ISSN 0029-3865

CNPq

CBPF - CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Notas de Física

CBPF-NF-010/94
January 1994

On the Central Charge in 3D–Supersymmetry

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CBPF-NF-010/94

Abstract

A matter self-interacting model with $N = 1$-supersymmetry in 3D is discussed in connection with the appearance of a central charge in the algebra of the supersymmetry generators. The result is extended to include gauge fields with a Chern-Simons term.

Key-words: Current algebra; Supersymmetry; Central charge.

Ordinary and supersymmetric Abelian gauge models in three-dimensional space-times have recently been fairly-well investigated [1]. Besides their relevance in connection with the possibility of getting non-perturbative results more easily, the ultraviolet finiteness of Yang-Mills (and gravity) Chern-Simons models is a remarkable feature of field theories defined in $D = (1+2)$ [2]. Also, $3D$ gauge theories seem to be the right way to tackle exciting topics of Condensed Matter Physics such as high-T_c superconductivity and fractional quantum Hall effect [3].

Our purpose in this paper is to assess a typical three-dimensional gauge model with $N = 1$ supersymmetry from the point-of-view of the algebra of supersymmetry generators. We actually wish to present here a few remarks on the connection between topologically non-trivial solutions, the Chern-Simons term, and the presence of a central charge operator in the supersymmetry algebra.

The super-Poincaré algebra in (2+1) dimensions is generated by a real two component spinorial charge Q_a, where the operatorial relations are denoted by

$$\{Q_a\,,\,Q_b\} = 2P_{ab} \quad \text{e} \quad [Q_a\,,\,P_{ab}] = 0\,, \tag{1}$$

where P_{ab} is the translation generator. We shall represent vectors in twofold way: for Lorentz indices we will use greek letters and for bi-spinorial indices we will use latin letters, bearing in mind the mapping $V_{ab} = V_\mu(\gamma^\mu)_{ab}$ [4]. The super-Poincaré algebra (1) for an extended supersymmetry for more than one flavour, is generalized to [5]

$$\begin{aligned} \{Q^i_a\,,\,Q^j_b\} &= 2\delta^{ij}P_{ab} + A^{ij}\epsilon_{ab}\,, \\ [Q^i_a\,,\,P_{ab}] &= 0\,, \\ [Q^i_a\,,\,A^{jk}] &= 0\,, \end{aligned} \tag{2}$$

with $i,j,k = 1,...,N$, ϵ_{ab} the Levi-Civita tensor and $A^{ij} = -A^{ji}$ the central charge, which is an internal symmetry Lie group generator. To recognize if a quantum field theory is consistent with supersymmetry we need to obtain the supersymmetry generators (supercharges) for the specific model we are working with. What we will emphasize in this paper is that though the algebra (2) is always valid at a classical level, it is avery formal relation. The local features of a system are presented by the current algebra, that depends, as we will see, on the details of the model [6]. With this point of view, we will analyse in detail how the various terms (consistent with the internal and external symmetries, which can appear in a $(2+1)$ dimension supersymmetric model) contribute to the equal time current algebra. The next step, which will be pursued in a next work, is to verify the commutation and anticommutation relations about the other components of the current, where we we will find the so called Schwinger terms after quantisation. These are the basic issues in the quantization program, in terms of the Green's functions. It describes the anomalies present in many particle processes.

We will use the following representation for the matrices γ, having $\eta^{\mu\nu} = (-;+,+)$ signature of the metric tensor

$$(\gamma^0)^{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; (\gamma^1)^{ab} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \; (\gamma^2)^{ab} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \tag{3}$$

Using the charge conjugation matrix C_{ab}[1] that works like a netric tensor in the spinorial space. It may be written as

$$C_{ab} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \tag{4}$$

where we have $C_{ab} = -C_{ba} = C^{ab}$ e $C_{ab}C^{cd} = \delta^{c}{}_{[a}\,\delta^{d}{}_{b]}$. Then the Clifford algebra for the γ matrices, that will be important for the computation of the current algebra is

$$\begin{aligned}
(\gamma^{\mu})^{ab}(\gamma^{\nu})_{ab} &= \eta^{\mu\nu} \quad ,\\
(\gamma^{\mu})^{ab}(\gamma^{\nu})_{bd} &= \eta^{\mu\nu}\delta^{a}{}_{d} - i\,\epsilon^{\mu\nu}{}_{\rho}(\gamma^{\rho})^{a}{}_{d} \quad ,\\
(\gamma^{\mu})_{a}{}^{b}(\gamma^{\nu})^{a}{}_{b} &= -\,\eta^{\mu\nu}\,,\\
(\gamma^{\mu})^{ab}(\gamma^{\nu})^{b}{}_{d} &= -\,\eta^{\mu\nu}C_{da} - i\,\epsilon^{\mu\nu\rho}(\gamma_{\rho})_{ad}\,,\\
(\gamma^{\mu})_{a}{}^{b}(\gamma^{\nu})_{b}{}^{d}\,(\gamma^{\rho})_{d}{}^{a} &= -\,i\,\epsilon^{\mu\nu\rho}\,,\\
(\gamma^{\mu})_{ab}(\gamma^{\nu})^{b}{}_{c}\,(\gamma^{\rho})^{c}{}_{d} &= i\,\epsilon^{\mu\nu\rho}C_{da} + \eta^{\rho\nu}(\gamma^{\mu})_{da} - \eta^{\rho\mu}(\gamma^{\nu})_{da} - \eta^{\mu\nu}(\gamma^{\rho})_{da}\,.
\end{aligned} \tag{5}$$

This paper is outlined as follows: in Section 1, a self-interacting scalar model is presented and the SUSY algebra is written down with the explicit form for the centarl charge operator; the introduction of the gauge sector is discussed in Section 2. Finally in Section 3, one discusses the supersymmetric version of a Chern-Simons term and the connection it bears with the central charge is investigated. Some general conclusions follow.

1 Self–Interacting Scalar Model

The representation of the scalar superfield expanded in θ Taylor series, where θ is a two conponent Majorana grassmanian variable is given by

$$\Phi(x,\theta) \;=\; A(x) + \theta^{a}\,\psi_{a}(x) - \theta^{2}\,F(x)\,; \tag{6}$$

where $A(x)$ is a physical scalar field, $\psi_a(x)$ is a fermion field and $F(x)$ is an auxiliar field. The supersymmetric covariant derivative is obtained knowing that it commutes with the supersymmetry generator Q_a. It is given by

$$D_{a} \;=\; \partial_{a} + i\,\theta^{b}\,\partial_{ab}\,, \tag{7.a}$$

$$[D_{a}\,,\,D_{b}] \;=\; 2P_{ab}\,, \tag{7.b}$$

Now it's possible to write an action, that is a scalar with respect to the symmetries (supersymmetry, Lorentz). This is

$$S_{escalar} \;=\; \int d^{3}x\,d^{2}\theta\left\{-\frac{1}{2}(D_{a}\,\Phi)^{2} + \frac{1}{2}m\,\Phi^{2} + \frac{\lambda}{8}\,\Phi^{4}\right\}\,, \tag{8}$$

where this expression is invariant by the following transformation in its component fields:

$$\begin{aligned}
\delta A &= -\epsilon^{a}\,\psi_{a}\,,\\
\delta\psi_{a} &= -\epsilon^{b}\,(C_{ab}\,F + i\partial_{ab}A)\,,\\
\delta F &= -\epsilon^{b}\,i\,\partial_{b}{}^{a}\,\psi_{a}\,.
\end{aligned} \tag{9}$$

[1]We will use the notation of the reference [4]

As we want to work with an "on shell" model, and using the definition of the Berezin integral ($\int d^2\theta = D^2|$), we get the component action:

$$\begin{aligned} S_{escalar} &= \int d^3x \left\{ \tfrac{1}{2} \left[\tfrac{1}{2}(\partial_{ab} A)(\partial^{ab} A) + \psi^a i \partial_a{}^b \psi_b \right] + m\psi^2 + \tfrac{3}{2}\lambda\psi^2 A^2 \right. + \\ &\quad \left. - \tfrac{1}{2} m^2 A^2 - \tfrac{1}{2} m\lambda A^4 - \tfrac{1}{8}\lambda^2 A^6 \right\}, \end{aligned} \tag{10}$$

and we can verify that this action is invariant under the "on shell" transformation on the component fields

$$\begin{aligned} \delta A &= -\epsilon^a \psi_a, \\ \delta \psi_a &= -\epsilon^b \left[C_{ab} \left(-\frac{1}{2} m A - \frac{1}{2}\lambda A^3 \right) + i\partial_{ab} A \right], \end{aligned} \tag{11}$$

As we know the supersymmetry is a symmetry of the action and not of the Lagrangean. Thus we have, in fact, two terms for the current: one originated by action variation and other originated by the Noether theorem. We will obtain the current in components and then introduce the equations of motion of the fields on it, to obtain the "on shell" currents.

Using the projection notation, we get

$$\delta S = \int d^3x\, d^2\theta\, \delta L = \epsilon^c \int d^3x \left[-\partial_{ab}(\delta_c{}^a D^b L) \right] \Big| = \epsilon^c \int d^3x \left[-\partial_{ab}(\Lambda_c^{ab}) \right], \tag{12}$$

Making a straightforward computation in the "off shell" action, the SUSY current results

$$\Lambda_c^{ab} = \frac{i}{2} \delta^a{}_c \left\{ iA\, \partial^{bd} \psi_d + m A \psi^b + \frac{\lambda}{2} A^3 \psi^b \right\}. \tag{13}$$

Now using the equations of motion for the expression (10) and then substituing the transformation (11), we get

$$\left(\frac{\partial \mathcal{L}}{\partial\, \partial_{ab} \Phi} \right) \delta\Phi \Big|_{\Phi=\psi, A} = \epsilon^c \left[-\frac{i}{2} \delta^a{}_c \psi^b \left(m A + \frac{\lambda}{2} A^3 \right) + \frac{1}{2} \psi^b \partial^a{}_c A + \frac{1}{2} \psi_c \partial^{ab} A \right], \tag{14}$$

In this way, using vectorial notation we obtain the Noether current:

$$\begin{aligned} J^\mu{}_c &= -i\psi^a (\gamma^\mu)_{ac} \left(m A + \frac{\lambda}{2} A^3 \right) - \frac{i}{2} \varepsilon^{\mu\nu\rho} \psi^b (\gamma_\rho)_{bc} \partial_\nu A + \\ &\quad + \frac{1}{2} \psi_c \partial^\mu A + \frac{1}{2} A \partial^\mu \psi_c - \frac{i}{2} \varepsilon^{\mu\nu\rho} A \partial_\nu \psi^a (\gamma_\rho)_{ac}, \end{aligned} \tag{15}$$

The supercharge is defined

$$\begin{aligned} Q_c &= \int d^2x J^0{}_c \\ &= \int d^2x \left\{ -i\psi^a (\gamma^0)_{ac} \left(m A + \frac{\lambda}{2} A^3 \right) + \frac{1}{2} \psi_c \partial^0 A + \frac{1}{2} A \partial^0 \psi_c + \right. \\ &\quad \left. - \frac{i}{2} \varepsilon^{0\nu\rho} \psi^a (\gamma_\rho)_{ac} \partial_\nu A + \frac{i}{2} \varepsilon^{0\nu\rho} A \partial_\nu \psi^a (\gamma_\rho)_{ac} \right\}, \end{aligned} \tag{16}$$

As we will use the canonical graded commutation relation for computation of the current algebra, we take the canonical conjugated momenta of the fields for this specific model. They are

$$\begin{aligned}
\Pi_{\psi_a} &= \frac{\partial L}{\partial \dot{\psi}_a} = \frac{i}{2}\frac{\partial \psi^d (\gamma^\mu)_d{}^b \partial_\mu \psi_b}{\partial \partial_0 \psi_a} = \frac{i}{2}\psi^b (\gamma^0)_b{}^a , \\
\Pi_A &= \frac{\partial L}{\partial \dot{A}} = \frac{1}{4}\frac{\partial (\partial^\mu A)(\partial_\mu A)}{\partial \partial_0 A} = \frac{1}{2}\partial^0 A ,
\end{aligned} \tag{17}$$

The canonical relations are seen to be

$$\begin{aligned}
\{\psi_a(x) , \psi^c(y)\} &= 2i\,(\gamma^0)_a{}^c \delta^2(x-y) , \\
[A(x) , \partial^0 A(y)] &= 2\,\delta^2(x-y) ,
\end{aligned} \tag{18}$$

A lengthy calculation yields the following expression for the charge anti-commutator:

$$\begin{aligned}
\{Q_a , Q_b\} &= \int d^2x \times \\
&-2i\left\{\frac{1}{4}\left[\, 2i\,\psi^a (\gamma^0)_a{}^b \partial^0 \psi_b + (\partial^0 A)(\partial^0 A) + 2i\,\psi^a (\gamma^i)_a{}^b \partial^i \psi_b + (\partial^i A)(\partial^i A) + \right.\right. \\
&- \left.\left. \psi^2 \left(m + \tfrac{3}{2}\lambda A^2\right) + \left(\tfrac{1}{2}m^2 A^2 + \tfrac{1}{2} m \lambda A^4 + \tfrac{1}{4}\lambda^2 A^6\right) \right] \right\} (\gamma^0)_{ab} + \\
&-2i\left\{\frac{1}{4}\; 2i\,\psi^a (\gamma^{(0})_a{}^b \partial^{i)} \psi_b + (\partial^{(0} A)(\partial^{i)} A) \right\} (\gamma_i)_{ab} .
\end{aligned} \tag{19}$$

If we compare this expression with the $0\,\mu$ component of the "improved" energy-momentum tensor

$$T_{\mu\nu} \equiv \frac{1}{e}\frac{\delta S}{\delta e^{(\mu}_{a'}} e_{\nu)a'} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} , \tag{20}$$

where S in this expression indicates the action (10), we may rewrite (19) as

$$\{Q_a , Q_b\} = 2i\,P^\mu (\gamma_\mu)_{ab} + 2i\,\epsilon^{ij} \int d^2x\, (\partial_i A)(\gamma_j)_{ab} \left(m\,A \frac{\lambda}{2} A^3\right) , \tag{21}$$

Observe that the mass and selfinteraction $\lambda\,\phi^4$ terms contribute to a central charge of the supercharge algebra, in its two component Majorana representation. In fact, when we take the chiral components of the charges we get

$$\begin{aligned}
\{Q^+ , Q^+\} &= 2i\,(P^0 + P^1) - 2\int d^2x \left(mA + \frac{\lambda}{2}A^3\right)\partial_2 A , \\
\{Q^- , Q^-\} &= 2i\,(P^0 - P^1) - 2\int d^2x \left(mA + \frac{\lambda}{2}A^3\right)\partial_2 A , \\
\{Q^+ , Q^-\} &= -2i\,P^2 - 2\int d^2x \left(mA + \frac{\lambda}{2}A^3\right)\partial_1 A ,
\end{aligned} \tag{22}$$

where we note that the central term contributes also in the $++$ and$--$ anticommutation relation, unlike the bidimensional case, where it appears only in the $+-$ relation. This case is more complex that one studied by Witten and Olive in their paper [7], because that in three dimension the γ^2 matrix mix the chiral components. The last terms in r.h.s. of equation present a bidimensional solitonic solution, whose analisis is in course.

Bearing in mind the last issue, we will only use the relevant terms for the Majorana central supercharge for the gauge field model.

2 On the Abelian Gauge Model

We construct a minimal coupled supersymmetric model by complexifying the scalar fields in the expression (8). Then we will have a phase symmetry (gauge), and when its parameter becomes local the gauge symmetry becomes local too, provided that the derivatives are covariantized. As we have, in fact two derivatives, that represent the translations in the two sectors (bosonic and fermionic), we will have to covariantize these two derivatives:

1) The Majorana one is written as

$$\nabla_a \equiv D_a \mp i\,\Gamma_a\,, \tag{23}$$

where Γ_a is a gauge superconnection with a super helicity $h = \frac{1}{2}$, and the signs − e + means if the derivatives is acting in the (scalar) superfields Φ ou $\bar{\Phi}$ respectively, $\bar{\Phi}$ being the comolex conjugated of Φ. Γ_a is represented by θ Taylor expansion series

$$\Gamma_a = \chi_a + \theta^b(C_{ab}B + iV_{ab}) + \theta^2(2\lambda_a - i\partial_a{}^b\chi_b)\,, \tag{24}$$

where λ_a is the (gaugino) spinorial field, V_{ab} is the usual gauge field, B a auxiliar scalar field and χ_a another auxiliar spinorial one, by supersymmetry. We will eliminate the auxiliar fields using the suitable gauge fixing, so called *Wess-Zumino gauge*, that explicitly breaks the supersymmetry, but stress the physical content of the multiplet Γ_a. We show the gauge field and its supersymmetric transformation, that will be usefull for later computation. These are

$$\Gamma_a = i\,\theta^b V_{ab} - 2\,\theta^2\,\lambda_a\,, \tag{25}$$

and

$$\begin{aligned} \delta V_{ab} &= i\,\epsilon_{(b}\,\lambda_{b)}\,, \\ \delta\lambda_a &= \tfrac{1}{2}\,\epsilon^c\,\partial_{c(a}\,V_{c)b}\,. \end{aligned} \tag{26}$$

2) The covariantized vectorial derivative is written as

$$\nabla_{ab} = D_{ab} \mp i\Gamma_{ab}\,, \tag{27}$$

where Γ_{ab} is the vectorial gauge superconnection. As we know, in order to have irreducible representations of the symmetry we need constraints in our model. In the supersymmetric case we have the so called convenctional constraint, that acts in such away that the supersymmetric algebra of the Majorana derivatives $\{\nabla_a\,,\,\nabla_b\} = 2\,i\,\nabla_{ab} + F_{ab}$ will have $F_{ab} = 0$. Then we easily compute that

$$\Gamma_{ab} = -\frac{i}{2}\,D_{(a}\,\Gamma_{b)}\,, \tag{28}$$

implying that in the *Wess–Zumino gauge* we have

$$\Gamma_{ab} = V_{ab} + i\,\theta_{(a}\,\lambda_{b)} - \frac{i}{2}\,\theta^2\,\partial_{c(a}\,V_{b)}{}^c\,, \tag{29}$$

By the graded Bianchi Identity we redefine the gauge field as

$$W_a = \frac{1}{2}\,D^b D_a \Gamma_b\,, \tag{30}$$

with the constraint $D^a W_a = 0$ ($D^a D_b D_a = 0$), implying as in the usual Lorentz gauge that exists only one independent component of the field W_a. Using the projector method we write

$$W_a| = \lambda_a \quad , \quad D_a W_b| = \frac{1}{2}(\partial_{ca} V_b{}^c + \partial_{cb} V_a{}^c) \equiv f_{ab} , \tag{31}$$

with f_{ab} the usual gauge field strenght. Another relations that will be very important and that may straightforwardly be obtained are (conf. ([4])):

$$\nabla_a \nabla^2 = i \nabla_a{}^b \nabla_b \pm i W_a \quad \text{e} \quad (\nabla^2)^2 = \Box \mp i W^a \nabla_a , \tag{32}$$

where $\Box$ means the covariant by Γ_{ab} d'Alembertian. Now we are in position to obtain terms which respect the symmetries to built a scalar action.

2.1 Scalar Superaction with a Background Gauge Field

We work, for simplicity with a background gauge field, but all the assumptions can be extended to a dynamical one. Our action is obtained by complexifying and minimally coupling with the gauge field the action (8), resulting in

$$S_{escalar} = -\frac{1}{2} \int d^3x \, d^2\theta \left\{ (\nabla^a \bar{\Phi})(\nabla_a \Phi) \right\} , \tag{33}$$

Redefining the component field (we can always do this), by the projector

$$\begin{array}{rcl} \Phi| = A & , & \bar{\Phi}| = \bar{A} , \\ \nabla_a \Phi| = \psi_a & , & \nabla_a \bar{\Phi}| = \bar{\psi}_a , \\ \nabla^2 \Phi| = F & , & \nabla \bar{\Phi}| = \bar{F} . \end{array} \tag{34}$$

Thus in the "on shell" component fields, in the *Wess-Zumino gauge*, it turns

$$S = \frac{1}{2} \int d^3x \left\{ \bar{\psi}^a \, i \, D_a{}^b \, \psi_b + \psi^a \, i \, D_a{}^b \, \bar{\psi}_b + A \Box \bar{A} + \bar{A} \Box A \right\} , \tag{35}$$

with the "on shell" supersymmetric transformation:

$$\begin{array}{rcl} \delta \bar{\psi}^a = \epsilon_b \, i \, D^{ab} \bar{A} & , & \delta \psi_b = -\epsilon^c \, i \, D_{bc} A , \\ \delta A = -\epsilon^a \, \psi_a & , & \delta \bar{A} = -\epsilon^a \, \bar{\psi}_a , \end{array} \tag{36}$$

Analogously to scalar case we can obtain

$$\Lambda_c^{ab} = -i \, \delta_c{}^a \, \nabla^b L \Big| . \tag{37}$$

we get

$$\Lambda_c^{ab} = \frac{1}{2} \delta_c{}^a \left\{ \bar{A} \, D^{db} \, \psi_d + A \, D^{db} \, \bar{\psi}_d \right\} . \tag{38}$$

And by using the equations of motion, we have

$$\left(\frac{\partial \mathcal{L}}{\partial \, \partial_{ab} \, \Phi} \right) \delta \, \Phi \Bigg|_{\Phi = \psi, A} = \epsilon^c \left\{ -\frac{1}{2} \left[\bar{\psi}^a D^b{}_c A + \psi^a D^b{}_c \bar{A} + \psi_c (D^{ab} \bar{A}) + \bar{\psi}_c (D^{ab} A) \right] \right\} . \tag{39}$$

Then the Noether current in the vectorial notation is

$$\begin{aligned} J^{\mu}{}_{c} &= \frac{i}{2}\varepsilon^{\mu\nu\rho}[\bar{\psi}^{a}(\gamma_{\rho})_{ac}\partial_{\nu}A + \psi^{a}(\gamma_{\rho})_{ac}\partial_{\nu}\bar{A}] - \frac{1}{2}(\psi_{c}\partial^{\mu}\bar{A} + \bar{\psi}_{c}\partial^{\mu}A) + \\ &- \frac{1}{2}(\bar{A}\partial^{\mu}\psi_{c} + A\partial^{\mu}\bar{\psi}_{c}) + \frac{i}{2}\varepsilon^{\mu\nu\rho}(\bar{A}\partial_{\nu}\psi^{a} + A\partial_{\nu}\bar{\psi}^{a})(\gamma_{\rho})_{ac}, \end{aligned} \tag{40}$$

resulting the supercharge

$$\begin{aligned} Q_{c} &= \int d^{2}x J^{0}{}_{c} = \\ &= \int d^{2}x\frac{1}{2}\left\{-(\psi_{c}\partial^{0}\bar{A} + \bar{\psi}_{c}\partial^{0}A) + i\varepsilon^{0ij}[\bar{\psi}^{a}(\gamma_{j})_{ac}\partial_{i}A + \psi^{a}(\gamma_{j})_{ac}\partial_{i}\bar{A}] + \right. \\ &\left. -(\bar{A}\partial^{0}\psi_{c} + A\partial^{0}\bar{\psi}_{c}) + i\varepsilon^{0ij}(\bar{A}\partial_{i}\psi^{a} + A\partial_{i}\bar{\psi}^{a})(\gamma_{j})_{ac}\right\} \end{aligned} \tag{41}$$

The canonical conjugated momenta that will be necessary for supercharge algebra are

$$\begin{aligned} \Pi_{\psi_{d}} = -i\bar{\psi}^{a}(\gamma^{0})_{a}{}^{d} \quad &, \quad \Pi_{\bar{\psi}_{d}} = -i\psi^{a}(\gamma^{0})_{a}{}^{d}, \\ \Pi_{A} = (D^{0}\bar{A}) \quad &, \quad \Pi_{\bar{A}} = (D^{0}A), \end{aligned} \tag{42}$$

giving the canonical commutation and anticommutation relations

$$\begin{aligned} \{\psi^{d}, \bar{\psi}^{a}\} = i(\gamma^{0})^{ad}\delta^{2}(x-y) \quad &, \quad \{\bar{\psi}^{d}, \psi^{a}\} = i(\gamma^{0})^{ad}\delta^{2}(x-y), \\ [A, D^{0}\bar{A}] = \delta^{2}(x-y) \quad &, \quad [\bar{A}, D^{0}A] = \delta^{2}(x-y). \end{aligned} \tag{43}$$

Making a long computation using the γ matrices Clifford algebra we reach the result

$$\{Q_{a}, Q_{b}\} = -2i\,P^{\mu}(\gamma_{\mu})_{ab} - \epsilon_{ij}\int d^{2}x\left\{A\bar{A}(\partial^{j}V^{i})C_{ab}\right\}, \tag{44}$$

where the P^{μ} is the $(0\,\mu)$ component of the improved energy-momentun tensor $T^{\mu\nu}$. The second term in r.h.s. of the above expression is a central Majorana (super)charge. Observe that this term depends on the magnetic field, as it was a infinite line of magnetic flux in a four dimensional space.

3 The Supersymmetric Chern–Simons Term

For this model we will add a supersymmetric Chern–Simons (CS) term and we will verify how it modifies the supercharge algebra. For this purpose, we begin with the gauge invariant CS term definition

$$S_{CS} = \frac{M}{g^{2}}\int d^{3}x\,d^{2}\theta\,\gamma^{a}W_{a}, \tag{45}$$

where M is the mass parameter, which is generated by this proper term, and g is the gauge field coupling constant. This expression in components, using the *Wess–Zumino gauge* becomes

$$S_{CS} = \frac{M}{g^{2}}\int d^{3}x\left[i\,V^{ab}(\partial_{ac}V^{c}{}_{b}) + 4\lambda^{2}\right], \tag{46}$$

where the first term in r,h.s. is the known CS term. Now including the term (45) in the action (35), and then calculating it "on shell", i e, taking into account the equations of motion for the F , $\bar{F}$ fields (which it are not affected by the CS term) and of the λ^d field for which the equation of motion allow us to choose

$$\begin{cases} \partial_a{}^d \lambda^a = 0\,, \\ \lambda^d = \frac{ig^2}{4M}(\psi^d \bar{A} - \bar{\psi}^d A)\,. \end{cases} \tag{47}$$

Then the "on shell" Lagrangean is

$$\begin{aligned} L_{din} &= \frac{i}{2}\bar{\psi}^a(\gamma^\mu)_a{}^b\partial_\mu\psi_b + \frac{i}{2}\psi^a(\gamma^\mu)_a{}^b\partial_\mu\bar{\psi}_b - \frac{1}{2}(\partial^\mu\bar{A})(\partial_\mu A) + \frac{i}{2}(\partial^\mu\bar{A})V_\mu A + \\ & -\frac{i}{2}V^\mu\bar{A}(\partial_\mu A) + \frac{iM}{g^2}\epsilon^{\mu\nu\rho}V_\mu\partial_\nu V_\rho\,, \end{aligned} \tag{48}$$

with the "on shell" supersymmetric transformation:

$$\begin{aligned} \delta\bar{\psi}^a &= i\epsilon_b D^{ab}\bar{A} \quad , \quad \delta\psi_b = -i\epsilon^c D_{bc} A\,, \\ \delta A &= -\epsilon^a\psi_a \quad , \quad \delta\bar{A} = -\epsilon^a\bar{\psi}_a\,, \\ \delta V_{ab} &= i\epsilon_{(a}\lambda_{b)}\,. \end{aligned} \tag{49}$$

In the same sense that we already do, we have

$$(\Lambda_{CS})^{ab}_c = \frac{i}{2}\delta^a{}_b(\psi^d\bar{A} - \bar{\psi}^d A)V^b{}_d\,, \tag{50}$$

and for equations of motion,

$$\left(\frac{\partial \mathcal{L}_{CS}}{\partial\,\partial_{ab}\,\Phi}\right)\delta\,\Phi\bigg|_{\Phi=\psi,A,V_{ab}} = \epsilon^c\left\{\frac{i}{4}\left[V^a{}_c\left(\psi^b\bar{A} - \bar{\psi}_b A\right) - \delta^b{}_c V^{ad}\left(\psi_d\bar{A} - \bar{\psi}_d A\right)\right]\right\}\,. \tag{51}$$

and, finally the vectorial Noether current is

$$(J_{SCS})^\mu_c = \frac{i}{2}\,V^\mu\left(\psi_c\bar{A} - \bar{\psi}_c A\right)\,. \tag{52}$$

with the component supercharge

$$(Q_{SCS})_c = \int d^2x (J_{SCS})^0_c = \int d^2x \left\{\frac{i}{2}V^0\left(\psi_c\bar{A} - \bar{\psi}_c A\right)\right\}\,. \tag{53}$$

Summing in the supercharge (41), and using the same canonical representation as the last case, we get the supercharge algebra that involves only the CS supercharge as

$$\{Q_a\,, Q_b\}_{CS} = 2\,i\,P^\mu(V^0)(\gamma_\mu)_{ab}\,. \tag{54}$$

What we observe is that the V^0 potencial field is completely eliminated from the algebra, implying that the Chern-Simons "corrected" $T^{0\mu}$ component of energy-momentum tensor, defined as "new" P^μ becomes independent on the potencial gauge field. It is possible to say that the the conjugated momenta of the A an $\bar{A}$ fields are in fact "corrected" by the CS term to become $\Pi_A\,\alpha\,\partial^0\bar{A}$ and $\Pi_{\bar{A}}\,\alpha\,\partial^0 A$. This indicates that the CS term play a role similar to a partial gauge fixing, eliminating one degree o freedom of the gauge field, refering to the algebra.

4 Conclusions

The basic motivation of this paper was to analyse the 3-dimensional counterpart of a well-known result by Olive and Witten [7], namely, the appearance of a central charge in the supersymmetric algebra as originated from non-trivial topological field configurations. Here, in thepresence and in the absence of gauge fields, we could conclude that vortex-like field configurations are reponsible for a central charge in the supersymmetry algebra, even in the case of a $N = 1$-supersymmetry. It is worthwhile to mention the results obtained by Lee, Lee and Weinberg [9] where a central charge comes out in context of an $N = 2$ extended supersymmetric model. We would like to pint out that the calculations of Section 3 recall that the Chern-Simons term for the gauge field does not give contribution to the central charge appearing in the algebra. The latter arises exclusively from the matter sector and its existence to the vortex-configurations of the scalar fields. Clearly, the rôle of the gauge fields is to render finite the vortex energy [8].

Next, we would like to analyse the presence of central charges in the models recently proposed by Dorey and Mavromatos [10] to study P, T conserving superconducting gauge models whwnever the latter are supermmetrised. One could perhaps understand whether or not centralcharge may be related to some physical aspects of superconductivity.

References

[1] A. Linde, *Rep. Progr. Phys.* **42** (1979) 389.
S. Deser, R. Jackiw and S. Templeton, *Phys. Rev. Lett.* **48** (1982) 975 ;
S. Deser, *Three Topics in Three Dimensions*, in the proceedings of the Trieste Conference on Supermenbranes and Physics in 2+1 Dimensions, eds. M.J. Duef *et al.*, World Scientific (Singapore, July 1989);

[2] F. Delduc, C. Lucchesi, O. Piguet and S.P. Sorella, *Nucl.Phys.* **B346** (1990) 313 ;
A. Blasi, O. Piguet and S.P. Sorella, *Nucl.Phys.* **B356** (1991) 154 ;
C. Lucchesi and O. Piguet, *Nucl.Phys.* **B381** (1992) 281;

[3] J. Schonfeld, *Nucl. Phys.* **B185** (1981) 157;

[4] S. J. Gates, M. T. Grisaru, M. Roček and W. Siegel, *Superspace*, Benjamin/Cummings Publishing Company (1983);

[5] S. Coleman e J. Mandula, *Phys. Rev.* **159** (1967) 1251;
R. Haag, J. T. Lopuszanski e M. Sohnius, *Nuc. Phys.* **B88** (1975) 257;

[6] R. Jackiw, *Field Theoretic Investigation in Current Algebra, Current Algebra and Anomalies*, S. B. Treiman, R. Jackiw, B Zumino e E. Witten (eds.), World Scientific Publishing Co. (1985);

[7] E. Witten e D. Olive, *Phys. Lett.* **B78** (1978) 97;
P. Di Vecchia e S. Ferrar, *Nuc. Phys.* **B130** (1977) 93;
A. D'Adda e P. Di Vecchia, *Phys. Lett.* **B73** (1978) 162;

[8] Lewis H. Ryder, *Quantum Field Theory* - Cap. 9, Cambridge University Press, 1985;

[9] C. Lee, K. Lee e E. Weinberg, *Phys. Lett.* **B243** (1990) 105;

[10] N. Dorey and N. E. Mavromatos, *Nuc. Phys.* **B386** (1992) 614;
N. E. Mavromatos, *Superconducting Gauge Theories in (2+1)-Dimensions*, CERN preprint, CERN-TH.6331/91 (1991).