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Area density of localization-entropy I: the case of wedge-localization

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Abstract

Using an appropriatly formulated holographic lightfront projection, we derive an area law for the localization-entropy caused by vacuum polarization on the horizon of a wedge region. Its area density has a simple kinematic relation to the heat bath entropy of the lightfront algebra. Apart from a change of parametrization the infinite lighlike length contribution to the lightfront volume factor corresponds to the short-distance divergence of the area density of the localization entropy. This correspondence is a consequence of the conformal invariance of the lightfront holography combined with the well-known fact that in conformality relates short to long distances. In the explicit calculation of the strength factor we use the temperature duality relation of rational chiral theories whose derivation will be briefly reviewed. We comment on the potential relevance for the understanding of Black hole entropy.

1 Introduction

Localization-entropy is a thermal manifestation of vacuum polarization, i.e. different from the standard heat bath thermality of classical statistical systems it is of purely quantum-physical origin. As the prize of quantum mechanical localization is paid by an uncertainty in momentum, localization in relativistically causal QFT is not possible without causing thermal manifestations of the localized vacuum state or in other words: localization aspects of (vacuum) QFT are inexorably related to thermal statistical mechanics properties. These localization-caused thermal manifestations turn out to have a very subtle relation to the standard heat bath thermality. But whereas uncertainty relations where identified as the characteristic properties of QM shortly after the discovery of the latter, the understanding of the thermal signature of vacuum fluctuations as a characteristic property of local quantum physics[1] is a recent observation; in the course of this article the reader will understand why it took such a long time.

Vacuum fluctuations as an unavoidable attribute of local quantum physics were first noted by Heisenberg [1] when he computed what we nowadays would call a "partial charge" by integrating the Wick-ordered zero component of the bilinear conserved current density over a finite spatial volume. Heisenberg noticed that the current conservation law does not control the infinitely strong particle-antiparticle vacuum fluctuations at the boundary of the volume. Later conceptual and mathematical refinements of QFT[2] showed that these fluctuations can be kept finite by allowing a region of "fuzzy" localization in a surface of finite thickness [2]. In the infinite volume (thermodynamic) limit the dependence of the so defined "partial charge" on the chosen smoothing prescription disappears and the partial charge converges against the unique global conserved charge.

In the presence of interactions, these quadratic vacuum fluctuations (particle/antiparticle pairs) of interaction-free partial charges change into vacuum polarization "clouds" involving an unlimited number of particle-antiparticle pairs. This phenomenon which in the perturbative context (where the number of pairs increases with the order of perturbation) was first noticed with some surprise by Furry and Oppenheimer [3]. These very early observations showed the limits of Dirac's view of QFT as some kind of relativistic particle quantum mechanics[3]. Placed into a more modern conceptual setting this

[1] In the spirit of Haag's book [9] we prefer the term *local quantum physics (LOP)* or *algebraic QFT* (AQFT) whenever we want to de-emphasize the use of field coordinatizations in favor of a more intrinsic local operator-algebraic presentation of QFT.

[2] These refinements resulted from a better understanding of the operator-valued distribution nature of fields which led to a test function smoothing in the definition of partial charges which includes a compact smearing in time.

[3] The relativistic particle interpretation of quantum field theory was finally abandoned when it became clear that Dirac's hole theory although successful in low orders (see Heitler's book) cannot cope with renormalization.

observation can be backed up by a powerful theorem stating that the existence of a PFG in a subwedge-localized algebra forces the theory to be free. Here PFG is the acronym for vacuum **p**olarization-**f**ree **g**enerator which is an operator whose application to a vacuum vector creates a one-particle state without any vacuum polarization contribution; the main content is that in the presence of interactions it is not possible that local PFG (which are affiliated with operator algebras $\mathcal{A}(\mathcal{O})$ for subwedge regions $\mathcal{O}$)) exist. The borderline case for the existence PFGs is the wedge[4] region; this can be established with the help of modular localization [4]. In this sense the wedge region offers the best compromise between local algebras corresponding to the interacting field concept and Wigner particle states [5]. Wedge-localized operator algebras also turn out to present the simplest "theoretical laboratory" for understanding the thermal manifestations of localization.

The central issue of this paper is the observation that vacuum polarization on individual localized operators can be related to collective properties of causally localized operator algebras to which they are affiliated. In particular it will be shown that the area proportionality of localization-entropy is a generic property of local quantum matter.

The idea that such localized algebras may exhibit thermal properties came from two different sources. There is the famous physically well motivated observation by Hawking on thermal radiation of quantum matter enclosed in a Schwarzschild black hole [6]. Closely related is Unruh's Gedankenexperiment [7] involving a uniformly accelerated observer whose world-line is restricted to a Rindler wedge in Minkowski spacetime. In this case the thermal manifestations of vacuum fluctuations are detached from the presence of strong curvature effects of general relativity. Independently there is the structural observation by Bisognano and Wichmann [8] which established that the restriction of the global vacuum state to a wedge-localized subalgebra becomes a thermal KMS (Kubo-Martin-Schwinger) state for arbitrary interacting matter content of the QFT model. In fact they found this thermal manifestation as a side result of their application of the modular Tomita-Takesaki theory of operator algebras (which was discovered a decade before with important independent contributions coming from physicists doing quantum statistical mechanics directly in infinite space (open systems) [9]). The special nature of the wedge region is that the associated modular objects have a well-known physical interpretation in terms of geometric symmetries (wedge-preserving Lorentz boost and TCP transformation into the opposite wedge). The connection between the Hawking-Unruh and the Bisognano-Wichmann thermal manifestation of vacuum polarization was first pointed out by Sewell [10].

Once it can be argued that Heisenberg's observation about vacuum polarization placed within the context of localized operator algebras leads to thermal manifestations, the surface nature of this local quantum phenomenon suggests that the thermal aspect is different from that of the standard heat bath situation. One would expect that on top of the standard extensive bulk contributions to entropy and energy from a heat bath which are proportional to the volume (and which are absent in case one starts with a global vacuum), there are additional area contributions caused by vacuum fluctuations at the causal horizon which is the boundary of the localization region. In the course of proving this assertion in this paper in the context of a Rindler wedge, we find an unexpected connection between localization thermality and an auxiliary standard heat bath thermal system associated with the lightfront-extended holographic projection of the original system which apparently had not been noticed before. In this correspondence the lightlike length factor R of the heat bath volume passes to the inverse of a distance[5] which measures the thickness ε of vacuum polarization "collar" around the boundary. One obtains for the localization entropy for wedge localization (the area A factor refers to the edge of the wedge)

$$s_{loc}(\varepsilon) \underset{\varepsilon\to 0}{=} A\,|ln\varepsilon|\,\frac{c\pi}{6} + o(\varepsilon) \tag{1}$$

$$R \simeq |ln\varepsilon|\,,\; A\times R = V \tag{2}$$

where c is a measures the degree of freedoms of the holographically projected matter which is related to the constant with was denoted by the same latter in a chiral theory. In particular the logarithmic divergence for $\varepsilon \to 0$ has nothing to do with the description of QFT in terms of particular singular field

[4] A general wedge W is a Poincaré transform of the standard wedge $W_0 = \{x_1 > |x_0|\,, x_{2,3}\ arbitrary\}$ and a subwedge region $\mathcal{O}$ is any region which can be enclosed in a wedge $W \supset \mathcal{O}$.

[5] In the conformally invariant holographic projection short distances are conformal related to long distances.

coordinatizations and hence it cannot be renormalized away or dumped into a physical parameter. In fact, and this is perhaps the most novel and surprising aspect of the present work, there exists a map which relates the heat bath entropy of the global lightfront to the localization entropy in such a way that the lightray length contribution to the standard heat bath volume factor equals the logarithm of a shrinking distance factor (2). This map takes the thermodynamic limit sequence (which is interpreted as approximating the global algebra by box-localized algebra form of the inside) into a *funnel*[6]. The crucial property which leads to the existence of such a map is the conformal invariance of the lightfront projection; it is well known that conformal covariance relates short to long distances. It shows that, contrary to popular opinion, it is not correct to link localization entropy (and its possible physical manifestations in black hole physics) with short distance cutoffs. Our observation goes into the same direction as that made before by Unruh showing that certain thermal manifestations of black hole analogs are insensitive against short distance modifications of the dispersion law (as long as those modifications maintain the covariance associated with the KMS state). Different from the Bekenstein [11] area law which is a consequence of special differential geometric properties classical tensor fields, (in particular in the setting of the Einstein-Hilbert general relativity), the area law for the energy and entropy caused by quantum localization is a *general property of matter in local quantum physics*.

Although, as will be seen, the mathematical setting of modular localization permits clear definitions and rigorous derivations, one faces serious problems when one tries to convert the thermal manifestations of localization into observational consequences. These difficulties also explain why it took such a long time after having noticed the presence of localization-caused vacuum polarization to become also aware of their thermal consequences. In order to explain this important point let us first recall that the notion of temperature in the standard heat bath setting of statistical mechanics is related to the time translation and the corresponding Hamiltonian. In terms of this Hamiltonian one defines a finite volume tracial Gibbs state at inverse temperature β. To arrive at thermodynamic equilibrium in which the boundary effects become insignificant, one performs the thermodynamic infinite volume limit in which the appropriately normalized Gibbs state converges towards a KMS state associated with the Hamiltonian automorphism. Independent of any details, KMS states associated with a Hamiltonian H are known to fulfill an abstract form of the second law [12] which can be expressed in terms of the following inequalities

$$\begin{aligned} E_H \equiv \langle U\Omega_\beta \left| H \right| U\Omega_\beta \rangle \geq \left\langle U\Omega_\beta \left| 1 - e^{-H} \right| U\Omega_\beta \right\rangle \geq \langle U\Omega_\beta \left| 1 \right| U\Omega_\beta \rangle - \langle U^*\Omega_\beta \left| 1 \right| U^*\Omega_\beta \rangle \geq 0 \qquad (3) \\ \left\langle A\Omega_\beta \left| e^{-H} \right| B\Omega_\beta \right\rangle = \langle B^*\Omega_\beta \left| 1 \right| A^*\Omega_\beta \rangle, \ A, B, U \in \mathcal{A} \end{aligned}$$

Here U denotes any unitary operator associated with the global observable algebra and the second inequality uses the KMS property in the form as written in the second line. The unitary operator applied to the thermal state represents the change caused by an external force which acts during a finite time. The positive sign expresses the impossibility to extract energy ($E_H < 0$) without causing a permanent change of the external conditions (impossibility of a perpetuum mobile). Standard assumptions about the form of the Hamiltonian allow to convert this abstract form of the second thermodynamic law into the more concrete quantified form in terms of an entropy function.

Modular theory permits to repeat these arguments word for word in case the operator algebra is a localized algebra $\mathcal{A}(\mathcal{O})$ and Ω_β is any vector on which this algebra acts in a cyclic and separating manner e.g. the vacuum state $\Omega = \Omega_\infty$ (if β is the inverse temperature). The modular substitute for H is the generator K of the so-called modular group $\Delta^{it} = e^{iKt}$ offen referred to as the modular “Hamiltonian”. Although modular theory guaranties the existence of the modular objects, it does not provide a physical interpretation of the modular Hamiltonian K and the modular “time” t. Only in the fortunate case in which Δ^{it} admits a geometric interpretation one can think of a Gedankenexperiment for really observing the thermal consequences in terms of thermal radiation. The only case in Minkowski space QFT is the wedge-localized operator algebra relative to the vacuum state; in this case Bisognano and Wichmann showed that the modular automorphism is the wedge-preserving Lorentz-boost. In curved space time there are more possibilities to find Killing symmetries which leave subregions invariant. In those cases it does not matter whether this occurs in the context of general relativity or in analog situations in acoustics, hydrodynamics or optics [13] where such situations are generated by encoding microscopic properties into

[6] A funnel is an increasing sequence of type I operator algebras which converges against a (not necessarily type I) limiting operator algebra. In our case the latter is the monade (see appendix).

an "effective" spacetime metric and the resulting effective description is a quantum theory with a finite propagation speed. Even if one does not share the optimism about its experimental accessibility [14], the subject is of sufficient intrinsic theoretical interest.

The derivation of the area density formula (1) which will be the main topic of this paper is based on the combination of three facts

- Holographic projection of the wedge algebra onto a transversely extended chiral algebra
- The possibility (limited to 2-dim. conformal theories) to pass from localization-caused thermal behavior to global heat bath thermality and vice versa.
- The existence of a "natural" thermodynamic limit sequence (which preserves the spacetime covariances) in conformal QFT.

In order to obtain concrete limiting formulas from the last fact one uses again the chiral nature of the conformal theories which permits to use asymptotic estimates of the Cardy-Verlinde type.

The three points have previously appeared in different contexts. They will be re-derived in a form adapted to the present purpose. The next section reviews the lightfront holography whereas the remaining two points are presented in section 3.

2 Reviewing lightfront holography and consequences of absence of transverse vacuum polarization

The study of thermal aspects of localization is greatly simplified by using instead of the original operator algebra its holographic projection. In case of the wedge algebra, the holographic projection leads to the lightfront algebra. The latter is related to what used to be called "lightcone quantization", in fact it could be seen as a conceptual and mathematical rescue operation to save some of the intuitive content of the latter. Different from the old approach it should not be viewed as a new quantization, but rather as a different spacetime encoding of a given QFT. In other words it is a concept which reprocesses the spacetime affiliation of the algebraic substrate indexed in terms of spacetime regions in the ambient space to a radically different one in which subalgebras of the same global algebra are indexed by localized regions on a lightfront (which is a manifold which contrary to the ambient manifold is neither globally nor even locally hyperbolic) [5]. If there are no interactions this can be done directly in terms of free fields. The steps are as follows:

$$
\begin{aligned}
&A(x) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int (e^{ipx}A^*(p) + h.c.)\frac{d^3p}{2p_0},\ \ A_W(r,\chi,x_\perp) \equiv A(x)|_W \qquad (4)\\
&\curvearrowright A_W(r,\chi,x_\perp) = \int (e^{i(m_{eff}rch(\chi-\theta)+p_\perp x_\perp}A^*(p) + h.c.)dp_\perp \frac{d\theta}{2}\\
&[A(p), A^*(p'] = 2p_-\delta(p_- - p'_-)\delta(p_\perp - p'_\perp),\ \ H = H_-\otimes H_\perp
\end{aligned}
$$

The first line is the representation of the local free field in terms of Wigner particle creation/annihilation operators whose commutation relations are written in the third line (where only the nonvanishing commutators have been written down). They act in a Fock space, which for convenience has been written in a tensor product notation adjusted to the momentum space decomposition into lightlike and transverse components. The restriction to the (right) wedge is conveniently done in terms of x- and p-space rapidity parametrization

$$
\begin{aligned}
&x^0 = rsh\chi, x^1 = rch\chi \qquad (5)\\
&p^0 = m_{eff}ch\theta, p^1 = m_{eff}sh\theta
\end{aligned}
$$

which leads to the second line. The restriction to the (upper) horizon $H(W)$ is done in terms of a limiting process $r \to 0, \chi \sim |\ln r| \to \infty$ such that $x_+ = re^{\chi}$ remains finite and $x_- = re^{-\chi} \to 0$. The formal expression for the limiting singular operator is

$$A_{H(W)}(x_+, x_\perp) = \lim_{r\to 0} A_W(r,\chi,x_\perp) = \frac{1}{(2\pi)^{\frac{3}{2}}}\int \frac{d\hat{\theta}}{2}\int \left(A^*(p)e^{im_{eff}p_- x_+ + ip_\perp x_\perp} + h.c.\right) d^2p_\perp$$

$$= \frac{1}{(2\pi)^{\frac{3}{2}}}\int_0^\infty \frac{dp_-}{2p_-}\int \left(A^*(p_-,p_\perp)e^{ip_-^- x_+ + ip_\perp x_\perp} + h.c.\right) d^2p_\perp \tag{6}$$

$$\left\langle A_{H(W)}(x_+,x_\perp), A_{H(W)}(x'_+,x'_\perp)\right\rangle_0 = \delta(x_\perp - x'_\perp)\int e^{ip_-(x_+ - x'_+)}\frac{dp_-}{2p_-}$$

$$\left[A_{H(W)}(x_+,x_\perp), A_{H(W)}(x'_+,x'_\perp)\right] = \varepsilon(x_+ - x'_+)\delta(x_\perp - x'_\perp)$$

$$\left[\partial_+ A_{H(W)}(x_+,x_\perp), \partial_+ A_{H(W)}(x'_+,x'_\perp)\right] = \delta'(x_+ - x'_+)\delta(x_\perp - x'_\perp) \tag{7}$$

In the transition from the first to the second line we have changed the $p_\perp$ dependent pre-factor of $p_- = e^{-\theta}$. In order to see that this is allowed one has to remember that this change does not modify the operator after integrating with the relevant limiting class of smearing functions (which vanish at the origin $p_- = 0$)[7]. Note that the terminology "lightfront restriction" only agrees with its naive geometric meaning $x_- = 0$ in this mass shell representation but doing this in correlation functions would give nonsensical results[8]. The linear extension of the horizon yields the lightfront. The formula for the lightfront fields $A_{LF}(x_+, x_\perp)$ extends that for $A_{H(W)}$ from $x_+ > 0$ to all x_+.

If one wants unrestricted test function spaces on the lightfront one should start from the derivative field $\partial_+ A(x)$ which creates the same Hilbert space as A. By a process called Haag-dualization the local operator algebra generated by $\partial_+ A(x)$ are known to be the same as those generated by A (this is similar to [15]). The lightfront restriction of the derivative field has the algebraic structure of a transversely extended chiral theory for an abelian current (7) where the δ' function represents the chiral aspect. and also (after Haag-dualization) the same local algebras.

The algebraic structure of the commutation relation reveals another interesting (and for our purpose important) information: the lightfront projection places a new infinite dimensional symmetry group into evidence which is of the Bondy-Metzner-Sachs type[9]. Such infinite-dimensional groups arose first in the investigation of asymptotic behavior of zero mass theories and in particular for asymptotically flat classical spacetimes (in the sense of Penrose). In our local quantum physics setting they simply arise from transverse extensions of the Diff(S^1) symmetry of chiral theories

$$(x_{\perp,}x_+) \to (x'_{\perp,}x'_+) = (Ex_\perp, \psi(x_{\perp,}x_+)) \tag{8}$$
$$\psi(x_{\perp,}\cdot) \in Diff(S^1)$$

where $Ex_\perp$ is a Euclidean transformation in the transverse direction and $x'_+ = \psi(x_{\perp,}x_+)$ an $x_\perp$-dependent diffeomorphisms of the compactified lightray coordinate $x_+ \in \dot{R} \equiv S^1$. This symmetry of the algebraic structure is unitarily implemented on the operator-algebraic level. It was already present in the ambient setting but went unnoticed because it does not have the form of a quantum Noether symmetry. This is because in the ambient bulk setting it belongs to an infinite group of "fuzzy" symmetry transformations, i.e. algebraic covariances similar to the localization preserving modular automorphism.

Since the issue of emergence of infinite symmetry groups in holographic projections is of no direct importance for the thermal manifestations of modular localization in this paper, the appearance of quantum B-M-S like groups and their use in the quantum aspects of the conformal infinity in the sense of Penrose will be deferred to a separate publication. Although the Poincaré group continues to act on the lightfront operators, the "visible" part consists only of a 7-parametric subgroup: the 3 parametric subgroup in the wedge plane (Boost, 2 lightlike translations), the 3-parametric transverse Euclidean group and the edge-changing (but lightfront preserving) 1-parametric subgroup of the Wigner little group.

[7] This is well-known for the zero mass scalar free field in two-dimensions. In that case the exponential contains a engineering dimension-setting mass parameter which has no bearing on the energy-momentum spectrum and which drops out after test function smearing within the appropriate space of functions [5].

[8] This means one has to do the GNS reconstruction of the operators acting in a Hilbert space and recover the mass shell representation.

[9] In the case of double cone holography [18] (treated in a separate paper) the group becomes actually identical to the BMS group in the limit of infinitely large double cones.

In the presence of interactions the lightfront restriction suffers from the same problem as the derivation of equal time canonical commutation relation; the obstacle in both cases is the infinite wave function renormalization (the divergence of the integral over the Kallen-Lehmann spectral function) [5].

The analog of the mass-shell representation (4) for interacting fields is fairly involved since it requires the apparatus of LSZ scattering theory. The latter leads to the following so-called Glaser-Lehmann-Zimmermann expansion of interacting Heisenberg fields in terms of incoming free fields

$$A(x) = \sum \frac{1}{n!} \int_{H_m} ... \int_{H_m} e^{i\sum_{i=1}^{n} p_i x_i} a(p_1, ..., p_n) : A_{in}(p_1)...A_{in}(p_n) : \Pi_i \frac{d^3 p_i}{2p_{i0}} \overset{LF}{\rightarrow} \tag{9}$$

$$A_{LF}(x_+, x_\perp) = \sum \frac{1}{n!} \int_{H_m} ... \int_{H_m} e^{i\sum_{i=1}^{n}(p_{i-}x_{+i} - p_{i\perp}x_{i\perp})} a(p_1, ..., p_n) \times$$
$$\times : A_{in}(p_1)...A_{in}(p_n) : \Pi_i \frac{dp_{i-}}{p_{i-}} dp_{i\perp} \tag{10}$$

Here the coefficient functions are mass shell restrictions of Fourier transforms of retarded correlation functions. Besides being extremely formal (the convergence properties of such representations are unknown), the use of formulas based on scattering theory would defeat the whole motivation for the use of the *lightfront formalism* which was to *simplify certain aspects of the original dynamical problem.* A different spacetime encoding cannot accomplish dynamical miracles; the best one can hope for is that certain aspects of vacuum polarization and their thermal manifestation which one is interested in become simpler at the prize of a more complicated particle and S-matrix properties.

A conceptually and mathematically superior approach consists in avoiding field coordinatizations altogether in favor of the modular localization formalism of operator algebras. In the presence of interactions this is the only approach to holography.

The prototype situation of an operator algebraic approach in this paper will be that of a Rindler-Unruh [7] wedge algebra whose holographic projection is the (upper) causal horizon which covers half a lightfront. The starting point is the equality of the wedge algebra with its holographic projection (the horizon $H(W)$ is the (upper) causal horizon of W and LF denotes its linear lightfront extension)

$$\mathcal{A}(W) = \mathcal{A}(H(W)) = \mathcal{A}(LF(W))|_{H(W)} \tag{11}$$

which in the absence of interactions follows from our free field computations whereas in general it is considered as part of the definition of what constitutes a causal and local quantum field theory[10] i.e. it belongs to those structural properties which remain unaffected by interactions. Although the wedge algebra is equal to that of its lightfront horizon, this does not apply to the substructures; in fact the simpler spacetime localization and vacuum-polarization aspects of the right hand side facilitates greatly the computation of certain quantities as the entropy. In several investigations it has been noted that the localization structure along the unique lightray contained in the lightfront (the longitudinal direction) is that of a chiral QFT [10][16][17], whereas the local resolution of the transverse directions (i.e. the directions into the edge of the wedge) is the result of more recent investigations [18]. These results show that the holographic lightfront projection has no transverse vacuum polarization, a fact which is related to the radical change of the spacetime interpretation in the re-processing of the ambient algebraic substrate to its holographic projection. In other words the holographic projection leads to a system which behaves as a transverse decupled quantum mechanics; the vacuum state tensor-factorizes in the transverse and fluctuates in the lightlike direction. In algebraic terms the global lightfront algebra tensor-factorizes under transverse subdivisions and this factorization is inherited by any longitudinal finitely extended subalgebra (see below). Although a detailed derivation of the localization-structure on the horizon of the wedge requires a substantial use of theorems about modular inclusions and intersections (for which we refer to [18][21]), the tensor factorization of the horizon algebra $\mathcal{A}(H(W)) \equiv A(E \times (0, \infty))$ under transverse subdivisions $E = E_\perp \times (E_\perp \backslash E)$ (by cutting the horizon into half cylinders $E \times (0, \infty)$ along the lightlike direction with E a subset of the transverse edge) relies only on the following structural theorem in operator algebras[11]:

[10] It is the limiting case of the "causal shadow property" of spacelike surfaces.

[11] This is not the only argument for the absence of transverse fluctuations [18] but it is the most general one.

Theorem 1 *(Takesaki [19]) Let* $(\mathcal{B},\Omega)$ *be a von Neumann algebra with a cyclic and separating vector* Ω *and* $\Delta_{\mathcal{B}}^{it}$ *its modular group. Let* $\mathcal{A}\subset\mathcal{B}$ *be an inclusion of two von Neumann algebras such that the modular group* $Ad\Delta_{\mathcal{B}}^{it}$ *leaves* $\mathcal{A}$ *invariant. Then the modular objects of* $(\mathcal{B},\Omega)$ *restrict to those of* $(\mathcal{A}e_{\mathcal{A}},\Omega)$ *where* e_A *is the projection* $e_{\mathcal{A}}H=\overline{\mathcal{A}\Omega}$ *as well as to those of* $(\mathcal{C}e_{\mathcal{C}},\Omega)$ *with* $\mathcal{C}=\mathcal{A}'\cap\mathcal{B}$ *the relative commutant of* $\mathcal{A}$ *in* $\mathcal{B}$ *and* $e_{\mathcal{C}}H=\overline{\mathcal{C}\Omega}$*. Furthermore* $\mathcal{A}$ *and* $\mathcal{C}$ *tensor-factorize i.e.* $\mathcal{A}\vee\mathcal{C}\simeq\mathcal{A}\otimes\mathcal{C}$ *on* $\Omega\otimes\Omega$*.*

In the application to lightfront holography the abstract algebras are identified with concrete subalgebras of the global algebra of a QFT model: $\mathcal{B}\equiv\mathcal{A}(W)=\mathcal{A}(LF(W))$ and $\mathcal{A}\equiv\mathcal{A}(E\times(0,\infty))$, $\mathcal{C}\equiv\mathcal{A}(E^{\perp}\times(0,\infty))$. The modular group of $\mathcal{A}(W)$ is the Lorentz-boost $\Lambda_W(-2\pi t)$ which in the holographic projection becomes a dilation. The dilation invariance of the half cylinder algebras $\mathcal{A}(E\times(0,\infty))$ is geometrically obvious. These half-cylinder algebras can be shown to be *monade* subalgebras [20] (hyperfinite III_1 von Neumann factors[12]) of the type I_∞ tensor subfactors corresponding to the full two-sided cylinder. The transverse subdivisions can be refined ad infinitum. This factorization into statistically-independent half cylinders forces the localization entropy to be additive in transverse direction under arbitrary subdivisions. In d=1+3, the transverse directions carry the dimension of the area of the edge. The area density of entropy is therefore the localization entropy of an auxiliary chiral theory of the lightray or (using its Moebius invariance) of its $\dot{R}\simeq S^1$ circular compactification. Whereas the area behavior of localization entropy was known [18], the actual computation for a localization interval remained an open problem which we will address in the sequel.

There is another way (also based on modular operator algebra theory) by which the transverse tensor factorization can be obtained [21]; it uses the lightlike energy positivity and cluster factorization in the vacuum state. The interesting aspect of the above theorem is that it does not use explicitly the vacuum properties so that it could be useful in generalizing the thermal manifestations of fluctuations near the causal horizon to global KMS states (not considered in this paper).

Note that the conformal invariant chiral structure along the lightray does not imply that the ambient theory is massless. Whereas the short-distance limit (leading to critical universality classes) changes the theory, the holographic projection takes place in the same Hilbert space as the ambient theory since the particle creation/annihilation operators of the massive particles and the representation of the Poincaré group has not changed; but the interpretation in terms of localization of A_{LF} is radically different from that of A, in particular they are relatively nonlocal (which is linked to the fact that certain Poincaré transformations, including the opposite lightray translation, act non-covariantly on A_{LF}).

Different from the equal time canonical structure which breaks down for interacting properly renormalizable fields, there is no such short distance restriction on the generating field of the holographic projection; whereas the short distance behavior of canonical fields must remain close to that of free fields, fields on the lightray exist for arbitrary high anomalous dimensions. The only problem is that one cannot get to those anomalous dimensional lightfront fields by the above pedestrian restriction procedure based on the mass shell representation. In view of the fact that lighfront holography involves a very radical spacetime re-processing of the algebraic substrate, this is not surprising. In (4) the dependence of the commutator on the transverse coordinates $x_\perp$ is encoded in the quantum mechanical derivative-free delta function which is directly related to the transverse factorization of the vacuum i.e. to the factorization of the algebra of a cylinder (finite transverse extension) in lightray direction into tensor products upon subdivision into sub-cylinders. Any extensive quantity as an entropy, which behaves additively for independent subsystems is then additive in transverse direction and hence follows an area law [18][21].

There are two methods which lead to the holographic projection. One is an algebraic method in which one starts from a wedge algebra and obtains the longitudinal and transverse locality structure on the lightfront by forming modular inclusions and modular intersection of algebras. The other method is more formal (less rigorous) and requires to know the expansion of the ambient field in terms of incoming particle operators i.e. a substantial amount of scattering theory. For the first method we refer to [18]; here we opt for the more pedestrian second method which consists in taking $x_+=0$ inside the expansion in terms of the incoming fields of scattering theory Here the integration goes over the upper/lower part of the mass hyperboloid H_m and the corresponding components of $A_{in}(p)$ denote the creation/annihilation operators of the (incoming) particles. Formally, i.e. modulo problems of convergence this series defines a field on

[12] The reason for using the shorter terminolgy "monade" for the unique Murray von Neumann "hyperfinite type III_1 factor" is that one can build up fulfledged QFTs by the modular relative positioning of a finite number (3 for chiral theories, 6 for d+1+3 QFT) of monades which is a perfect QFT analog of Leibnitz view of reality.

the lightfront. For consistency reasons the commutation relation of the formal pointlike generators must be of the form[13]

$$[A_{LF}(x_+, x_\perp), B_{LF}(x_+, x'_\perp)] = \delta(x_\perp - x'_\perp) \sum_n \delta^{(n)}(x_+ - x_+) C_{nLF}(x_+, x_\perp) \tag{12}$$

where the sum goes over a finite number of derivatives of delta functions and C_{nLF} are (composite) operators of the model. The presence of the quantum mechanical δ-function and the absence of transverse derivatives expresses the transverse tensor-factorization of the vacuum i.e. all the field theoretic vacuum polarization has been compressed into the x_+ lightray direction. In view of the fact that in chiral theories the existence of pointlike field generators follows from the covariance structure of the algebraic setting [22], there seems to be no problem to construct pointlike fields in this transverse extended chiral theory along the same lines; the local structure of the commutation relations is then a consequence of locality and Moebius covariance [18].

The absence of transverse vacuum fluctuations leads to the additivity of entropy under transverse subdivisions i.e. to the notion of area density of entropy. The holography reduces the calculation of the area density to a calculation of localization entropy of a hypothetical chiral QFT on the lightray which is the Moebius covariant chiral algebra before the local resolution in transverse direction. The basic problem, which will be addressed in the next section, is how to assign a localization entropy to an interval on the (compactified) lightray.

If the absence of transverse vacuum fluctuations and the ensuing area behavior would be limited to wedge-localization, the present conceptual setting would not be so interesting. As a confidence-building extension one would like to establish these facts at least for the compact causally closed double cone region $\mathcal{D}$ which for convenience we chose symmetric around the origin. In this case there is no geometric candidate for the modular group of $(\mathcal{A}(\mathcal{D}), \Omega)$ when the underlying QFT is not conformal invariant. For conformal covariant QFTs[14] on the other hand the modular group consists of a one-parameter conformal subgroup which involves a chiral 2-fixed point Moebius transformation in the radial variables $r_\pm$ [23]. There are convincing but not rigorous arguments [24] to the extend that the modular group close to the boundary $\partial\mathcal{D}$ becomes asymptotically equal to the action of this conformal group. The upper boundary ∂D_+ is a causal horizon for $\mathcal{D}$ and the angular rotations would correspond to the transverse translations on the wedge i.e. to the directions which are free of vacuum polarization. Though analogies are helpful, the double cone situation is sufficiently different and warrants a separate presentation to which we hope to return in a separate paper.

3 The interpretation of localization-caused thermality on the horizon in terms of heat bath thermal behaviour on the lightfront

It has been known for some time that under special conditions the distinction between heat bath and localization thermality becomes blurred. One such situation has been studied in conformal two-dimensional models and named appropriately "Looking beyond the Thermal Horizon" [25] whereas a similar situation in higher dimensions was presented as "a converse Hawking-Unruh effect" [26]. The basic question is under what circumstances a heat bath KMS state associated to the translation automorphism on a global operator algebra may be interpreted as the restriction of a vacuum state on an appropriately constructed extended global algebra ("behind the horizon"); i.e. the original global algebra is viewed as a subalgebra of an extended algebra to which the vacuum state on the extended algebra is being restricted. In general the commutant $\mathcal{A}'$ of a global algebra $\mathcal{A}$ in a KMS state is antiisomorphic to the global algebra but there is no geometric interpretation of $\mathcal{A}'$. So using a somewhat colorful terminology the question is: under what circumstances can the abstract commutant $\mathcal{A}'$ be viewed as "virgin territory" behind an imagined causal horizon i.e. $\mathcal{A}_{KMS} = \mathcal{A}(\mathcal{O})_{vac}$ and $\mathcal{A}'_{KMS} = \mathcal{A}(\mathcal{O}')_{vac}$?

[13] The field generators of local transverse factorizing operator algebras must have the claimed form of the spacetime commutation relations for reasons of consisteny; in particular the appearance of derivatives in the transverse delta functions would destroy the factorization.

[14] $\mathcal{D}$ is in fact conformally equivalent to W [9].

For our purpose it is sufficient to understand this for a translational KMS state at temperature β on a chiral algebra on the lightray R with $\alpha_t(\cdot)$ implementing the linear translation on R

$$\omega_\beta(A) = (\Omega_\beta, A\Omega_\beta)\,,\ \ A, B \in \mathcal{A}(R) \tag{13}$$
$$F_{A,B}(t) = \omega_\beta(\alpha_t(A)B),\ F_{A,B}(t+i\beta) = \omega_\beta(B\alpha_t(A)) \tag{14}$$

Here the first line denotes the content of the GNS construction which associates to a state on a C^* algebra a concrete operator algebra acting cyclically on a vector Ω_β in a Hilbert space (where in the usual physicists manner we retain the same notation for the abstract operator A and its concrete Hilbert space representations). The second line is the definition of the KMS property of the state ω_β which consists in the existence of an analytic function for every pair of operators $A, B \in \mathcal{A}(R)$. Assuming that the ground state theory exists, one knows sufficient conditions under which the existence of the KMS state associated with the time translation follows; these criteria are related to quantum field theoretical phase space properties. In the case of chiral theory these properties have the simple Gibbs form $tre^{-\beta L_0} < \infty$ where L_0 is the standard notation for the rotational generator in the 3-parametric Moebius group. The theorem which geometrizes the abstract thermal commutant of the KMS representation of $\mathcal{A}(R)$ reads

Theorem 2 *The operator algebra associated with the heat bath representation of $\mathcal{A}(R)$ at temperature $\beta = 2\pi$ is identical to the vacuum representation restricted to the half-line chiral algebra*

$$(\mathcal{A}(R), \Omega_{2\pi}) = (\mathcal{A}(I), \Omega) \tag{15}$$
$$(\mathcal{A}(R)', \Omega_{2\pi}) = (\mathcal{A}(I'), \Omega)$$
$$R \to I = (0, \infty)$$
$$t \to e^{-t} \in I \tag{16}$$

The equality extends to the subalgebras if the localization structure defined in terms of the translational parametrization of R on the left hand side is mapped to the dilatational parametrization of I (16)

For the validity of this assertion it is important to be aware of the fact that (different from the groundstate representation of the global algebra which as all global vacuum representations are always of quantum mechanical type I_∞) global KMS representations are of the same type as restricted (localized) vacuum representations which are of hyperfinite type III_1.

It is straightforward to extend this theorem to translative KMS states at temperature β in which case the real line is mapped onto a semiinfinite line whose starting point depends on β. In the two dimensional version the plane is mapped into the forward light cone and the abstract thermal commutant is mapped onto the algebra of the backward light cone. The computational side of these generalizations can be found in [27]

These isomorphisms play the crucial role in mapping the type I_∞ boxed Gibbs systems of the thermodynamic limit sequence into a type I_∞ sequence which pictorially speaking approximates the semi-axes from the inside. The appealing aspect of these approximations is that whereas in higher dimensional massive theories the approximands are corresponding to other quantizations within basically the same theory[15], chiral QFTs offer to do this within the same C^* algebra by forming Gibbs states with an interpolating sequence which have an R-dependent rotational L_0 like discrete spectrum. For $R \to \infty$ the spectrum becomes dense and converges against the continuum of the translation Hamiltonian

$$H_R := L_{0,R} \underset{R\to\infty}{\to} H \tag{17}$$
$$L_{0,R} \equiv \frac{1}{R} Dil(\frac{1}{2R}) L_0 Dil(\frac{1}{2R})^{-1} = H + \frac{1}{4R^2}K \tag{18}$$

where K is obtained from H by applying the conformal inversion $K = IHI$ and L_0 corresponds to the value $R = \frac{1}{2}$. In a very interesting recent paper [28] it was shown that such a "relativistic box" interpolation is always possible for conformal theories in arbitrary spacetime dimensions and that it is deeply related to Irving Segal's attempt to use the R-extended Dirac-Weyl compactification of Minkowski

[15] Quantization boxes of different sides define different C^* algebras even though "morally" they belong to the same system.

spacetime for cosmological purposes. In the n-dimensional case H is the zero component of the energy-momentum operator and K the zero component of its conformal reflected counterpart.

The previous theorem on equivalence of the heat bath thermal with the localization thermal setting together with the transversal extended chiral nature of the lightfront algebras leads to the desired result relating the lightfront heat bath algebra at $\beta = 2\pi$ (the arguments can be generalized to chiral intervals in general position which require $\beta \neq 2\pi$) to the restriction of the vacuum to the localization algebra of the horizon of the wedge; in particular the relation between the global heat bath entropy and the localization entropy associated with the wedge horizon is

$$\begin{aligned} S_{2\pi}(LF) &= (Area \times length)\, s_{2\pi} = Area \times s_{area}(\varepsilon) \\ &\curvearrowright s_{area}(\varepsilon) \equiv |ln\varepsilon|\, s_{2\pi} \end{aligned} \tag{19}$$

where the apparent short distance singular behavior in $\varepsilon \to 0$ is just the exponentially parametrized thermodynamic length factor $\varepsilon = e^{-l}$ in the above theorem. There remains the calculation of the thermodynamic limit $s_{2\pi}$ for chiral theories which will be carried out in the next section. If one's only interest is the entropy function $s_{area}(\varepsilon)$ one may ignore the unitary transformation associated with the R-dependent dilation since the trace is invariant under unitary transformations.

One note of caution: thermodynamic KMS states on the original massive bulk matter have no simple relation to the massless case; they are belonging to different theories (apart from the case where the original bulk matter is conformal). It is only through the matter substrate-maintaining holographic encoding that the chiral conformal theory enters the discussion. The remaining question to what extend the ε has an intrinsic meaning (i.e. with an interpretation which is not added on but comes from the theory itself) will be commented on in the section and in the appendix.

4 Modular temperature-duality and the leading behavior of localization entropy

In the previous section it was shown that for chiral theories the global heat bath thermal entropy at KMS temperature $\beta = 2\pi$ and the localization entropy for an (arbitrary) interval are two sides of the same coin; the only difference is the parametrization which changes from a long distance $R \to \infty$ via $\varepsilon = e^{-R}$ to short distances. Hence The object which remains to be computed is the partition function (for computational convenience a factor 2π has been split off from β, the subscript r in the remaining β_r stands for "reduced")

$$Z_\alpha(R) \equiv tr|_{H_\alpha} e^{-2\pi\beta L_0},\ 2\pi\beta = \frac{1}{R} \tag{20}$$

Here we assumed that the chiral theory which appears in the holographic projection is "rational" i.e. its observable algebra only admits a finite number of unitarily inequivalent representations with associated representation spaces H_α From this partition function the entropy follows in the standard way

$$\begin{aligned} \rho_\alpha &\equiv \frac{e^{-2\pi\beta L_0}|_{H_\alpha}}{tr|_{H_\alpha} e^{-\beta L_0}},\ 2\pi\beta = \frac{1}{R} \\ S_\alpha(\beta) &= -tr\rho_\alpha ln\rho_\alpha = \left(1 - \beta\frac{d}{d\beta}\right) lntr|_{H_\alpha} e^{-2\pi\beta L_0} \end{aligned} \tag{21}$$

The remainder of the computation is done with the help of the temperature duality relation which maps the partition function for large temperature into one with small temperature (large β). According to Verlinde and Cardy this is done with the help of the temperature duality relation for the partition function which holds for an appropriately shifted L_0

$$\begin{aligned} \hat{Z}_\alpha(\beta) &= \sum_\gamma S_{\alpha\gamma}\hat{Z}_\gamma(\frac{1}{\beta}) \\ \hat{Z}_\alpha(\beta) &\equiv tr|_{H_\alpha} e^{-2\pi\beta\hat{L}_0},\ \hat{L}_0 \equiv L_0 - \frac{c}{24} \end{aligned} \tag{22}$$

Relations of this kind first emerged from the Kac-Peterson study of characters of loop-groups and geometrical structural arguments in favor of their general validity for rational chiral models were proposed by Verlinde. The Verlinde matrix $S_{\alpha\gamma}$ which appears in these relations is a priori not the same as Rehren's "statistics character" i.e. a numerical matrix related to the braid group statistics data. There exists however a derivation based on modular operator theory which shows that this is the case. For the convenience of the reader this derivation will be sketched below.

The remaining limit calculation $\beta \to \infty$ is almost trivial since the leading term for Z_α comes solely from the numerical $\frac{c}{24}$ contribution. The contribution of the charged sectors differ from the vacuum contribution only in the nonl-eading terms. In fact the constant term in $o(\varepsilon)$ in the entropy in the α sector turns out to be $lnS_{\alpha 0}$ [36]. Hence the holographic matter content enters the leading entropy term only through its *algebraic structure* and has no dependence on the superselected charges. Rewriting everything in terms of the ε-parametrization one obtains the result of the introduction (1) where the charge index has been omitted (since the leading behavior only involves the vacuum sector).

The best way to understand this temperature duality relation in an operator setting is to view the partition function as the zero-point correlation function in a unnormalized thermal state and to use modular theory in order to perform an *angular Euclideanization* [20]. The crucial formula is[16]

$$e^{-\tau L_0} = \Delta^{\frac{1}{4}} \tilde{\Delta}^{i\frac{\tau}{2\pi}} \Delta^{-\frac{1}{4}} = \tilde{\Delta}_c^{i\frac{\tau}{2\pi}} \tag{23}$$

in words: the modular Euclideanization of the modular group $\tilde{\Delta}^{i\tau} \subset S(2,\mathbb{R})$ associated with the subalgebra $\mathcal{A}(-1,1)$ for $\tau > 0$ is equal to contraction defined in terms of the rotational generator L_0. The latter is in turn identical to the modular group $\tilde{\Delta}_c^{i\tau} \subset SU(1,1)$ which is the $\tilde{\Delta}^{i\tau}$ group transformed into the compact z-picture ($z = e^{2\pi i\tau}$) description i.e. with a changed star-operation [20]. $\tilde{\Delta}^{i\tau}$ acts on the algebra $\mathcal{A}(0,\infty)$ for $t > 0$ as a two-sided compression. define the following rotational Gibbs state correlation functions of chiral fields Φ_k

$$\langle \Phi(\tau_1, ...\tau_n) \rangle_{\alpha,\beta_r} \equiv \frac{1}{tr|_{H_\alpha} e^{-2\pi\beta_r \hat{L}_0}} tr|_{H_\alpha} e^{-2\pi\beta_r \hat{L}_0} \prod_k e^{i\tau_k 2\pi\beta_r L_0} \Phi_k(0) e^{-i\tau_k 2\pi\beta_r L_0} \tag{24}$$

The temperature duality relation in terms of these correlations are the [20]

$$\langle \Phi(i\tau_1, ..i\tau_n) \rangle_{\alpha,2\pi\beta_r} = \left(\frac{i}{\beta_r}\right)^a \sum_\gamma S_{\alpha\gamma} \left\langle \Phi(\frac{1}{\beta_r}\tau_1, ..\frac{1}{\beta_r}\tau_n) \right\rangle_{\gamma,\frac{2\pi}{\beta_r}} \tag{25}$$

$$\langle \Phi(\tau_1, ..\tau_n) \rangle_{\alpha,2\pi\beta_r} := tr_{H_{\rho_\alpha}} e^{-2\pi\beta_r \left(L_0^{\rho_\alpha} - \frac{c}{24}\right)} \pi_{\rho_\alpha}(\Phi(\tau_1, ..\tau_n))$$

$$a = \sum_i dim\Phi_i$$

If the model has only one sector (the vacuum sector) as it is the case for multi-component current models whose maximal extensions are based on selfdual lattices, the matrix S degenerates to the identity and the terminology "temperature duality" acquires its literal meaning [20]. In case of existence of several sectors the matrix S enters through a cocycle (a charge transport around the circle [29]) and the contribution of the various sectors is governed by the matrix S [20].

For more general chiral models beyond minimal models the temperature may not be the only parameter which enters the description of thermal behavior. In theories with a rich charge structure one may need the chiral analog of chemical potentials.

Note that the localization entropy in the $\varepsilon \to 0$ limit of the auxiliary chiral theory does not depend on the length of an interval. In the presence of several intervals corresponding to stochastically independent systems, the partition functions factorized and the entropy is simply as expected the sum of the contributions from the individual intervals.

Some auto-critical remarks about our calculation of localization entropy are in order. The existence of a conformal Hamiltonian with discrete spectrum permits to replace the extrinsic quantization boxes used in the standard thermodynamic limit by intrinsic relativistic boxes i.e. sequences of states on the

[16]This formula was derived in collaboration with Wiesbrock [37], but its physical role in Euclideanization was not explored.

same C* algebra. But the picture that the smaller relativistic boxes are sitting inside the bigger ones is extrinsic. This is also a shortcoming of the usual thermodynamic limit $V \to \infty$ approach. It has been known for some time that the so-called split property allows within the QFT setting to construct localized thermodynamic limit sequences as well as their analogs for localization caused thermal aspects. These are so-called "funnel sequences" of increasing type I_∞ algebras N_i which converge against a monade

$$\mathcal{N}_1 \subset \mathcal{N}_2 \subset ... \subset \mathcal{A} \tag{26}$$

In fact the field theoretic setting permits to construct a continuous sequence $\mathcal{N}_\varepsilon$ where the type I_∞ algebra $\mathcal{N}_\varepsilon$ is canonically associated to a pair of monades $\mathcal{A}(\mathcal{O}_\varepsilon) \subset \mathcal{A}(\mathcal{O})$ where $\mathcal{O}_\varepsilon \subset \mathcal{O}$ are causally complete spacetime regions (with the larger one having a nontrivial causal complement $\mathcal{O}'$) such that ε measures the security distance (minimal spacelike distance) between the smaller inside the bigger region. The restriction of the vacuum to $\mathcal{N}_\varepsilon$ turns out to be a thermal Gibbs-like state. In the case of chiral theory ε is simply the minimal distance of a smaller interval from the two endpoints of the bigger. Certain aspects of such a funnel approximation are implicit in the work of Buchholz and Junglas [30] on proving the existence of KMS temperature states from the assumption that one knows the vacuum representation. The split method is obligatory if one wants to compute the energy or entropy for a finite split distance, whereas the above relativistic box method is expected to be restricted to the leading term. Contrary to a momentum cut-off, which catapults the theory outside any conceptual control[17], the split property creates a physical distance ε within a given local theory. But since a local theory has no elementary length, ε is not fixed by the theory. Needless to say that setting it equal to the Planck length, leads to the Bekenstein formula.

The problem with this totally intrinsic split method is that it is easy to show the existence of a funnel approximation [43] but it has turned out to be extremely hard to do computations. So the description of this method (for more details see the appendix) may be important for the future development.

5 Concluding remarks

Thermal aspects caused by the quantum field theoretic vacuum polarization at boundaries of causally complete localization regions are in several aspects different from the classical heat bath thermal behavior. In the case of a wedge region one finds that the vacuum polarization leads to an area law for energy and entropy where the area refers to the edge of the wedge. The conformal invariance of the lightfront projection reveal however an unexpected relation between a global heat bath thermal system at a fixed temperature (in our case $\beta = 2\pi$) and the thermal aspects of a system caused by vacuum fluctuations as a result of localization in the direction of the lightray; apart from the fact that the increasing lightray length contribution to the thermodynamic volume factor passes to an appropriate $|ln\varepsilon|$ factor which measures the size of the "vacuum polarization collar" as in (2), the two thermal systems are identical. The apparent short distance behavior turns out to be of pure kinematical origin since it is conformally related (together with the transverse area factor which remains unchanged under conformal transformations along the lightray) to the standard volume factor. Conformal theories inexorably intertwine short and long distances and there is no short-distance "hiding place" for new physics behind an ultraviolet cutoff as long as the conformal invariance (which is inexorably related to holography onto a horizon) is maintained. In a forthcoming second part of this paper it will be shown that these conclusions continue to hold for compactly localized regions; this will be explicitly shown for double cone localizations.

Our result shows in particular that the conceptual basis of previous calculations of localization entropy as entanglement entropy associated with the energy levels of the standard time translation Hamiltonian [32] is not sustainable. In those calculations a free scalar field is restricted to the exterior of a sphere (the model of a black hole) by simply factorizing the quantum mechanical energy levels into their inside/outside contribution. The infinity caused by vacuum fluctuations as a result of the sharp factorization is parametrized in terms of a momentum space cutoff. Choosing the latter of the order of the Planck scale the

[17] A momentum space cut-off is an extremely ill-defined concept. Even in those cases in which the local theory (e.g. through its correlation functions) is explicitly known (chiral theories, factorizing models), nobody has an idea to what manipulation on the local theory this corresponds (note that in order to stay within quantum theory positivity must be maintained).

resulting formula is consistent with the quantum interpretation of Bekenstein's classical area formula. The conceptual antinomies to the present approach to localization entropy are

- The localization-entropy should be associated with the modular group of the pair $(\mathcal{A}(\mathcal{O}),\Omega)$ and this is not the Hamiltonian which implements the usual time translation but rather an intrinsically determined $\mathcal{O}$-dependent modular "Hamiltonian" (a Lorentz boost in the case of a $\mathcal{O}$ = wedge region). The quantum mechanical interpretation of QFT in the rigid sense of filling energy levels is not supported by the property of local covariance of QFT which attributes dynamical properties to the vacuum. A particular spectacular case of a misleading use of QFT in the spirit of a kind of relativistic quantum mechanic is the computation of the vacuum fluctuation contribution to the cosmological term, for a fundamental criticism see [31]. It is very important that the interpolating discrete spectrum "Hamiltonians" (those in the funnel) interpolate the limiting $(\mathcal{A}(\mathcal{O}),\Omega)$ modular "Hamiltonian".

- The above derivation [32] does not reveal that the origin of the area factor is the accumulation of vacuum polarization at the horizon, the pivotal localization aspect was not taken into account.

- The [32] calculation fails to relate the divergence of the entropy (after splitting off the transverse area factor) to the volume factor of a heat bath system on the lightfront and in this way mystifies the short distance aspect.

Instead of explaining the a kinematical $|ln\varepsilon|$ divergence in the given local QFT in terms of an inverse thermodynamic limit length factor the above calculation unnecessarily mystifies the situation by invoking a hypothetical nonlocal cutoff theory.

In most of the work on black hole entropy the quantum interpretation of the classical Bekenstein area law has been the point of departure and the local quantum aspects had to be adjusted in accordance with this classical presetting. In this work we have inverted this order. In the context of a Schwarzschild black hole this would mean that we start with a stationary state which is stationary with respect to the Schwarzschild time development and regular on the event horizon whose restriction to the outside is a KMS state at the Hawking temperature. The split property would in the previous sense would then lead to a one-parametric family of area densities for energy and entropy and the requirement of identifying the entropy with the Bekenstein classical area formula (whose origin is classical differential geometry and not quantum physics) can be interpreted as fixing one particular ε. The main problem is to search for a new theory in which this extrinsic fixing becomes an intrinsic necessity. In order to say that certain string theory solutions contribute to this question one has to make sure that they have the KMS thermal property otherwise the solution has nothing to do with thermal aspects of black holes.

A prerequisite for the applicability of localization thermality to black holes is that the latter can be treated as stationary so that one stays within equilibrium thermodynamics. But since black holes originate from collapsing stars this is strictly speaking not true. For a rotational symmetric collapse Fredenhagen and Haag have been able to show the existence of a Hawking radiation at lightlike infinity which originates from the onset of collapse. The state which describes such a history is certainly not a KMS state and the fact that there is a Hawking radiation with an energy spectrum associated with a Hawking temperature does not automatically imply that the above entropy considerations are valid. Perhaps the Fredenhagen-Haag arguments can be extended to include entropy.

Appendix – Some relevant facts about modular theory and the split property

For the convenience of the reader we mention some mathematical concepts concerning modular aspects of operator algebras [33] which have been freely used in the main text. Modular theory associates to a "standard" pair $(\mathcal{A},\Omega)$ of an operator in a Hilbert space and a vector Ω on which it acts in a cyclic and separating (the only annihilator of Ω in $\mathcal{A}$ is the zero operator) a (one-parametric) unitary modular group Δ^{it} and an antiunitary idempotent operator J which result from the densely defined closable antilinear Tomita S-operator by polar decomposition according to

$$SA\Omega = A^{*}\Omega \tag{27}$$
$$S = J\Delta^{\frac{1}{2}},\ \ \sigma_t(\mathcal{A}) = Ad\Delta^{it}\mathcal{A} = \mathcal{A},\ J\mathcal{A}J = \mathcal{A}',\ J^2 = 1$$

where the second line contains in addition to the polar decomposition formula also the two main statements of the Tomita-Takesaki theory: the Ad action of the modular unitary defines the *modular automorphism* of $\mathcal{A}$ and the modular inversion J an antiunitary isomorphism onto the commutant (thus showing that in a standard situation the algebra is antiisomorph with its commutant, which excludes the irreducibility of a standard situation). Certain aspects of the spectrum of the modular group determine the "type" (isomorphism class) of the operator algebra; in particular for type III_1 the spectrum of the infinitesimal generator is purely continuous and covers the positive semi-axis.

From case studies and general structural arguments one knows that the local algebras of QFT are isomorphic to the unique hyperfinite type III_1 factor von Neumann algebra (which specifically in this quantum field theoretic setting) for brevity as well for more profound reasons (see introduction) is referred to as a *monade*; they are standard with respect to the vacuum (the Reeh-Schlieder property). The global algebras of QFT on the other hand are of type I_∞, but they loose this quantum mechanical property in thermal states; in this case they acquire the same algebraic structure as the local algebras namely hyperfinite type III_1. Whereas the commutant in the heat-bath thermal situation remains an abstract thermal shadow world, the commutant in the localized vacuum situation is geometric and in typical cases equal to the algebra of the causal disjoint (Haag duality). The only case in which the modular group acts geometrically independent of the particular model of QFT is the wedge situation $(\mathcal{A}(\mathcal{W}),\Omega_{res})$; there are however many "partially geometric" situations in which the reference state is different from the vacuum and the corresponding modular group acts as a diffeomorphism if restricted to the subalgebra [20].

The modular theory was significantly enriched by the concept of modular inclusion and of modular intersection [34][35]. These structures are related to a generalization of the Takesaki theorem in section 2, instead of requiring that the modular group of the larger algebra acts as a one-parametric automorphism group on the smaller, one only assumes that it contract the algebra in one direction ($\pm$ halfsided modular inclusions). These structures can then be used to show a QFT with all its structural richness including its covariances and geometric aspects can be obtained from the pure algebraic modular positioning of a finite number of copies of representations of the monade in a joint Hilbert space [38]. This view seems to be very powerful for a better understanding about the relation between algebraic properties, geometry and thermal aspects and may well lead to a third path towards quantum gravity (a modular path).

A closely related modular concept which permits to implement many ideas (which in the Lagrangian approach required to imagine momentum space cutoff[18]) within the given local QFT is the so called *split property*. Although it will not be used in this article for computations (because it belongs to those modular properties which still resist computational attempts) it provides by far the best conceptual setting for localization entropy. Let us finally close this section with some remarks on the split property and closely related previous attempts to generalize the notion of entropy beyond the time-honored von Neumann entropy definition.

There are several candidates for such a definition with a similar physical-intuitive content. One attempt employs the framework of the Connes-Narnhofer-Thirring entropy [39][40] which is a kind of relative entropy [41][42]; it associates (adapted for the present purpose) to an inclusion of two localized algebras and a state ω denoted as $(\mathcal{A}\subset\mathcal{B},\omega)$ an entropy $H_{\mathcal{B}}(\omega,\mathcal{A})$. The definition is such that in case of $A=B$ being quantum mechanical type I_∞ it agrees with the von Neumann entropy where ω represented by a density matrix. A closely related idea which is based on the more restrictive assumption of a "split" inclusion $(\mathcal{A}\subset\mathcal{B},\omega)$ is due to Doplicher and Longo [43]. This split property, which later turned out to be the consequence of a physical phase space degree of freedom behavior [45] in local quantum physics, gives rise to a functorial related type I_∞ algebra $\mathcal{N}$

$$
\begin{aligned}
&\mathcal{A}\subset\mathcal{N}\subset\mathcal{B} \qquad (28)\\
&\mathcal{A}=\mathcal{A}(\mathcal{O}),\mathcal{B}=\mathcal{A}(\hat{\mathcal{O}})
\end{aligned}
$$

where the second line specifies the localization properties of the algebras in terms of spatial geometric inclusions $\mathcal{O}\subset\hat{\mathcal{O}}$. The $\mathcal{N}$ (which can be explicitly written in terms of modular data [43]) describes a sharply $\mathcal{O}$-localized algebra $\mathcal{A}(\mathcal{O})$ surrounded by a halo of "fuzzy localization" which extends up to $\hat{\mathcal{O}}$

[18] A ultraviolet cutoff is a formal devise which tries to represent the actual theory as a limit of mathematically controllable theories, but even in those low-dimensional cases where this can be achieved the physical status of the approximands is not known.

in which the vacuum-polarization-"atmosphere" is permitted to thin out softly[19]. The vacuum state ω restricted to the type I_∞ operator algebra $\mathcal{N}$ is a density matrix $\rho(\mathcal{N},\omega)$ (in terms of the tracial weight formalism) to which the von Neumann definition of the entropy may be applied ([43] page 511).

This split property is intimately linked with the notion of correlation-free product states. Its functorial construction starts from the assumption that the uncorrelated factor state (the split state) defined by

$$\omega_{sp}(AB') := \omega(A)\omega(B') \ for \ A \in \mathcal{A} = \mathcal{A}(\Lambda),\ B' \in \mathcal{B}' = \mathcal{A}(\bar{\Lambda}) \tag{29}$$

is a normal state[20] (i.e. has natural continuity properties with respect to the involved operator algebras); such a state according to modular theory possesses a distinguished vector representative $\eta \in \mathcal{P}(\mathcal{A} \vee \mathcal{B}', \Omega)$ in the natural cone associated with the algebra generated by $\mathcal{A}$ and $\mathcal{B}'$ and the vacuum i.e. $\omega_{sp}(AB') = \langle \eta \,| AB' |\, \eta \rangle$. The properties of this vector lead to the unitary equivalence W of the vacuum representation of the algebra $\mathcal{A} \vee \mathcal{B}'$ with the tensor product representation $\mathcal{A} \otimes \mathcal{B}'$ on $H \otimes H$; the functorial related type I_∞ factor algebra $\mathcal{N}$ turns out to be simply $W(B(H) \otimes 1)W^*$ whereas $P(\Lambda) = W(1 \otimes P_\Omega)$ is the projector onto the factor space $\overline{N\,|\eta\rangle}$.

Here we used local commutativity of $\mathcal{A}$ and $\mathcal{B}'$ algebras in order to arrive at stochastic independence in the sense of existence of correlation-free product states. It is interesting to note that with a stronger notion of absence of correlation [44] one is able to characterize the commutativity of algebras in terms of existence of correlation-free states. This shows that local commutativity is inexorably linked with stochastic independence for causally disjoint observation i.e. that some form of relativistic causality is not an option. The split construction is very important to reconcile a KMS localization temperature with a finite localization entropy. Strictly speaking the spatial interpretation of the thermodynamic limit sequence (and the related analogous inner exhaustion of the vacuum state restricted to a sharply localized algebra by a sequence of type I_∞ algebras) is "metaphorical". But the split property allows to replace this argument by a completely autonomous one in which the box sequence is replaced by a sequence of fuzzily localized type I_∞ algebras which form a genuine inclusive exhausting sequence ("funnel") inside the "monade " i.e. one for which the approximating systems are spatially increasing towards the open thermal system.

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