CBPF-NF-011/05

Gravitational eternally-collapsing compact objects

Herman J. Mosquera Cuesta[1,2], José M. Salim[1], N. O. Santos[1,3,4]
[1] *Instituto de Cosmologia, Relatividade e Astrofísica (ICRA-BR), Centro Brasileiro de Pesquisas Físicas Rua Dr. Xavier Sigaud 150, CEP 22290-180 Urca, Rio de Janeiro, RJ, Brazil; hermanjc@cbpf.br*
[2] *Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, Miramare 34014, Trieste, Italy*
[3] *LRM-CNRS/UMR 8540, Université Pierre et Marie Curie, ERGA Boîte 142, 4 place Jussieu, 75005 Paris, Cedex 05, France*
[4] *Laboratório Nacional de Computação Científica, CEP 25651-070, Petrópolis, RJ, Brazil* *

(Dated: April 28, 2005)

Over the late years a growing discussion has taken on on whether or not there exists a solution of Einstein's equations which describes the formation of eternally-collapsing astrophysical compact objects other than black holes. Here we address the issue using a general relativistic perturbation theory assessment where dissipation through heat flow produces a null radiation. It is shown that a purely gravitational eternally-collapsing compact object (GECO) can indeed be formed only during the collapse of a supermassive star, whose radius after collapse is larger than the Schwarzschild's. The self-consistently computed opacity of the material composing the supermassive star, which radiates at the Eddington luminosity limit, is in agreement with stellar evolution and nuclear physics standards. Yet the perturbed mass responsible for the GECO luminosity is determined by the heat flux and the work done by the gas pressure restraining the collapse.

PACS numbers: 04.40.Dg, 04.70.-s, 97.60.Lf

The precise nature of the central compact star in the astrophysical sources known as Galactic Black Hole Candidates (GBHCs) is yet a matter of controversy. The issue is on the verge as most observations of GBHCs seem to rule out neutron stars (NSs) and even quark stars as their primaries, and more specially stellar-mass black holes because of the lack of evidence for event horizons in those objects [1].

Motivated in part by this conundrum, a search, within Einstein's general relativity, for exotic collapsed compact objects as stars with nontrapped surfaces [5], vacuum condensate stars [4], and magnetospheric eternally-collapsed compact objects (MECOs) [3, 7], and gravastars [8] has started on. It has been suggested that the general relativistic field equations do admit alternative extreme collapsed objects other than black holes. Nonetheless, there seems to be no clear understanding about the inextricable intricacy between the very restrictive constraint on the eternally-collapsing objects emission having to take place at the Eddington luminosity limit (ELL) and the nearly-null acceleration requirement for the collapse, which strictly speaking would demand a perturbation theory apraisal. Our purpose here is to describe the evolution to collapse, upon a perturbation of its hydrostatic equilibrium, of a massive star which is radiating at the ELL. As the power emitted by the star is so huge, one must expect that its subsequent collapse takes place at an extremely slow radius-shrinking rate with almost vanishing inward acceleration.

Based on these premises, we bring in a general relativistic perturbative treatment to evolve the star shrinking. As a prove of its self-consistency, we demonstrate that after imposing the Eddington limit to the luminosity escaping from the collapsing star, one obtains values of the opacity parameter $\mathcal{K}$ of the star material that are in agreement with what is expected from stellar evolution and atomic physics, for a wide star mass spectrum [14]. We also show that there exist a lower limit for the star mass that allows for the collapse not to end at the Schwarzschild radius, but rather at one a bit larger. Thus, our analysis is able to produce a GECO instead of a standard black hole, as far as $\mathcal{K}$ is consistently checked.

To model the collapse of a radiating star we consider a sphere described by a timelike three-space Σ. It divides spacetime into two distinct four-dimensional domains ν^- and ν^+. The interior spacetime ν^- is described by the general spherically symmetric metric with shear-free fluid motion in comoving coordinates

$$ds^2_- = -A^2(t,r)dt^2 + B^2(t,r)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \tag{1}$$

while the exterior spacetime ν^+ is described by Vaidya's metric, which represents an outgoing radial flux of unpolarized radiation,

$$ds^2_+ = -\left[1 - \frac{2M(v)}{R}\right] dv^2 - dvdR + R^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{2}$$

where $M(v)$ is the total energy inside Σ and is an arbitrary function of the retarded time v.

We assume that the source in Einstein's field equations is given in the interior spacetime ν^- by

$$G^{-}_{\alpha\beta} = \kappa T_{\alpha\beta} = \kappa[(\Delta + P)w_\alpha w_\beta + P g_{\alpha\beta} + q_\alpha w_\beta + q_\beta w_\alpha], \quad (3)$$

where κ is the gravitational coupling constant, Δ is the energy density of the fluid, P the isotropic pressure, w_α the four-velocity and $q_\alpha = q\delta^r_\alpha$ the radial heat flux vector which has to satisfy $q_\alpha w^\alpha = 0$.

Using Darmois´ junction conditions [9, 10] we obtain together with the field equations the relations

$$R(v) \stackrel{\Sigma}{=} rB, \quad (4)$$

$$\frac{dv}{1+z} \stackrel{\Sigma}{=} Adt, \quad (5)$$

$$M(v) \stackrel{\Sigma}{=} \frac{r^3 B\dot{B}^2}{2A^2} - \frac{r^3 B'^2}{2B} - r^2 B'. \quad (6)$$

$$P \stackrel{\Sigma}{=} qB \stackrel{\Sigma}{=} \frac{2}{\kappa R^2} L, \quad (7)$$

where $\stackrel{\Sigma}{=}$ means that both sides of the equation are evaluated on Σ, and z is the redshift given by

$$1 + z \stackrel{\Sigma}{=} \left(1 - \frac{2M}{R} + 2\frac{dR}{dv}\right)^{-1/2}, \quad (8)$$

and L is the luminosity,

$$L \stackrel{\Sigma}{=} -\frac{dM}{dv}(1+z)^2. \quad (9)$$

For the details of these calculations see [11, 12]. Relations (4) and (5) describe the radius of the fluid distribution and the time in both coordinates systems (1) and (2), respectively. While (6) describes the total energy entrapped inside the surface Σ where the dot and prime represent differentiation with respect to t and r, respectively. Relation (7) represents the continuity of the radial flux of momentum across Σ and shows that the pressure at the surface of the sphere does not vanish unless $q_\Sigma = 0$. If $q_\Sigma = 0$ there is no dissipation of heat from the source and the spacetime in ν^+ is that of Schwarzschild.

Slow radiating collapse.— We suppose that the fluid undergoes slow dissipative collapse, in an almost hydrostatic equilibrium background. Further we suppose that the perturbation on Δ and P have the same time dependence. Hence we have for the quantities so far defined,

$$A(t,r) = A_0(r) + \epsilon a(r)T(t), \quad (10)$$

$$B(t,r) = B_0(r) + \epsilon b(r)T(t), \quad (11)$$

$$\Delta(t,r) = \Delta_0(r) + \epsilon\delta(t,r), \quad (12)$$

$$P(t,r) = P_0(r) + \epsilon p(t,r), \quad (13)$$

and the radial heat flow q being of the order of ϵ, with $0 < \epsilon \ll 1$. The subscript zero denotes quantities describing equilibrium.

Considering Einstein´s field equations together with (10 - 13) we obtain

$$\kappa p = -2\kappa P_0 \frac{b}{B_0} T + 2\frac{b}{A_0^2 B_0}\left[\alpha(r)T - \ddot{T}\right], \quad (14)$$

$$\kappa q = 4\frac{b\beta(r)}{A_0^2 B_0^2}\dot{T}, \quad (15)$$

where

$$\alpha(r) = \frac{A_0^2}{B_0}\left[\left(\frac{A_0'}{A_0} + \frac{B_0'}{B_0} + \frac{1}{r}\right)\left(\frac{b}{B_0}\right)' + \left(\frac{B_0'}{B_0} + \frac{1}{r}\right)\left(\frac{a}{A_0}\right)'\right]\frac{1}{b}, \qquad \beta(r) = \frac{A_0^2}{2}\left(\frac{b}{A_0 B_0}\right)' \stackrel{\Sigma}{=} \frac{1}{b}, \quad (16)$$

and because of the pressure isotropy condition $a(r)$ and $b(r)$ have to satisfy

$$\left[\left(\frac{a}{A_0}\right)' + \left(\frac{b}{B_0}\right)'\right]' - 2\left(\frac{A_0'}{A_0} + \frac{B_0'}{B_0}\right)\left[\left(\frac{a}{A_0}\right)' + \left(\frac{b}{B_0}\right)'\right] - \frac{1}{r}\left[\left(\frac{a}{A_0}\right)' + \left(\frac{b}{B_0}\right)'\right] + 4\frac{A_0'}{A_0}\left(\frac{a}{A_0}\right)' = 0. \quad (17)$$

For anisotropic systems see [15]. The junction condition (7) with $P_0 \stackrel{\Sigma}{=} 0$ allows us to obtain a differential equation for $T(t)$,

$$\ddot{T} - 2\beta\dot{T} - \alpha T \stackrel{\Sigma}{=} 0. \quad (18)$$

We choose non-oscillatory solutions of (18), by assuming that $a(r)$ and $b(r)$ are such that $\alpha_\Sigma > 0$ and $\beta_\Sigma \leq 0$. Then

$$T(t) \stackrel{\Sigma}{=} -\exp[(-\beta + \sqrt{\alpha+\beta^2})t], \quad (19)$$

which represents a system that starts collapsing at $t = -\infty$ when $T(-\infty) = 0$ and is static, and goes on collapsing, diminishing its luminosity radius, while t increases.

Since from (15) $\beta = 0$ implies $q = 0$, then the solution for $b(r)$ can be assumed $b(r) = k[1 + \xi f(r)]A_0 B_0$, where k and ξ are constants. Thus, from (15) we have

$$\kappa q = \frac{2k\xi f'}{B_0^2}\dot{T}. \quad (20)$$

As $q > 0$ and the fluid is collapsing, $\dot{T} < 0$, then we assume $1 \gg \xi > 0$ and $f' < 0$.

After taking into account (10) and (11), the total energy entrapped inside Σ, given by (6), becomes

$$M(v) \stackrel{\Sigma}{=} M_0(r) + \epsilon m(t, r). \quad (21)$$

Considering that (21) can be used for spheres of radius r inside Σ [16] and using the field equations we obtain the novel result

$$M_0(r) = \frac{\kappa}{2}\int_0^{B_0 r} \Delta_0 (B_0 r)^2 d(B_0 r), \qquad m(t,r) = -\frac{\kappa}{2}(B_0 r)^2 \left[(-\beta + \sqrt{\alpha+\beta^2})_\Sigma\, qA_0B_0\left(1 - \frac{2M_0}{B_0 r}\right)^{1/2} + P_0 brT\right]. \quad (22)$$

We show that the equilibrium mass in (22) entrapped up to r is Newtonian-like, that is: density times proper volume, and is clearly positive. While the perturbed mass in (22) contains both a negative quantity because of the dissipation (due to q), which is the energy flowing onto the proper spherical surface $\kappa(B_0 r)^2/2$, which dominates during the initial stages of collapse, and a positive quantity, of purely relativistic origin, representing the work done by P_0 along the proper collapsing distance brT. Notice that for $m(t, r_\Sigma)$ there is only a contribution stemming from the dissipative term because $P_0 \stackrel{\Sigma}{=} 0$.

Eddington limit.— For the Eddington limit, in the slow collapsing regime, the collapsing fluid radiates at Σ as

$$L_{Edd} \stackrel{\Sigma}{=} \frac{4\pi Gc}{\mathcal{K}} M(1+z), \quad (23)$$

where $\mathcal{K}$ is the plasma opacity. From (5), (9), and (23) we have

$$-\frac{\dot{M}}{A} \stackrel{\Sigma}{=} \frac{4\pi Gc}{\mathcal{K}} M. \quad (24)$$

Considering that during this regime the radiation of M is supplied by $m(t,r)$, then (24) with (22) becomes

$$-\frac{\dot{T}}{A_0} \stackrel{\Sigma}{=} \frac{4\pi Gc}{\mathcal{K}} T, \quad (25)$$

which together with solution (19) imposes the remarkable relation

$$-\beta + \sqrt{\alpha+\beta^2} \stackrel{\Sigma}{=} \frac{4\pi Gc}{\mathcal{K}} A_0 , \quad (26)$$

linking the internal structure of the star to its opacity.

As we shall discuss briefly, the Newtonian limit is appropriate to describe the dynamics of the collapse of a supermassive star [13]. In that case $A_0 = 1$ and $B_0 = 1$ allowing (17) to be integrated and (16) implies $\alpha = \mathit{constant} > 0$, which can be chosen as the dynamical time scale of the system $\alpha = (GM_0/r^3)_\Sigma$, and further (16) gives also $\beta = \xi f'/2$.

Taking into account these limits, including the static limit of T in order to keep with the calculation up to first order in the perturbation parameter ϵ, one can write (9) and (20) for the heat flux as

$$\frac{L}{4\pi r^2} \equiv q = \frac{2}{\kappa}\xi f' \sqrt{\alpha} T. \quad (27)$$

By noting that in (16) one can approximate $\alpha \simeq \xi f'/r$, then the heat flux relation becomes

$$L = \frac{8\pi}{\kappa}\alpha^{3/2} r^3 T. \quad (28)$$

As we are interested in the evolution of the star being governed by the Eddington luminosity, then one can replace the luminosity L in (28) by its Eddington counterpart: $L_{\rm Edd} = 3.28 \times 10^4 (M_\gamma/M_\odot) L_\odot$, where $L_\odot = 4\pi\sigma_{\rm SB} R_\odot^2 T_\odot^4 = 3.83 \times 10^{33}$ erg s^{-1} is the solar luminosity, with $R_\odot$ and $T_\odot$ representing the radius and surface temperature of the Sun, respectively, and σSB the Stefan-Boltzmann constant. Besides, M_γ represents the actual mass responsible for the radiation emitted by the star [17], as seen by a comoving observer, and in our case is $m(t,r)$ given by (22), which then reduces to ($M_\odot$ units)

$$m(t,r) = -3.41 \times 10^{-6} \left(\frac{\kappa q}{k} B_0^2\right) r^2 \left(\frac{T}{\dot{T}}\right) \ . \qquad (29)$$

Hence, relation (28) becomes

$$0.445 L_\odot \left(\frac{\kappa^2 q}{k} \frac{B_0^2}{\dot{T}}\right) = \alpha^{3/2} r. \qquad (30)$$

The full calculation leads to $\alpha = 5.19 \times 10^{-16}$ s^{-1}, which is obtained by using the radius of equilibrium for supermassive stars (our starting point to follow the star collapse) $R_0 = 1.7 \times 10^{11} \left(M/M_\odot\right)^{1/2}$ cm. By substituting this result into (22), one obtains the value for the opacity $\mathcal{K}$ of the material that should constitute the star from the beginning of its evolution, it then reads

$$\mathcal{K} = 5.41 \times 10^{-2} \left(\frac{M}{10\ M_\odot}\right)^{-1/4} \text{cm}^2\ \text{g}^{-1}. \qquad (31)$$

This value is exactly of the same order of magnitude as expected from fundamental stellar astrophysics [14], even if the actual supermassive star mass spreads over an interval which may encompass several orders of magnitude.

Up to here, our reasoning mainstream has been that in order to stand on against its own gravitational pull and avoiding the formation of a black hole after a quasi-static evolution to collapse, the progenitor star has to be a supermassive star emitting at the ELL and possessing an opacity as computed above. Supermassive stars are relativistic polytropes [13], that is, stars in where pressure and density satisfy $P = \bar{K} \rho^{1+1/n}$, with polytropic index $n = 3$, and $\bar{K}$ the polytropic constant. Next we describe the dynamics of a collapsing $n = 3$ polytrope to show that such a star could indeed give origin to a GECO, as there exists a minimum radius for such configuration that is larger than the Schwarzschild radius for a given mass.

On the physical properties of GECO progenitor stars.— The theory of stellar structure and evolution [14] shows that a supermassive radiation-pressure-dominated star must be a polytrope of index $n = 3$. Supermassive $n = 3$ polytropes have a free polytropic constant $\bar{K}$, which means that the star mass M can be chosen arbitrarily (in constrast to the relativistic degenerate polytrope of the same index, where $\bar{K}$ and M are fixed). In the case where the mass is given, there still exists an infinite number of models for different radii R. This is possible because $\bar{K}$ is independent of R. For this reason, a supermassive configuration is in neutral equilibrium: one needs to take no work at all for dispersing it to infinite or compress it to its minimum radius, which is not the Schwarzschild's (see below). Therefore the polytrope $n = 3$ is indifferent to radial changes. In our view, it is this *bizarre* behavior of $n = 3$ polytropes that may allow for the appearance of relativistic stellar configurations of equilibrium like a GECO at the end of a pure gravitational collapse.

If one now perturbes that equilibrium configuration by suddenly reducing slightly the pressure, because the constant $\bar{K}$ is thus also slightly reduced, then the gaseous sphere begins to (homologously) contract following, in the Newtonian approximation that we have been using since (26), the modified Emdem's dynamics for the spatial dependence [14]

$$\frac{1}{z^2}\frac{d}{dz}\left(z^2 \frac{d\omega}{dz}\right) + \omega^3 = \lambda, \qquad (32)$$

which is obtained from the Poisson's equation by using the definition of variables: $r(t) = R(t) z$, $\rho = \rho_c \omega^3(z)$, with $R(t)$ and $\omega(z)$ as scaling factors for the radial distance and density, respectively, z is a dimensionless length-scale, and ρ_c is the star central density. Here $\lambda (> 0)$ measures the star deviation from its hydrostatic equilibrium. Meanwhile, its temporal evolution (also obtained from Poisson's equation) follows the relation [14]

$$\frac{3}{4} \left(\frac{\pi G}{\bar{K}^3}\right)^{1/2} R^2 \ddot{R} = -\lambda \ . \qquad (33)$$

Hereby, a collapse that starts at R_0 for $T(t = -\infty) = 0$ evolves its scaling factor $R(t)$ in time as [14]

$$R^{3/2}(t) = R_0^{3/2} + \sqrt{6\lambda} \left(\frac{\bar{K}^3}{\pi G}\right)^{1/4} T(t). \qquad (34)$$

As the equilibrium is independent of radius, it follows that a star that is contracting toward its equilibrium position may reach a configuration of equilibrium for any radial value that is greater than, or equal to [13]

$$R_{\rm min} \simeq 0.18\ R_{\rm Schw} \left(\frac{M}{M_\odot}\right)^{1/2} , \qquad (35)$$

providing an energy source exists to replace the energy lost to space. Here $R_{\rm Schw} \equiv 2GM/c^2$ defines the

Schwarzschild radius. We stress in passing that an equilibrium radius does exist for ELL stars which generate energy from burning hydrogen via the Carbon-Nitrogen-Oxigen (CNO) cycle (typical nuclear energy-source of high-metallicity supermassive stars [13]). Notice, besides, that most of the energy derived from the gravitational collapse goes into supporting the star, and very little is available to supply the Eddington luminosity. This is because the total energy of these stars is small compared to the gravitational energy.

Discussion and Conclusions.— Apart from (22) and (31), the existence of a minimum radius (35) other than the Schwarzschild's is the most important result of our Letter. It states that there is a minimum mass $M_{\rm min} \simeq 31\ {\rm M}_\odot$ (a kind of lower limit; as several effects may render it higher) for which the resulting collapsed compact object produced by a pure gravitational collapse restrained by the ELL can become an actual GECO instead of a standard Schwarzschild black hole. If we then bring in this result to the discussion on the nature of the compact object in GBHCs, we are forced to conclude that as the mass in those objects is $M_{\rm GBHC} \lesssim 10\ {\rm M}_\odot$, each of those can be a canonical Schwarzschild black hole rather than a GECO, as far as no other interaction is called for, as for instance the intrinsic magnetic fields in [7].

Financial support from Brazil agencies FAPERJ (HJMC) and CNPq (JMS, NOS) is acknowledged.

* Electronic address: `hermanjc@cbpf.br`

[1] M. Abramowicz, W. Kluźniak, and J.-P. Lasota, Astron. Astrophys. 396, L31 (2002)

[2] T. Baumgarte, S. L. Shapiro, Astrophys. J. 585, 930 (2003)

[3] M. Malheiro, S. Ray, H. J. Mosquera Cuesta, J. Dey, General relativistic effects of strong magnetic fields on the gravitational force: a driving engine for bursts of gamma-rays in SGRs? e-Print Archive: astro-ph/0411675 (2004)

[4] P. O. Mazur and E. Mottola, Proc. Nat. Acad. Sci. 111: 9545-9550 (2004). e-Print Archive: gr-qc/0407075

[5] Mitra, A., Found. Phys. Lett. 15: 439 (2002); Found. Phys. Lett. 13: 543 (2000).

[6] G. Pelletier and A. Markowith, Astrophys. J. 502, 598 (1998)

[7] S. L. Robertson and D. J. Leiter, Mon. Not. Roy. Astron. Soc. 350: 1391 (2004), Astrophys. J. 596: L203 (2003), Astrophys. J. 565: 447 (2002)

[8] M. Visser and D. L. Wiltshire, Class. Quant. Grav. 21: 1135 (2004). e-Print Archive: gr-qc/0310107

[9] G. Darmois, *Mémorial des Sciences Mathématiques* (Paris: Gauthier-Villars) Fasc. 25 (1927)

[10] W. B. Bonnor and P. A. Vickers, *Gen. Rel. Grav.* 13 29 (1981)

[11] N. O. Santos, Mon. Not. R. Astron. Soc. **216** 403 (1985)

[12] W. B. Bonnor, A. K. G. de Oliveira, N. O. Santos, Phys. Reps. 181, No. 5, 269 (1989)

[13] W. A. Fowler, Rev. Mod. Phys. 36, 1104 (1964); Astrophys. J. 144, 180 (1966)

[14] R. Kippenhahn, A. Weigert, *Stellar structure and evolution*, Springer-Verlag, Berlin (1990)

[15] L. Herrera and N. O. Santos, Phys. Rep. **286**, 53 (1997)

[16] M. E. Cahill and G. C. McVittie, J. Math. Phys. **11**, 1382 (1970)

[17] Notice that $L_{\rm Edd}$ by itself determines the time scale for collapse, nearly the dynamical time scale rather than the Kelvin-Helmholtz time scale.