

# The Cosmic Microwave Background Spectrum and an Upper Limit for Fractal Space Dimensionality

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## Abstract

The possibility to constrain fractal space dimensionality from Astrophysics and other areas is briefly reviewed. Using data from FIRAS instrument aboard COBE satellite and assuming space dimensionality to be  $3 + \epsilon$ , we calculate  $\epsilon = -(0.957 \pm 0.006) \times 10^{-5}$  and an absolute temperature  $2.726 \pm 0.003 \times 10^{-2}$  K by fitting the cosmic microwave background radiation spectrum to Planck's radiation distribution. The upper limit for  $|\epsilon|$ , shown to be similar to what was found based on microphysics, is interpreted as the maximum fluctuation space dimensionality should have undertaken in a very large spatial and temporal scale compared to that of the decoupling era.

*Subject headings:* cosmic microwave background

## 1 Introduction

The general idea that Astrophysics has something to say about space dimensionality can be traced back to Kant's conjecture (Handyside 1929) that three-dimensionality of space can be related to the structure of Newtonian gravitational law. As a second step, one can quote the work of William Paley (1802), which can be considered the first attempt to shed light on the space dimensionality problem clearly from Anthropic arguments. In his work, Paley analyzes the consequences of changes in the form of Newton's gravitational law and of the stability of the solar system on human existence. His speculations take into account a number of mathematical arguments for an anthropocentric design of the World, which rest all upon the stability of the planetary orbits in our solar system and on a Newtonian mechanical *Weltanschauung*, as should be expected at that time.

A scientific approach to this issue was introduced by Ehrenfest (1917, 1920) who tried to understand the threefold nature of space by formulating the question: "*How does it become manifest in the fundamental laws of Physics that space has 3 dimensions?*". Following this approach, the first investigations concerning General Relativity and a heuristic model of a pulsating Universe were made by Tangherlini (1963, 1986), who tried to impose constraints to integer space dimensionality by searching for bound stable states of the Universe.

The above question can obviously be reversed and we can try to answer the following: “*How do the fundamental laws of Physics entail space dimensionality?*” (Caruso & Moreira, 1997). In this alternative approach, space dimensionality may be taken as an unknown quantity or can be admitted to have a non integer value  $d = 3 + \epsilon$ ,  $\epsilon$  being a parameter to be experimentally determined. Since the introduction of the concept of fractal dimension by Mandelbrot (1977) this became an interesting possibility to be explored.

## 2 The first predictions

Following the general aforementioned idea, with just one exception, several authors have determined limits for  $|\epsilon|$ , as shown in Table 1.

Reference	Measured Quantity	$ \epsilon $
Zeilinger & Svozil, 1985	anomalous (g-2) electron factor	$(5.3 \pm 2.5) \times 10^{-7}$
Jarlskog & Ynduráin, 1985	periastron of Mercury and binary pulsar PSR 1913 + 16	$< 1.5 \times 10^{-9}$
Grassi, Sironi & Strini, 1986	3K radiation frequency spectrum	$< 0,02$
Schäfer & Müller, 1986	Lamb shift in hydrogen	$< 4 \times 10^{-11}$
	electron Compton wave length	$\leq 5 \times 10^{-7}$
	anomalous magnetic moment of the muon	$< 10^{-5}$
Müller & Schäfer, 1986	perihelion of Mercury	$\approx 10^{-9}$
Torres & Herrejón, 1989	3K radiation frequency spectrum	$\leq 10^{-3}$

Table 1: First predictions for deviations  $|\epsilon|$  from the integer value 3 for space dimensionality.

From a general point of view, it is important to stress that all these results cover a very large length scale from micro to macro cosmos, suggesting that space dimensionality did not fluctuated significantly from the central value  $d = 3$  for a very large range of spatial scales. Except form the two almost similar constraint concerning the cosmic background radiation, the upper limits for  $|\epsilon|$  vary four order of magnitudes for a typical length scale going from  $10^{-8}$  (atomic scale) to  $10^{20}$  meters.

So far as astronomical arguments are used, we see from Table 1 that different data from the motion of Mercury gave results of the same order of magnitude, namely,  $\epsilon \simeq 10^{-9}$ . A quick inspection of this table shows that the two upper limits on  $|\epsilon|$  which follows from analysis of the cosmic background radiation are not so stringent as the others. The result  $|\epsilon| < 0.02$  (Grassi, Sironi & Strini, 1986) was obtained by comparing the available data at that time for the Relic Radiation against a laboratory black-body source. The limit  $|\epsilon| \leq 10^{-3}$  (Torres & Herrejón, 1989) was achieved estimating the cosmic background radiation experimental errors for both the radiance and the spectrum to be of the order of ten percent (Woody & Richards, 1981; Smoot et al., 1983; Meyer & Jura, 1984; Uson & Wilkinson, 1984). The general idea underlining these two papers is that precise measurements of the *shape* of the black-body radiation frequency spectrum can be used to set upper limits to deviations of space dimensionality from the integer value 3. In this direction, it was shown by Mather et al. (1990) that the deviation of the shape of the cosmic microwave background spectrum (CMBR) from that of a black-body can be determined with an accuracy much greater than the measurement of the absolute temperature of the sky. In addition, it was

concluded, in this paper, that “*since the data show a good null both when FIRAS is looking at the external calibrator and at the sky, one can determine from the interferograms alone that the spectrum of the sky is close to a blackbody, regardless of the details of the data reduction and calibration*”. Therefore, we should investigate how the more precise 1996 COBE – COsmic Background Explorer – results (Fixsen et al. 1996) can improve the aforementioned limits.

### 3 The fit of COBE data

The FIRAS (Far InfraRed Absolute Spectrophotometer) on COBE satellite has been designed to measure extremely small deviations of the CMBR from a black-body spectrum. However, as all previous experimental results on the frequency spectrum of the Relic Radiation, the COBE data do not provide us with absolute measurements.

This experimental restriction is a consequence of the fact that, at the present moment, the anisotropy of the CMBR is not large enough to provide us with a clear choice of some particular dark region of the sky which could help us with the calibration (Richards, 1982). However, there is a hope that, at least for the present generation of spectral measurement apparatus, the WMAP (Wilkinson Microwave Anisotropy Probe) data, which provide an accurate enough measurement of the dipole spectrum, could lead to a recalibration of the absolute measurement of the FIRAS COBE using this data (D.J. Fixsen, 2008).

Nevertheless, they led to the conclusion that the CMBR agrees with a black-body spectrum to a high accuracy. The published results (Fixsen et al, 1996) are obtained by a comparison against an almost ideal (emissivity  $\approx 0.98$ ) black-body source placed inside the satellite and calibrated to a second external black-body (emissivity  $\approx 0.99997$ ). Actually, FIRAS, which is a rapid-scan polarized Michelson interferometer, provided a continuous differential comparison with a reference black-body adjusted to null the input signal. Unfortunately, there is no way to measure fluxes independently. From the theoretical point of view, this comparison is accomplished by assuming the Planck distribution and presupposing space to be three-dimensional.

For a  $d$ -dimensional space, Planck’s radiation law for the spectral density energy  $u_\nu$ , as a function of temperature  $T$  and frequency  $\nu$ , generalizes to (Caruso & Oguri, 2006)

$$u_\nu = \frac{2(d-1)\pi^{d/2}}{\Gamma(d/2)} \left(\frac{\nu}{c}\right)^d \frac{h}{e^{h\nu/kT} - 1} \quad (1)$$

For the moment, let us attribute any deviation from the ideal three-dimensional black-body radiation law to be due to the hypothesis that the number of dimension is actually  $d = 3 + \epsilon$  instead of just 3.

Thus, the data from Fixsen et al. (1996), complemented with those given at the site of COBE Collaboration (2008), are fitted, using the MINUIT package running the CERN ROOT program, not directly by eq. (1) but by a function of three parameters ( $N$ ,  $T$ ,  $\epsilon$ )

$$u_\nu = N \frac{\nu^d}{e^{h\nu/kT} - 1} \quad (2)$$

since what is relevant to our analysis is the shape of the spectrum. The result of the fit is shown in Figure 1.

The values which emerge from the fit are:

$$\begin{cases} \epsilon = -(0.957 \pm 0.006) \times 10^{-5} \\ T = 2.726 \pm 0.003 \times 10^{-2} \text{ K} \end{cases} \quad (3)$$

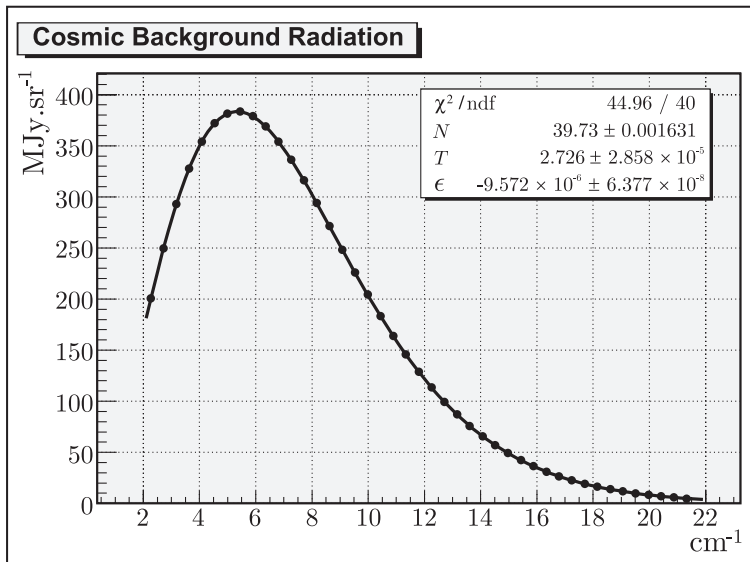


Figure 1: Fit of the intensity of cosmic microwave background spectrum as a function of the inverse of wavelength by Planck's radiation law generalized to  $d$  dimensions.

Notice that the FIRAS experimental result for the absolute temperature of CMBR,  $2.728 \pm 0.004$  K (95% confidence level), found in Fixsen et al. (1996), is dominated by systematic errors. It is important to stress that in our fit we have one more parameter besides a slightly different normalization, which, in practice, did not change the quality of the fit as can be shown by comparing our formal  $\chi^2$  per dof = 1.124 to the 1.15 value found in the COBE analysis. Actually, the statistical error to be compared to our result for the absolute temperature is 0.00001, given in Table 2 of Fixsen et al. (1996). By other side, the FIRAS-measured CMBR residuals strongly support the quality of their data and their fit. Thus, it follows that also our result is statistically significant.

Clearly, the assumption that all deviations from Planck's radiation law might be due to its dependence on dimensionality gives an overestimated value for  $\epsilon$ . Other sources for these deviations indeed exist and were analyzed Fixsen et al. (1996), such as Bose-Einstein and Compton distortions. In both cases the upper limits found for the involved parameters indicate a small effect. Based on these results, Wright et al. (1994) estimated that from the Bose-Einstein period ( $10^5 < z < 3 \times 10^6$ ) the ratio between the amount of energy converted from anything but the cosmic background to that of the background itself is  $\lesssim 6.4 \times 10^{-5}$ ; for the energy released after that period but before the decoupling era ( $z \simeq 10^3$ ) the limit is quite similar,  $\lesssim 6.4 \times 10^{-5}$ . Therefore, the value obtained for  $\epsilon$  should be seen also as an upper limit, namely  $|\epsilon| < 0.957 \times 10^{-5}$ , related to the epoch when the universe became entirely transparent: the decoupling era.

## 4 Discussions

Strictly speaking, since absolute measurements for the fluxes are not yet available, the physical meaning of the parameter  $\epsilon$  should be reinterpreted. Indeed, since the experimental device compares the background radiation to that of a local black-body with controlled temperature, at least in principle, one can figure out that the value of  $\epsilon$  could be locally (in this case, where the reference black-body is placed) different from the fractal dimensions of space on the horizon scales. In this case,  $\epsilon$  should be interpreted as a difference between

these two fractal dimensions at two far away spatial scales, as did Grassi, Sironi & Strini (1986). Alternatively, one can suppose that locally space is just three-dimensional and, in this case, we are putting a limit on how much space dimensionality could be different from three in the farthest corner of the universe one can look into.

In this paper, we established a bound for  $\epsilon$  by fitting data on CMBR taking the absolute temperature  $T$ ,  $\epsilon$ , and the normalization as free parameters in eq. (2). The overestimated value we get for  $\epsilon$ , equation (3), is two order of magnitude less than the more accurate upper limit value estimated by Torres & Herrejón (1989) with small errors. Another important difference found here, in respect to this last paper and to that of Grassi, Sironi & Strini (1986), is that we are able to fix also the *sign* of the  $\epsilon$  parameter, showing that it is *negative*. To the best of our knowledge, this is the first time that the sign of  $\epsilon$  can be determined by analyzing experimental results at astrophysical or cosmological scale. Up to now, the strongest constraint comes from microphysics. Indeed, Zeilinger & Svozil (1985) were able to determine not only the value of  $|\epsilon| = |d - 3|$  but the sign of this parameter too, from quantum field theory, getting  $\epsilon = -(5.3 \pm 2.5) \times 10^{-7}$ , which is approximately one order of magnitude less than our upper limit for  $|\epsilon|$ . It is important to remember that they pointed out that a value of space-time dimension less than 4 (assuming just one time dimension) would render all the logarithmic divergences in relativistic quantum field theory finite. Their result seems to resolve the discrepancies between the theoretical and experimental values of the anomalous magnetic moment of the electron (*cf.* also Svozil & Zeilinger, 1986). Another evidence in favor of arbitrarily small non-zero  $|\epsilon|$  comes from the study of Ising gauge theories in non-integer dimensions (Bhanot & Salvador, 1986). This was a first evidence, based on a numerical analysis, that for abelian gauge theories, the space-time dimension  $d + 1 = 4$  may, in some sense, be identified as an upper critical dimension.

In the concluding remarks of their 1985 paper, Zeilinger & Svozil wrote: “*It is certainly a challenge for future research to investigate whether or not the deviation of the dimension of space-time from four can be made more statistically significant than the present work suggests. Furthermore, the question of possible evidence for such a small deviation in other areas of Physics deserves attention*”. If we assume time to be one-dimensional, as usually done, the main result of the present paper can be seen as an answer to both challenges.

Finally, since the small deviation from three-dimensionality of space obtained here is extract directly from the CMBR data, it actually suggests that space dimensionality did not vary significantly in a huge temporal scale, once this background radiation is expected to be related to the Big Bang. This time scale can be safely put on the later epoch where the universe was about  $3 \times 10^5$  yr old ( $z \simeq 10^3$ ). In addition, due to its isotropy, this is the *only* experimental situation that, being confronted to *any* other local experiment on Earth, aimed to measure the number of space dimensions, drive to the conclusion that dimensionality did not vary also in a very large spatial scale (a cosmological scale).

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