dx_1'^{\mu}\, dx_2'^{\nu} \langle 0|T[\psi(x)\, A_\mu(ux_1')\, A_\nu(vx_2')\, \overline{\psi}(y)]|0\rangle +$$
$$+ (ie)^2 \int d^4x_1 \int_x^y dx_1'^{\mu} \langle 0|T[\psi(x)A_\mu(ux_1')\mathcal{L}_{int}(x_1)\,\overline{\psi}(y)]|0\rangle \qquad (9)$$

or

$$G^{(2)}_{NEW} = G^{(2)}_{OLD} + G^{(2)}_{I} + G^{(2)}_{II} \qquad (10)$$

Thus, the gauge invariant propagator englobes the usual one coming from (2). The possible new interpretation will be given through the pieces $G^{(2)}_{I}$ and $G^{(2)}_{II}$.

III. The expression for $G_I^{(2)}(x,y)$

Parametrizing (9) with a straight line as Fig. 1

$$G_I^{(2)}(x,y) = \frac{e^2}{2!}\int du\, dv\; S_F(x-y)\, D_{F_{\mu\nu}}\left[(x-y)(u-v)\right](x-y)^{\mu}(x-y)^{\nu} \tag{11}$$

and considering the Fourier transform

$$G^{(2)}(p-p') = \int d^4x\, d^4y\; e^{ipx}\, e^{-ip'y}\, G^{(2)}(x-y)$$

yields,

$$G^{(2)}(p-p') = -\frac{e^2}{2}\int \frac{d^4q\, d^4k}{du\quad dv}\, \frac{d^2}{dq_\mu dq_\nu}\left\{\delta\left[p-q-k(u-v)\right]\delta(p-p')\right\} \frac{1}{\not{q}-m}\, \frac{(-g_{\mu\nu}+\alpha k_\mu k_\nu/k^2)}{k^2} \tag{12}$$

Using the divergence theorem

$$G_I^{(2)}(p-p') = -\frac{e^2}{2}\int d^4q\, d^4k\, du\, dv\; \delta\left[p-q-k(u-v)\right]\delta(p-p')\, \frac{d^2}{dq_\mu dq_\nu}\left\{\frac{1}{\not{q}-m}\right\} \frac{\left(-g_{\mu\nu}+\alpha\dfrac{k_\mu k_\nu}{k^2}\right)}{k^2} \tag{13}$$

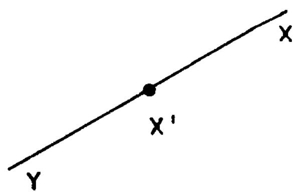

Fig. 1. The integral $\int_c A_\mu(x')dx'^{\mu}$ is parametrized for a straight line

Calculating the second derivative and making use of the delta function gives

$$G_I^{(2)}(p-p') = -e^2\,\delta(p-p')\int d^4k\,du\,dv\,\frac{1}{\not{p}-\not{k}(u-v)-m}\,\gamma^\mu\,\frac{1}{\not{p}-\not{k}(u-v)-m}\,\gamma^\nu\,\frac{1}{\not{p}-\not{k}(u-v)-m}\,\frac{-g_{\mu\nu}+\alpha\,k_\mu k_\nu/k^2}{k^2} \qquad (14)$$

The gauge dependent part in (14) is

$$G_I^{(2),G} = -\alpha\,e^2\,\delta(p-p')\int\frac{d^4k}{k^4}\left[\frac{1}{\not{p}-\not{k}-m} - \frac{1}{\not{p}-m}\right] \qquad (15)$$

IV. The expression for $G_{II}^{(2)}(x,y)$

From (9),

$$G_{II}^{(2)}(x,y) = (ie)^2\int d^4x_1\int_x^y dx'^\mu <0|T[\psi(x)A_\mu(x')\,\overline{\psi}(x_1)\,\gamma^\nu\psi(x_1)A_\nu(x_1)\overline{\psi}(y)|0> \qquad (16)$$

Parametrizing as Fig.1 and changing the variables to

$$x \equiv 0 \quad , -x_1 \equiv y$$

gives,

$$G_{II}^{(2)}(x) = ie^2\int du\,d^4y\; S_F(y)\;\gamma^\nu\;S_F(x-y)\;D_{F_{\mu\nu}}[y-ux]\,x^\mu \qquad (17)$$

Making the Fourier transform

$$G_{II}^{(2)}(p) = \frac{e^2}{(2\pi)} \int du\, d^4k\, d^4q\, d^4r\, \frac{\partial}{\partial r_\mu} \{ [\delta(p-q+ku) \}\, \delta[k+q-r] \frac{1}{\not{q}-m} \gamma^\nu \frac{1}{\not{p}-m} \frac{\left(-g_{\mu\nu} + \alpha \frac{k_\mu k_\nu}{k^2}\right)}{k^2} \tag{18}$$

integrating over d^4q and using the divergence theorem

$$G_{II}^{(2)}(p) = -\frac{e^2}{(2\pi)^4} \int du\, d^4k\, d^4r\, \delta(p-r+ku) \frac{\partial}{\partial r_\mu} \left[\frac{1}{\not{p}-\not{k}-m} \gamma^\nu \frac{1}{\not{p}-m} \right] \left[\frac{-g_{\mu\nu} + \alpha \frac{k_\mu k_\mu}{k^2}}{k^2} \right]$$

Performing the Integration over d^4k

$$G_{II}^{(2)}(p) = \frac{e^2}{(2\pi)^4} \int du\, d^4k \left[\frac{1}{\not{p}+\not{k}u-\not{k}-m} \gamma^\mu \frac{1}{\not{p}+\not{k}u-\not{k}-m} \gamma^\nu \frac{1}{\not{p}+\not{k}u-m} + \frac{1}{\not{p}+\not{k}u-\not{k}-m} \gamma^\nu \frac{1}{\not{p}+\not{k}u-m} \gamma^\mu \frac{1}{\not{p}+\not{k}u-m} \right] \left[\frac{-g_{\mu\nu} + \alpha\, k_\mu k_\nu / k^2}{k^2} \right] \tag{19}$$

(19) can be splitted in terms of a gauge and non gauge parts

$$G_{II}^{(2)}(p) = G_{II}^{(2),NG} + G_{II}^{(2),G} \tag{20}$$

V. Calculation of the non-gauge part in Z_2

$$Z_{2,I} = \lim_{\not p \to m} -i(\not p - m)\, G_I(p) \tag{21}$$

Studying first the non-gauge part contribution to (21) we get from (14) that the ratio between the numerator and denominator in the integrand is given by

$$\frac{N}{D} = \frac{[\not p - \not k(u-v) + m]\, \gamma^\mu\, [\not p - \not k(u-v) + m]\, \gamma_\mu\, [\not p - \not k(u-v) + m]}{[p^2 - m^2 - 2kp(u-v) + k^2(u-v)^2]^3}$$

Using the calculation process described in the Appendix A

$$N(u,v,\ldots) = N(0,0,\ldots) = (\not p + m)\,\gamma^\mu\,(\not p + m)\,\gamma_\mu\,(\not p + m)$$

and changing the variables to

$$x = u - v$$

$$y = u + v$$

gives

$$Z^{NG}_{2,I} = \lim_{\not p \to m} (-)\; i\,(\not p - m)\, e^2\, \delta(p - p')\, N(0,0,\ldots)\,. \int \frac{d^4k}{k^2} \int \frac{dx\,dy}{2} \frac{1}{[p^2 - m^2 - 2kpx]^3} \tag{22}$$

Integrating over x and y

$$Z^{NG}_{2,I} = i\,e^2\ \delta(p-p')\ \frac{2m^2 + m\not{p} - p^2}{2}\ \frac{1}{(2\pi)^4} \int \frac{d^4k}{k^2}\ \frac{1}{(k.p)^2}$$

In the Appendix B is calculated $\int d^4k$. It is regularized with $k^2 \to k^2 + \Lambda^2$ where $\Lambda^2 \to 0$. It gives

$$Z^{NG}_{2,I} = \frac{e^2}{(4\pi)^2}\ \frac{4m^2 + 2m\not{p} - 2p^2}{p^2}\ \left[\frac{1}{\varepsilon} + \frac{1}{2}(\gamma - \ell n 4\pi) + \frac{1}{2}\,\ell n\,\frac{4\Lambda^2}{p^2}\right] \tag{23}$$

VI. Calculation of the non-gauge part in $Z_{2,II}$

From (21) and (19)

$$Z^{NG}_{2,II} = \lim_{(\not{p}-m)\to 0} \frac{i\,e^2}{(2\pi)^4} \int \frac{d^4k}{k^2}\, du\ (\not{p}-m)\ \{L + M\}$$

where L and M are given by

$$L = \frac{\not{p} - u\not{k} + m}{\left[(p-uk)^2 - m^2\right]}\,\gamma^{\mu}\,\frac{\not{p} - u\not{k} + m}{\left[(p-uk)^2 - m^2\right]}\,\gamma_{\mu}\,\frac{\not{p} + (1-u)\not{k} + m}{\left[(\not{p} + (1-u)k)^2 - m^2\right]}$$

and

$$M = \frac{p - (1-u)\not{k} + m}{\left[p - (1-u)k\right]^2 - m^2}\ \frac{\left[(2-n)(\not{p} + u\not{k}) + nm\right]\left[\not{p} + u\not{k} + m\right]}{\left[(p+uk)^2 - m^2\right]^2} \tag{24}$$

The integral over u is evaluated through the method of the Appendix A. Considering the analytic continuation to n dimensions and symmetrizing the integrand with $k \to -k$,

$$Z^{NG}_{2,II} = \frac{i\,e^2}{(2\pi)^4} \int \frac{d^nk}{k^2}\ \frac{(\not{p}-m)(\not{p}+m)\left[(2-n)\not{p} + nm\right]\left[\not{p} - \not{k} + m\right]}{\left[(p-k)^2 - m^2\right]\ 2pk\ (p^2 - m^2)} \tag{25}$$

and

$$Z^{NG}_{2,\text{II}_n} = \frac{i\,e^2}{(2\pi)^4}\int \frac{d^nk}{k^2}\;\frac{(\not p-m)(\not p-\not k+m)\left[(2-n)\not p+nm\right]\left[\not p+m\right]}{\left[(p-k)^2-m^2\right]\,2pk\;(p^2-m^2)} \tag{26}$$

Adding (25) and (26)

$$Z^{NG}_{2,\text{II}} = \frac{i\,e^2}{(2\pi)^4}\int \frac{d^nk}{k^2}\;\frac{8m^2-2m\not k-2k.p}{\left[(p-k)^2-m^2\right]\,2pk} \tag{27}$$

Using the integrals of the Appendix B

$$Z^{NG}_{2,\text{II}} = -\frac{2e^2}{(4\pi)^2}\,\frac{m^2}{p^2}\left[\ell n\Lambda-1+i\pi\right]-\frac{e^2}{(4\pi)^2}\left(\frac{\not p}{m}+1\right)\left[\frac{2}{\varepsilon}+\gamma-\ell n\,4\pi+2\,\ell n\,m-2\right] \tag{28}$$

VII. The gauge part

The $G_{\text{I}}^{(2),G}$ is written in (15). In order to calculate the integral over du in (19), consider the identity

$$-\not k = \not p - u\not k - m - \left[\not p + (1-u)\not k - m\right]$$

it yields,

$$G_{\text{II}}^{(2),G}(p) = \alpha\,\frac{e^2}{(2\pi)^4}\int \frac{d^4}{k^4}k\;du\,\{N+P\}$$

where

$$N = -\frac{1}{\not p-u\not k-m}\;\not k\;\frac{1}{\not p+(1-u)\not k-m}+\frac{d}{du}\left(\frac{1}{\not p-u\not k-m}\right)$$

and

$$P = \frac{d}{du}\left(\frac{1}{\not p + u\not k - m}\right) + \frac{1}{\not p - u\not k - m}\not k\frac{1}{\not p + (1-u)\not k - m} \tag{29}$$

then,

$$G_{II}^{(2),G} = \alpha\frac{2e^2}{(2\pi)^4}\int\frac{d^4k}{k^4}\left\{\frac{1}{\not p - \not k - m} - \frac{1}{\not p - m}\right\} \tag{30}$$

The gauge depend part for $G_{OLD}^{(2),G}$ can be written as

$$G_{OLD}^{(2),G} = -\frac{\alpha e^2}{(2\pi)^4}\int\frac{d^4k}{k^4}\frac{1}{\not p - m}\not k\frac{1}{\not p - \not k - m}\not k\frac{1}{\not p - m} \tag{31}$$

Using the relation

$$\not k = \not p - m - (\not p - m - \not k)$$

and symmetrizing the integrand $k \to -k$, yields

$$G_{OLD}^{(2),G} = -\frac{\alpha e^2}{(2\pi)^4}\int\frac{d^4k}{k^4}\left\{\frac{1}{\not p - \not k - m} - \frac{1}{\not p - m}\right\} \tag{32}$$

(30), (14) and (29) in (10) gives

$$G_{NEW}^{(2),G} = 0 \tag{33}$$

Then the new renormalization coefficient Z_2^{NEW} will be given by (23), (28) and a part from the old propagator [1]. It yields,

$$Z_2^{NEW,(2)} = 1 - \frac{e^2}{(4\pi)^2}\left[\frac{2}{\varepsilon} + \gamma - \ell n\, 4\pi - 2i\pi\right] + \\ + \frac{e^2}{(4\pi)^2}\left[\frac{8}{\varepsilon} + 4\gamma - 4\ell n\, 4\pi + \ell n\,\frac{16\Lambda^2}{m^2}\right] \tag{34}$$

where the second term (34) is correlated with the ultraviolet divergence and the third with infrared.

VIII. Conclusion

At present the belief is that physical entities must be gauge invariant. Motivated by this a new propagator was defined in (3). We could interpret the phase factor in it as cable that controls messages that create and destroy particles in x and y. Equation (5) would be a way to install such cable. Mathematically, it means that the gauge rotation (1) redistributes the perturbation series as (8). There appears then two new terms besides the common one. Physically, this new serie arose from the fact that the photon and electron field are not independent. The gauge concept brought a connection between them through the parameter $\lambda(x)$.

The gauge rotation (1) and the property of gauge invariance (independence of the parameter α in the photon propagator (12)) reveal different aspects of a Gauge Theory. Our observation is that although these theories can generate a gauge invariant boson, it may depend on (1) perturbatively.

The so called Gauge Effect will appear through the influence of the new terms in the expansion in the calculations. For instance, the cross section σ in a Compton Scatering will suffer this Effect in order e^4 as in Fig. 2. In order to have a complete calculation

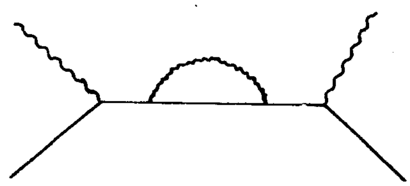

Fig. 2. A graph for the Compton effect in order e^4. The Gauge Effect will bring through (9) another two diagrams.

it is necessary to include graphs as in Fig. 3. They are based on the vertex function. The difference in the cross section using these new propagators must be evaluated. Its value will tell us about the meaning of (1) in the Compton scattering. However it is α independent. We also would observe the possible ap apearance of a new beta function.

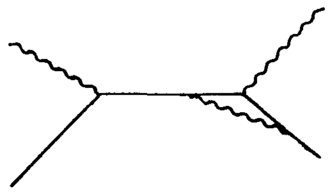

Fig. 3. The gauge effect in the vertex renormalization through

$$<0|T\,[\psi(x)\, e^{i\int_c A_\mu(x')\, d x'^\mu}\, \overline{\psi}(y)\, A_\nu(z)\, e^{i\int \mathcal{L}_{int}\, d^4x}\,]\,|0>$$

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Appendix A - The method of calculation

A usual expression in the text is

$$L = \lim_{a\to 0} \left[a \int_0^1 \frac{1}{(u+a)^2} f(u)\, du \right]$$

Expanding in Taylor series

$$L = \lim_{a\to 0} \left\{ f(a) \int_0^1 du \frac{1}{(u+a)^2} + f'(a) \int_0^1 du \frac{1}{u+a} + \ldots \right\}$$

that gives

$$L = \lim_{a\to 0} \left\{ f(a) + a\, f'(a)\, \ell n \frac{1+a}{a} + \ldots + \text{something } a^n \right\}$$

$$L = f(0) \tag{A1}$$

Example: $a = \not{p} - m$

$$\lim_{\not{p}\to m} (\not{p}-m) \int_0^1 du \frac{1}{(\not{p}-m+u)^2} = 1$$

Another kind of expression is

$$M = \lim_{a\to 0} \left[a \int_0^1 du \frac{1}{(u+b)^2} f(u) \right]$$

where $b = A_n a^n + A_{n-1} a^{n-1} + \ldots + \text{cte}$

giving

$$M = \frac{f(0)}{A_n} \tag{A2}$$

Similarly,

$$N = \lim_{a\to 0} \left[a \int_0^1 \frac{1}{(u^2+u+a)^2} f(u)\, du \right] = f(0) \tag{A3}$$

$$0 = \lim_{a\to 0} \left[a \int_0^1 \frac{1}{(u^2+du+a)^2} f(u)\, du \right] = \frac{f(0)}{d} \tag{A4}$$

$$P = \lim_{a\to 0} \left[a \int_0^1 \frac{1}{(u^2+du+b)^2} f(u)\, du \right] = \frac{f(0)}{A_n d} \tag{A5}$$

Appendix B - Some Basic Integrals

The first integral is

$$T = \frac{1}{(2\pi)^4} \int \frac{d^4k}{k^2+\Lambda^2} \frac{1}{(k.p)^2} \tag{B1}$$

Using Feynman parameters and integrating over d^nk

$$T = \frac{i}{(16\pi^2)^{n/4}} \Gamma(3-\frac{n}{2}) \left(\frac{p^2}{2}\right)^{n-6} \frac{1}{n-4} (-\alpha)^{\frac{n-4}{2}}$$

where $\alpha^2 = \frac{4\Lambda^2}{p^2}$

The other integral is

$$U = \frac{1}{(2\pi)^4} \int \frac{d^4k}{k^2+\Lambda^2} \frac{1}{\left[(p+k)^2 - m^2\right]} \frac{1}{2pk} \tag{B3}$$

Integrating over d^nk

$$U = -\frac{i}{(4\pi)^2} \int dx\, dy \frac{y}{\left[1+y(x-1)\right]} \frac{1}{a^2-y^2p^2} \tag{B4}$$

where

$$a^2 = \Lambda^2(1-y)\left[1+y(x-1)\right] = \Lambda^2 - y^2p^2$$

Evaluating over x gives the following integrals

$$I_1 = \int_0^1 dy\left[\ell n(1-y)+y\right]\frac{1}{y^2} = -1$$

$$I_2 = \int_0^1 dy\,\frac{y}{y^2-\Lambda^2} = -(\ell n\Lambda + i\pi)$$

that yields,

$$U = \frac{i}{(4\pi)^2}\,\frac{1}{4p^2}\left[-1+\ell n\Lambda + i\pi\right] \qquad (B5)$$

$$V = \frac{1}{(2\pi)^4}\int\frac{d^4k}{k^2+\Lambda^2}\,\frac{\not k}{\left[(p-k)^2-m^2\right](2pk)} \qquad (B6)$$

Using the identity

$$\not k = \frac{\not p}{2m^2}\,(2pk)$$

will result for (B6) in an integral that does not have infared divergences in the x integration. Then we can consider $\Lambda^2 = 0$ and through dimensional regularization gives

$$V = -\frac{i}{(4\pi)^2}\,\frac{\not p}{m^2}\left[\frac{2}{\varepsilon}+\gamma-\ell n4\pi+\ell n m^2-2\right] \qquad (B7)$$

where ε=4-n is the dimensional regularization parameter and γ is the Euler constant.

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