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REMARKS ON DECOUPLING PROCEDURES IN GREEN'S FUNCTION

THEORY OF FERROMAGNETISM

FOR  $S = 1/2$

by

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## REMARKS ON DECOUPLING PROCEDURES IN GREEN'S FUNCTION

## THEORY OF FERROMAGNETISM

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ABSTRACT

It is the purpose of this note to discuss for the  $S = 1/2$  ferromagnet how recently proposed decoupling schemes may be obtained, the validity of the involved assumptions and the consequences in the low temperature range. It is suggested that the spurious  $T^3$  term present in almost Green's functions approaches is connected to local kinematics violating treatments of the spin correlation.

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### I. DEFINITION OF THE PROBLEM AND CALLEN'S APPROXIMATION

Consider a ferromagnetic material described by  $N$  localized spins on a crystal lattice, and coupled through the isotropic Heisenberg Hamiltonian:

$$H = - \sum_{g,f} J_{gf} \vec{S}_g \cdot \vec{S}_f,$$

where  $\vec{S}_g$  is the spin operator (in units of  $\hbar$ ) of the lattice site  $g$  and  $J_{gf}$  is the exchange integral corresponding to the interaction between the spins on lattice sites  $g$  and  $f$ . The thermodynamics of the system (1) will be studied here within the framework of Green's function theory (ZUBAREV 1960 or TYABLIKOV 1967). The appropriate propagator  $\langle\langle S_g^+; S_m^- \rangle\rangle_E$  satisfies the following equation of motions:

$$E \langle\langle S_g^+; S_m^- \rangle\rangle_E = \frac{\langle S^z \rangle}{\pi} \delta_{gm} + 2 \sum_{f \neq g} J_{gf} (\langle\langle S_f^z S_g^+; S_m^- \rangle\rangle_E - \langle\langle S_g^z S_f^+; S_m^- \rangle\rangle_E) \quad (2)$$

where  $\langle\langle A; B \rangle\rangle_E$  is the time Fourier transform of the double-time Green's function  $\langle\langle A(t); B(t') \rangle\rangle$ . As usually, equation (2) involves more complicated propagators and one should introduce some sort of approximations in order to break the chain of equations.

For higher-order Green's functions Callen (1963) has proposed a new decoupling instead of the classical RPA approximation (TYABLIKOV 1959). To get his basic equation of decoupling, he proposed a representation of the  $S_g^z$  operator in terms of the transverse spin operators ( $S^-, S^+$ ) and of a temperature dependent parameter. This representation for  $S = 1/2$  is

$$S_g^z = \frac{\alpha}{2} + \frac{1}{2} (1-\alpha) S_g^+ S_g^- - \frac{1}{2} (1+\alpha) S_g^- S_g^+, \quad (3)$$

where one chooses  $\alpha = 2\langle S^z \rangle$ . With this choice for  $\alpha$ , the equation (3) becomes

$$S_g^z = \langle S^z \rangle + \frac{1}{2} (1 - 2\langle S^z \rangle) S_g^+ S_g^- - \frac{1}{2} (1 + 2\langle S^z \rangle) S_g^- S_g^+ \quad (4)$$

It follows from this representation that:

- i) for the low-temperature limit  $\langle S^z \rangle \approx 1/2$ , then eq. (4) reduces to

$$S_g^z = \frac{1}{2} - S_g^- S_g^+, \quad (5)$$

describing then small derivations from perfect order;

- ii) for the high-temperature limit  $\langle S^z \rangle \approx 0$ , then eq. (4) reduces to

$$S_g^z = \frac{1}{2} (S_g^+ S_g^- - S_g^- S_g^+), \quad (6)$$

which describes fluctuations of the z-component about  $\langle S^z \rangle = 0$ ;

- iii) the higher-order Green's functions involving  $S_g^z$  are transformed in higher order Green's functions involving only the  $S^+$  and  $S^-$  operators, that is, the equation of motion for  $\langle\langle S_g^+; S_m^- \rangle\rangle_E$  includes then only transverse propagations. To these propagators is then reasonable to apply the symmetric decoupling (Callen) where the fluctuations of transverse correlations are neglected;

- iv) introduces a supplementary information on the degree of order associated to the system at a certain temperature.

With the representation (4) the Green's functions involving the z-component in eq. (2) are written as:

$$\begin{aligned} \langle\langle S_g^z S_f^+; S_m^- \rangle\rangle_E &= \langle S^z \rangle \langle\langle S_f^+; S_m^- \rangle\rangle_E + \frac{1}{2} (1 - 2\langle S^z \rangle) \langle\langle S_g^+ S_g^- S_f^+; S_m^- \rangle\rangle_E - \\ &- \frac{1}{2} (1 + 2\langle S^z \rangle) \langle\langle S_g^- S_g^+ S_f^+; S_m^- \rangle\rangle_E. \end{aligned} \quad (7)$$

Decoupling in a symmetric way the higher-order Green's function of the right-side of eq. (7) we obtain:

$$\langle\langle S_g^+ S_f^- S_m^+; S_m^- \rangle\rangle_E \stackrel{g \neq f}{=} \langle S_g^+ S_g^- \rangle \langle\langle S_f^+; S_m^- \rangle\rangle_E + \langle S_g^- S_f^+ \rangle \langle\langle S_g^+; S_m^- \rangle\rangle_E \quad (8)$$

and

$$\langle\langle S_g^- S_f^+ S_m^+; S_m^- \rangle\rangle_E \stackrel{g \neq f}{=} \langle S_g^- S_g^+ \rangle \langle\langle S_f^+ S_m^- \rangle\rangle_E + \langle S_g^- S_f^+ \rangle \langle\langle S_g^+ S_m^- \rangle\rangle_E. \quad (9)$$

Substituting equations (8) and (9) in equation (7), we get

$$\begin{aligned} \langle\langle S_g^z S_f^+ S_m^- \rangle\rangle_E \stackrel{g \neq f}{=} \langle S^z \rangle \langle\langle S_f^+ S_m^- \rangle\rangle_E - 2 \langle S^z \rangle \langle S_g^- S_f^+ \rangle \langle\langle S_g^+ S_m^- \rangle\rangle_E + \\ + \frac{1}{2} \{ (1-2 \langle S^z \rangle) \langle S_g^+ S_g^- \rangle - (1-2 \langle S^z \rangle) \langle S_g^- S_g^+ \rangle \} \langle\langle S_f^+ S_m^- \rangle\rangle_E \end{aligned}$$

but the third term in the right-side of this expression is zero by eq. (4).

Hence

$$\langle\langle S_g^z S_f^+ S_m^- \rangle\rangle_E \stackrel{g \neq f}{=} \langle S^z \rangle \langle\langle S_f^+ S_m^- \rangle\rangle_E - 2 \langle S^z \rangle \langle S_g^- S_f^+ \rangle \langle\langle S_g^+ S_m^- \rangle\rangle_E. \quad (10)$$

One should note that for the high-temperature limit all the points in the lattice are uncoupled by the thermal agitation, then the second term in the right-side of equation (10) goes to zero more rapidly than the first one. This means that in the high-temperature limit the symmetric decoupling and RPA coincide. Equation (10) is the basic equation of Callen's decoupling (CALLEN 1963). We must remember that using this type of basic equation Callen has obtained goods results for the low-temperature magnetization for  $S > 1$ . However for  $S = 1/2$ , in this result is appears a spurious term in  $T^3$  and do not have Dyson's  $T^4$  term in the low-temperature magnetization. In order to discuss the reason for this discrepancy we firstly analyse the implications of the symmetric decoupling.

II. IMPLICATIONS OF THE SYMMETRIC DECOUPLING

The spin operators must satisfy the following subsidiary condition:

$$(S_m^-)^{2S+1} = (S_m^+)^{2S+1} = 0 . \tag{11}$$

This condition plays a fundamental rule in the  $S = 1/2$  case for the choice of the decoupling. In the Green's function method one can automatically determine certain correlation functions in terms of lower-order correlation functions, by using the decoupling. That is, in the Heisenberg case, the choice of a basic equation of decoupling for the higher-order Green's function  $\langle\langle S_g^z S_f^+; S_m^- \rangle\rangle (g \neq f)$  implies that the correlation function  $\langle S_m^- S_g^z S_f^+ \rangle$  can be calculated in terms of the correlation function of two operators. On the other hand, the  $S_g^z$  operator has the following exact representation (MORITA and TANAKA 1965):

$$S_g^z = S - \frac{1}{2S} S_g^- S_g^+ - \frac{1}{(2S)^2 (2S-1)} (S_g^-)^2 (S_g^+)^2 + \dots + a_{2S} (S_g^-)^{2S} (S_g^+)^{2S} , \tag{12}$$

where the largest power is  $2S$  because of the subsidiary condition (11). In that case, the correlation function  $S_m^- S_g^z S_f^+$  using eq. (12) is given by:

$$\langle S_m^- S_g^z S_f^+ \rangle_{g \neq f} = S \langle S_m^- S_f^+ \rangle - \frac{1}{2S} \langle S_m^- S_g^- S_g^+ S_f^+ \rangle - \dots \tag{13}$$

For the particular case where  $g = m$  the last term in eq. (13)  $\langle S_g^- (S_g^-)^{2S} (S_g^+)^{2S} S_f^+ \rangle$  is rigorously zero. Conversely, using the basic equation of decoupling in order to calculate  $\langle S_m^- S_g^z S_f^+ \rangle$  it turns out that almost always the subsidiary condition (11) is not satisfied. This in general is responsible for the spurious terms and discrepancies in the thermodynamics.

Now one can understand why the Callen's approximation had success for  $S > 1$ . Indeed, when  $S > 1$ , the approximation will be felt by correlation functions of higher order than four, where the symmetric decoupling does not apply. This means that the subsidiary condition (11) will be violated at temperatures higher than  $T^4$  and this is responsible by the spurious terms  $\propto \theta^{3S+1/2}$  and  $\propto \theta^{3S+5/2}$ .

The situation is rather different for the  $S = 1/2$  case. In this case the subsidiary condition (11) implies that  $\langle S_g^- S_g^- S_g^+ S_f^+ \rangle = 0$ , this means that the approximation over the Green's function  $\langle\langle S_g^- S_g^+ S_f^+; S_m^- \rangle\rangle$  must be done in such a way that, when  $g = m$ , the subsidiary condition (11) will be as much as possible incorporated.

Let us now analyse the approximation made over the correlation functions  $\langle (S_g^-)^2 S_g^+ S_f^+ \rangle$  and  $\langle S_g^- S_g^Z S_f^+ \rangle$  by the symmetric decoupling. Subtracting eq. (9) from eq. (8) and using eq. (6), which is the representation of  $S_g^Z$  in the high-temperature limit, one gets:

$$\langle\langle S_g^Z S_f^+; S_m^- \rangle\rangle_{E, g \neq f} = \langle S^Z \rangle \langle\langle S_f^+; S_m^- \rangle\rangle_E. \quad (14)$$

The eq. (14) shows that in the high-temperature limit the symmetric decoupling implies in neglecting the magnetization fluctuations (RPA), thus confirming the heuristic analysis developed above. Calculating the jump of the Green's functions (all jumps in this note are calculated for  $t = t'$ ) in eq. (14) and taking  $m = g$ , we find the correlation function  $\langle S_g^- S_g^Z S_f^+ \rangle$ , which is given by

$$\langle S_g^- S_g^Z S_f^+ \rangle_{g \neq f} = \langle S^Z \rangle \langle S_g^- S_f^+ \rangle, \quad (15)$$

in the high-temperature limit, in agreement with the R.P.A. spirit. On the other hand, taking  $m = g$  in the eq. (10), one gets:

$$\langle\langle S_g^Z S_f^+; S_g^- \rangle\rangle_E = \langle S^Z \rangle \langle\langle S_f^+; S_g^- \rangle\rangle_E - 2 \langle S^Z \rangle \langle S_g^- S_f^+ \rangle \langle\langle S_g^+; S_g^- \rangle\rangle_E \quad (16)$$

Calculating the jump of the Green's functions in eq. (16), one gets the correlation function  $\langle S_g^- S_g^Z S_f^+ \rangle$ , which is given by

$$\langle S_g^- S_g^Z S_f^+ \rangle_{g \neq f} = \langle S^Z \rangle (1 - 2 \langle S^- S^+ \rangle) \langle S_g^- S_f^+ \rangle \quad (17)$$

But, calculating the average of eq. (5), which is the  $S_g^Z$  representation in the low-temperature limit, one gets:

$$1-2 \langle S_g^- S_g^+ \rangle = 2 \langle S^z \rangle . \quad (18)$$

From eqs. (17) and (18) one concludes that the correlation function  $\langle S_g^- S_g^z S_f^+ \rangle$  in the low-temperature limit is given by:

$$\langle S_g^- S_g^z S_f^+ \rangle_{g \neq f} = 2 \langle S^z \rangle^2 \langle S_g^- S_f^+ \rangle . \quad (19)$$

(DEMBINSKI 1968) has shown that eq. (19) is consistent with Callen's results. Still in the low-temperature region and from eqs. (5) and (19) one concludes that the correlation function  $\langle (S_g^-)^2 S_g^+ S_f^+ \rangle$  is given by:

$$\langle (S_g^-)^2 S_g^+ S_f^+ \rangle_{g \neq f} = \left( \frac{1}{2} - 2 \langle S^z \rangle^2 \right) \langle S_g^- S_f^+ \rangle . \quad (20)$$

At low-temperature the magnetization  $\langle S^z \rangle$  and the correlation function  $\langle S_g^- S_f^+ \rangle$  are respectively given by (CALLEN):

$$\langle S^z \rangle \approx \frac{1}{2} - a_0 \Theta^{3/2} . \quad (21)$$

$$\langle S_g^- S_f^+ \rangle \approx a_0 \Theta^{3/2} \quad (22)$$

where  $a_0 = \rho(3/2)$  and  $\Theta = 3k_B T/4\pi J_0 S_\nu$  ( $\nu = 1, (3/4)2^{2/3}, 2^{1/3}$ -SC; bcc; fcc).

Substituting the eqs. (21) and (22) in the eq. (20) one gets:

$$\langle (S_g^-)^2 S_g^+ S_f^+ \rangle_{g \neq f} \approx 2 a_0^2 \Theta^3 , \quad (23)$$

showing that only at absolute zero  $\langle (S_g^-)^2 S_g^+ S_f^+ \rangle = 0$  rigorously.

The correlation function  $\langle S_g^- S_f^+ \rangle$  ( $g \neq f$ ) is connected essentially with effects of excitation of spin waves and since Callen's results are satisfactory when the subsidiary condition (11) is not strongly violated, one can admit that the difficulty is not introduced by this correlation function in the  $S = 1/2$  case.

For  $S = 1/2$  and  $g = m$  the eq. (13) reduces to:



$$\langle S_g^- S_g^z S_f^+ \rangle_{g \neq f} = \frac{1}{2} \langle S_g^- S_f^+ \rangle \quad (24)$$

On the other hand, substituting the eq. (21) in the eq. (19) one concludes that:

$$\langle S_g^- S_g^z S_f^+ \rangle_{g \neq f} \approx \frac{1}{2} \langle S_g^- S_f^+ \rangle - 2a_0^2 \theta^3, \quad (25)$$

Showing that the error of the Callen's approximation is  $2 a_0^2 \theta^3$  compared with the exact given by eq. (24) in the low-temperature region.

The high-temperature limit, given by eq. (15), is consistent with the RPA approximation introduced by TYABLIKOV (1959). However it is reasonable to propose that the relation (15) should also hold for the correlation function  $\langle S_g^- S_g^z S_f^+ \rangle$  in the low-temperature limit. Since, the system is "frozen" at low-temperatures, the correlation between  $f$  and  $g$  is expected to be not very much affected by the fluctuations of  $S_g^z$  which are supposed to be small. So let us propose a new decoupling that, for  $g = m$ , and low-temperature implies:

$$\langle S_g^- S_g^z S_f^+ \rangle_{g \neq f} = \langle S^z \rangle \langle S_g^- S_f^+ \rangle. \quad (26)$$

In these conditions, with aid of eqs. (21) and (22). one gets:

$$\langle S_g^- S_g^z S_f^+ \rangle_{g \neq f} \approx \frac{1}{2} \langle S_g^- S_f^+ \rangle - a_0^2 \theta^2, \quad (27)$$

showing that the error of the new decoupling is less than the Callen's one. By the new decoupling and using eq. (26) one also gets:

$$\langle (S_g^-)^2 S_g^+ S_f^+ \rangle_{g \neq f} \approx a_0^2 \theta^3, \quad (28)$$

where the subsidiary condition is better treated than in the Callen's approximation, eq. (23). We hope that in the new decoupling the discrepancies of the subsidiary condition (11) will be present in powers of temperature greater than four.

### III. DISCUSSION OF THE IMPROVED DECOUPLING

We must however emphasize that proposing some decoupling procedure for higher-order Green's function, that is, to express them in terms of lower-order Green's functions, enables us to solve the infinite chain of equations of motion in an approximated way. Thus we can get the lower-order Green's function as precise as the decoupling procedure is. But also the higher-order Green's functions are determined in the same approximation. This can be seen by the substitution of the lower-order Green's function, already determined, in the basic equation of decoupling procedure. In conclusion, a physical insight over the correlation functions is very desirable in proposing schemes. Among the various decoupling procedures we are going to fix one that gives the same result for the correlation function  $\langle S_g^- S_g^z S_f^+ \rangle$  at low and high-temperatures. We propose the following decoupling for the higher-order Green's functions, which appear in the right-side of eq. (7):

$$\langle\langle S_g^+ S_g^- S_f^+; S_m^- \rangle\rangle_E = \langle S_g^+ S_g^- \rangle \langle\langle S_f^+; S_m^- \rangle\rangle_E + \langle S_g^- S_f^+ \rangle \langle\langle S_g^+; S_m^- \rangle\rangle_E + S_{gm} A \langle\langle S_f^+; S_m^- \rangle\rangle_E \quad (29)$$

and

$$\langle\langle S_g^- S_g^+ S_f^+; S_m^- \rangle\rangle_E = \langle S_g^- S_g^+ \rangle \langle\langle S_f^+; S_m^- \rangle\rangle_E + \langle S_g^- S_f^+ \rangle \langle\langle S_g^+; S_m^- \rangle\rangle_E + \delta_{gm} B \langle\langle S_f^+; S_m^- \rangle\rangle_E, \quad (30)$$

where A and B must be such that the following conditions are satisfied:

- a) we must get the RPA approximation in the high-temperatures limit;
- b) we must get the expression (26) for the low-temperatures limit, when  $m=g$ .

We note that this decoupling reproduces Callen's one for  $m \neq g$ . The correlation term of the symmetric decoupling must be proportional to  $\langle\langle S_f^+; S_m^- \rangle\rangle$  since only when  $m$  is equal to  $g$  the problem of subsidiary condition appears. In the equations (29) and (30) two function of temperature involved (A and B); they

should be determined separately in the two temperature limits since there is a priori no reason for their equality in both temperature regions.

Substituting eqs. (29) and (30) in the eq. (7) one gets the general equation:

$$\begin{aligned} \langle\langle S_g^z S_f^+; S_m^- \rangle\rangle_E &= \langle S^z \rangle \langle\langle S_f^+; S_m^- \rangle\rangle_E - 2 \langle S^z \rangle \langle S_g^- S_f^+ \rangle \langle\langle S_g^+; S_m^- \rangle\rangle_E + \\ &+ \frac{\delta_{gm}}{2} \{ A(1-2\langle S^z \rangle) - B(1+2\langle S^z \rangle) - B(1+2\langle S^z \rangle) \} \langle\langle S_f^+; S_m^- \rangle\rangle_E. \end{aligned} \quad (31)$$

where again Callen's results are reproduced for  $g=f$ .

Subtracting eqs. (29) and (30) and using the high-temperature limit (6) for the representation of  $S_g^z$  one gets:

$$\langle\langle S_g^z S_f^+; S_m^- \rangle\rangle_E \quad (\langle S^z \rangle + \delta_{mg} \frac{A-B}{2}) \langle\langle S_f^+; S_m^- \rangle\rangle_E. \quad (32)$$

The condition (a) for A and B implies that we must have in eq. (32)

$$A = B \quad (33)$$

at high-temperature. Substituting equation (33) in the eq. (31) one gets:

$$\langle\langle S_g^z S_f^+; S_m^- \rangle\rangle_E = \langle S^z \rangle \langle\langle S_f^+; S_m^- \rangle\rangle_E - 2 \langle S^z \rangle \langle S_g^- S_f^+ \rangle \langle\langle S_g^+; S_m^- \rangle\rangle_E - 2 \delta_{gm} \langle S^z \rangle A \langle\langle S_f^+; S_m^- \rangle\rangle_E.$$

Taking  $m = g$  and calculating the associated correlation function one gets:

$$\langle S_g^- S_g^z S_f^+ \rangle_{g \neq f} = \langle S^z \rangle \langle S_g^- S_f^+ \rangle - 2 \langle S^z \rangle \langle S_g^- S_f^+ \rangle \langle S_g^- S_g^+ \rangle - 2A \langle S^z \rangle \langle S_g^- S_f^+ \rangle.$$

Since RPA must hold at high-temperature, then we must have

$$2 \langle S^z \rangle \langle S_g^- S_f^+ \rangle \langle S_g^- S_g^+ \rangle_{g \neq f} = - 2A \langle S^z \rangle \langle S_g^- S_f^+ \rangle. \quad (34)$$

Hence

$$A = B = - \langle S_g^- S_g^+ \rangle.$$

at high-temperature.

On the other hand, using the low-temperature limit (5) for the representa

tion of  $S_g^z$  and the expression (30) we get

$$\langle S^z \rangle \langle \langle S_f^+; S_m^- \rangle \rangle_E - \langle \langle S_g^z S_f^+; S_m^- \rangle \rangle_E = \langle S_g^- S_f^+ \rangle \langle \langle S_g^+; S_m^- \rangle \rangle_E \delta_{mg} + B \langle \langle S_f^+; S_m^- \rangle \rangle_E,$$

which for  $m = g$  implies:

$$\langle S^z \rangle \langle \langle S_g^- S_f^+ \rangle \rangle_{g \neq f} - \langle \langle S_g^- S_g^z S_f^+ \rangle \rangle_{g \neq f} = \langle S_g^- S_f^+ \rangle \langle \langle S_g^- S_g^+ \rangle \rangle + B \langle \langle S_g^- S_f^+ \rangle \rangle. \quad (35)$$

The condition (b) for A and B implies that we must have in eq. (35)

$$B = - \langle \langle S_g^- S_g^+ \rangle \rangle. \quad (36)$$

at low-temperature. With the aid of eq. (36), the eq. (31) becomes

$$\begin{aligned} \langle \langle S_g^z S_f^+; S_m^- \rangle \rangle_E &= \langle S^z \rangle \langle \langle S_f^+; S_m^- \rangle \rangle_E - 2 \langle S^z \rangle \langle \langle S_g^- S_f^+ \rangle \rangle \langle \langle S_g^+; S_m^- \rangle \rangle_E + \\ &+ \frac{\delta_{gm}}{2} \{ A(1-2\langle S^z \rangle) + (1+2\langle S^z \rangle) \langle \langle S_g^- S_g^+ \rangle \rangle \} \langle \langle S_f^+; S_m^- \rangle \rangle_E, \end{aligned}$$

which with the jumps of the Green's functions and taking  $m=g$ , reduces to

$$\begin{aligned} \langle \langle S_g^- S_g^z S_f^+ \rangle \rangle_{g \neq f} &= \langle S^z \rangle \langle \langle S_g^- S_f^+ \rangle \rangle - 2 \langle S^z \rangle \langle \langle S_g^- S_g^+ \rangle \rangle \langle \langle S_g^- S_f^+ \rangle \rangle + \\ &+ \frac{1}{2} \{ A(1-2\langle S^z \rangle) + (1+2\langle S^z \rangle) \langle \langle S_g^- S_g^+ \rangle \rangle \} \langle \langle S_g^- S_f^+ \rangle \rangle. \end{aligned} \quad (37)$$

Using again the condition (b) for A and B we must have in eq. (37)

$$A = - \langle \langle S_g^- S_g^+ \rangle \rangle, \quad (38)$$

at low-temperature. Hence we conclude that in the low-and high temperature limits.

$$A = B = - \langle \langle S_g^- S_g^+ \rangle \rangle. \quad (39)$$

The expressions (31) and (39) give the basic equation for the new decoupling in the low-and high-temperature limits which is

$$\langle \langle S_g^z S_f^+; S_m^- \rangle \rangle_E = \langle S^z \rangle \langle \langle S_f^+; S_m^- \rangle \rangle_E - 2 \langle S^z \rangle \langle \langle S_g^- S_f^+ \rangle \rangle \langle \langle S_g^+; S_m^- \rangle \rangle_E + \delta_{mg} 2 \langle S^z \rangle \langle \langle S_g^- S_g^+ \rangle \rangle \langle \langle S_f^+; S_m^- \rangle \rangle_E. \quad (40)$$

At this point we must remember that the representation (4) for  $S_g^z$  was proposed only to get a basic equation of decoupling. So after this basic equation

was established the following relations:

$$S_g^z = \frac{1}{2} - S_g^- S_g^+$$

and

$$S_g^z = \frac{1}{2} (S_g^- S_g^+ - S_g^+ S_g^-)$$

must hold independently of the temperature region. Hence in the eq. (40) we can substitute

$$\langle S_g^- S_g^+ \rangle = \frac{1}{2} - \langle S_g^z \rangle \quad (41)$$

and we get

$$\langle\langle S_g^z S_f^+; S_m^- \rangle\rangle_{g \neq f} = \langle S_g^z \rangle \langle\langle S_f^+; S_m^- \rangle\rangle_E - 2 \langle S_g^z \rangle \langle S_g^- S_f^+ \rangle \langle\langle S_g^+; S_m^- \rangle\rangle_E + \delta_{gm} \langle S_g^z \rangle (1 - 2 \langle S_g^z \rangle) \langle\langle S_f^+; S_m^- \rangle\rangle_E. \quad (42)$$

We must however emphasize that this basic equation of the new decoupling is only valid in the low-and-high-temperature limits. But for practical proposes only two temperatures regions has interest.

The expression (42) is the basic equation of decoupling proposed by Dembinski. Using eq. (42) he obtained the low-temperature magnetization without the  $T^3$  spurious term, the coefficient of  $T^4$  term is quite similar as the one obtained by DYSON (1956) and also obtained the Curie temperature which is the same obtained in the RPA approximation (TYABLIKOV 1959, TAHIR-KHELI and TER HAAR 1962).

## DISCUSSION

Finally we must emphasize, as pointed out by WORTIS (1965), that "the mechanics of the Heisenberg model is dominated by two properties: (a) It always obeys spin kinematics and (b) the low-lying states have a propagational, particle-like, "spin-wave" behaviour. These opposing properties must be reconciled in any formalism". In this point of view we must point out that RPA

(TYABLIKOV 1959, TAHIR-KHELI and TER HAAR 1962) obscures the property (b), because the eq. (15) holds independently of the temperature limits. The  $T^3$  spurious term in the low-temperature magnetization appears because that RPA does not treat well the spin correlations and because of this gives the worst renormalized energies of the excitations of the system. For  $S > 1$  the Callen's decoupling does not violated the properties (a) and (b) at low-temperature. This means that this decoupling treats better the spin correlations and getting a better renormalized energies of excitations than RPA and the subsidiary condition (11) is satisfied at low-temperature. For  $S = 1/2$  we can say that Dembinski's decoupling does not violates the properties (a) and (b), which was the main point of this note.

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