CBPF-NF-011/02

Comment on "Solution of the Relativistic Dirac-Morse Problem"*

Arvind Narayan Vaidya

Instituto de Física - Universidade Federal do Rio de Janeiro

Caixa Postal 68528 - CEP 21945-970, Rio de Janeiro, Brazil

Rafael de Lima Rodrigues†

Centro Brasileiro de Pesquisas Físicas (CBPF)

Rua Dr. Xavier Sigaud, 150,CEP 22290-180, Rio de Janeiro, RJ, Brazil

Abstract

We do not think that the relativistic Morse potential problem has been correctly formulated and solved by A. D. Alhaidari.

Key-words: Dirac equation; Dirac-Morse problem; Exact solution.

PACS numbers: 03.65.Pm 03.65.Ge

---

*To be submitted for publication in Phys. Rev. Lett.

†Permanent address: Departamento de Ciências Exatas e da Natureza, Universidade Federal da Paraíba, Cajazeiras - PB, 58.900-000 - Brazil. E-mail to RLR is rafaelr@cbpf.br or rafael@cfp.ufpb.br and e-mail to ANV is vaidya@if.ufrj.br.

In a recent letter Alhaidari claims to have formulated and solved the Dirac-Morse problem [1]. He starts with the Hamiltonian for a Dirac particle in an electromagnetic potential and aims to give a unified treatment of three problems: Coulomb, oscillator and Morse differing only in the choice of gauge. If this were possible gauge invariance would imply that the physical content of the theory is not altered by gauge changes. The known results for the energy spectrum for the first two problems as quoted by him contradict this expectation.

In any case if we consider a Hamiltonian that appears in the paper in a slightly different notation

$$H = \boldsymbol{\alpha} \cdot (\mathbf{p} - \hat{\mathbf{r}} W(r)) + \beta M + V(r) \tag{1}$$

where $\hat{\mathbf{r}} = \frac{\mathbf{r}}{r}$ and separate variables following [2] then defining $\Phi = \begin{pmatrix} G_{\ell j}(r) \\ F_{\ell j}(r) \end{pmatrix}$ we get the radial equation

$$(-i\rho_2 \frac{d}{dr} + \rho_1 \frac{\kappa}{r} - \rho_2 W - E + V) + M\rho_3)\Phi = 0 \tag{2}$$

where ρ_i are the Pauli matrices and $\kappa = \pm(j + \frac{1}{2})$ for $\ell = j \pm \frac{1}{2}$, which contradicts Alhaidari's equation (1).

Next, Alhaidari's Hamiltonian does not even include that of the relativistic oscillator as a special case [3]. The Hamiltonian that satisfies this condition is

$$H = \boldsymbol{\alpha} \cdot (\mathbf{p} - i\beta \hat{\mathbf{r}} W(r)) + \beta M + V(r). \tag{3}$$

The resulting radial equation

$$(-i\rho_2 \frac{d}{dr} + \rho_1 (W + \frac{\kappa}{r}) - E + V + M\rho_3)\Phi = 0 \tag{4}$$

does correspond to Alhaidari's equation (1) where the quantum numbers ℓ and j are omitted. Due to the matrix β accompanying W in the Hamiltonian, Alhaidari's interpretation of the vector $(V, \hat{\mathbf{r}}W)$ as an electromagnetic potential is incorrect.

Next, there is no reason for the functions $V(r)$ and $W(r)$ which appear in the Hamiltonian to depend on the angular quantum numbers which make their appearance only when we separate variables to solve the Dirac equation. Hence his choice of the constraint (in our notation)

$$W(r) = \frac{1}{S}V(r) - \frac{\kappa}{r} \tag{5}$$

with both V and W nonzero and S a constant cannot be satisfied. Alhaidari could have avoided the mathematical contradiction by taking the Hamiltonian to be

$$H = \boldsymbol{\alpha} \cdot (\mathbf{p} - i\beta\hat{\mathbf{r}}(W(r) + \frac{K}{r})) + \beta M + V(r) \tag{6}$$

where $K = \gamma^0(1 + \mathbf{\Sigma} \cdot \mathbf{L})$ is the Dirac operator, which leads to the radial equation

$$(-i\rho_2 \frac{d}{dr} + W\rho_1 - E + V + M\rho_3)\Phi = 0 \tag{7}$$

Applying the transformation $\Phi = e^{-i\rho_2\eta}\hat{\Phi}$ we get

$$[-i\rho_2 \frac{d}{dr} - (E - V) + \rho_1(W\cos 2\eta - M\sin 2\eta) + \rho_3(W\sin 2\eta + M\cos 2\eta)]\hat{\Phi} = 0 \tag{8}$$

Choosing $W = \frac{V}{\sin 2\eta}$, we get equations (4-5) of Alhaidari for $G_{\ell j}$ and $F_{\ell j}$ leading to energy levels degenerate in l, j, m which is physically uninteresting. In the nonrelativistic formulation [4] the radial equation for the Morse potential does contain the centrifugal barrier contribution for nonzero values of ℓ.

In conclusion we do not think that the relativistic Morse potential problem has been correctly formulated and solved.

REFERENCES

[1] A. D. Alhaidari, *Phys. Rev. Lett.* **87**, 210405 (2001).

[2] J. D. Bjorken and S. D. Drell, *Relativistic Quantum Mechanics* (Mc-Graw Hill Book Company,New York,1965).

[3] M. Moshinsky and A. Szczepaniak, *J. Phys. A: Math. Gen.* **22**, L817 (1989).

[4] P. M. Morse, *Phys. Rev.* **34**, 57 (1929).