ISSN 0029-3865

CBPF-NF-009/84

A POSSIBLE INTERACTION BETWEEN SPIN-1/2 AND SPIN-1/2 FIELDS*

by

N. Fleury[2] and J. Leite Lopes[1,2,3]

[1]Centro Brasileiro de Pesquisas Físicas- CBPF/CNPq
Rua Dr. Xavier Sigaud, 150
22290 - Rio de Janeiro, RJ - Brasil

[2]Centre de Recherches Nucléaires, Université Louis Pasteur
67037 - Strasbourg, France

[3]Instituto de Física
Universidade de São Paulo
Cidade Universitária
01498 - São Paulo, SP

* Dedicated to Mario Schönberg on his seventieth birthday.

ABSTRACT

A straightforward extension of the standard Weinberg-Salam model to Rarita-Schwinger formalism, is a heuristic way to obtain electroweak currents for spin-3/2 leptons. We postulate a new interaction between spin-1/2 and spin-3/2 particles that maintains the SU(2) x U(1) gauge invariance, and we obtain a possible form for the interaction involving these two types of particles and the gauge fields. This takes place through a coupling which involves derivatives of the electromagnetic field and the corresponding coupling constant is inversely proportional to the mass of spin-3/2 particles. Several reactions are possible with this new interaction such as the radiative decay of a charged Rarita-Schwinger particle. Other reactions are corrections to well-known processes : for instance, in the usual Compton effect, the fermionic virtual line can be replaced by the corresponding spin-3/2 one. But, due to the large mass of the spin-3/2 lepton, these corrections are significantly non negligible only at ultra relativistic energies. On the other hand, in the low energy limit this correction gives fortunately no contribution to the Thomson cross section because the coupling $\frac{3}{2} - \frac{1}{2} - \gamma$ is proportional to the photon's momentum. If one considers all the possible couplings of these fields, one can combine them in diagrams of lowest orders whose theoretical predictions have to be submitted to experimental pressure.

The invention of supersymmetric[1] theories is at the origin of a renewed interest in spin-3/2 fieds. Indeed, the fermion-boson symmetry postulated in these theories associates to a state of a particle of mass M and spin J, three other states with mass M and spin J-1/2, J, J+1/2 respectively. Spin-3/2 particles are thus predicted in the massive multiplet (1, 1, 1/2, 3/2) defined by $j = 1$ and in the multiplet (1, 2, 3/2, 3/2) corresponding to $j = 3/2$. Massless particles with helicity $\lambda = 3/2$ exist in multiplets ($\pm$ 3/2, $\pm$ 1/2).

Furthermore, the experimental discovery of a new spin-1/2 lepton, the tauon, with mass higher than that of baryons raises the question whether leptons are not composite states of sub-leptonic constituents which might also exist in excited spin-3/2 states[2].

From a theoretical point of view, for these high spin fields, we are actually left with remaining well-known difficulties concerning mainly the quantization of the field theory[3]. If the renormalizability of a theory can not be considered in any case as a criterion of truthfulness, it is however necessary condition for doing consistent calculations in a perturbative model. In our case, we consider the results obtained as a possible guide for experimental search of such spin-3/2 leptons, which so far have not yet been systematically undertaken. To illustrate our step, we keep in mind that the effective point like Fermi's model, although describing well β-decay, was later found out to be a non renormalizable theory ; higher-order corrections led to divergent calculations, and the intermediate boson theory, partially inspired with QED, could not solve this problem. It turned out that the solution came with the unification of weak and EM interactions in a gauge theory ã la Yang-Mills, the bosons having their mass through a spontaneous breaking of the gauge symmetry[4].

In this paper, we apply the standard SU(2) x U(1) model of Weinberg-Salam to the Rarita-Schwinger (abbreviated into R-S) formalism for spin-3/2 particles in order to obtain possible properties satisfied by these fields and the form of the corresponding weak and EM currents. Although in this case the SU(2) ⊗ U(1) theory will not be renormalizable it may present a heuristic interest.

We then postulate an interaction between a spin-3/2 and a spin-1/2 lepton of the same family. This interaction is built in such a way that the above gauge invariance of the theory is maintained. The exact form of the different couplings is given explicitely. This opens the way to straightforward calculations of cross sections for reactions involving spin-3/2 and spin-1/2 particles and we give some possible graphs which have to be considered.

THE FEYNMANN PROPAGATOR FOR SPIN-3/2 FIELDS

A spin-3/2 free field of masse M is described by a vector-spinor $\psi_a^\mu(x)$ which obeys Dirac's equation :

$$(i\,\gamma^\mu\,\partial_\mu - M)\psi^\nu(x) = 0 \tag{1a}$$

and two constraints :

$$\partial_\nu\,\psi^\nu(x) = 0 \tag{1b}$$

$$\gamma_\nu\,\psi^\nu(x) = 0 \tag{1c}$$

which give in a Lorentz invariant way the right number of independant components of the field $\psi^\nu(x)$ (we have omitted the spinor index a).

We have shown elsewhere[5)] that these field equations may be obtained form a unique lagrangian. The method used is similar to the one of the spin-1 field : we prove that there exists a scalar and hermitic lagrangian built from the available elements of the theory (the tensor-spinors, pseudo tensor-spinors plus the duals) which generates a first order equation in the R-S field and gauge invariant in the limit of a zero mass and which is the only one to satisfy these conditions.

This lagrangian is :

$$L_{RS} = - \bar{\psi}_\alpha \, \varepsilon^{\alpha\mu\nu\beta} \, \gamma^5 \, \gamma_\mu \, (\partial_\nu + i \, \frac{M}{2} \, \gamma_\nu)\psi_\beta \quad . \tag{2}$$

From which we obtain :

$$\varepsilon^{\alpha\mu\nu\beta} \, \gamma^5 \gamma_\mu \, (\partial_\nu + \frac{iM}{2} \, \gamma_\nu)\psi_\beta = 0 \tag{3}$$

This is equivalent to the system (1). These are the equations used in supergravity when $M = 0$ and ψ_β is a Majorana field.

The integral equation which describes the Feynman propagation of the vector spinor defined by equ. (3) is the following :

$$\theta(t_2 - t_1)\psi^\alpha_{(+)}(x_2) = i \int_{t_2>t_1} S_F^{\alpha\mu}(x_2 - x_1)\varepsilon_{\mu\nu\beta\lambda} \, \gamma^5 \, \gamma^\nu \, \delta^\lambda_o \, \psi^\beta_{(+)}(x_1) d^3x_1 \tag{4a}$$

for positive energy solutions and

$$\theta(t_1 - t_2)\psi^\alpha_{(-)}(x_2) = - i \int_{t_2>t_1} S_F^{\alpha\mu}(x_2 - x_1)\varepsilon_{\mu\nu\beta\lambda} \, \gamma^5 \, \gamma^\nu \, \delta^\lambda_o \, \psi^\beta_{(-)}(x_1) d^3x_1 \tag{4b}$$

for negative energy solutions. $S_F^{\alpha\mu}$ is the Feynman propagator for massive spin-3/2 fields given by[6] :

$$\begin{cases} S_F^{\mu\nu}(x_2 - x_1) = - i \, (i \, \gamma\cdot\partial_2 + m) \, \Delta_F^{\mu\nu}(x_2 - x_1) & \text{(5a)} \\ \Delta_F^{\mu\nu}(x_2 - x_1) = (g^{\mu\nu} - \frac{1}{3} \, \gamma^\mu \, \gamma^\nu - \frac{i}{3M} \, \gamma^\mu \, \partial^\nu + \frac{i}{3M} \, \gamma^\nu \, \partial^\mu + \frac{2}{3M^2} \, \partial^\mu \, \partial^\nu) \, \Delta_F(x_2 - x_1) & \text{(5b)} \end{cases}$$

We notice that in view of the ocurrence of the double differentiation of Δ_F in equ. (5b) the propagator $S_F^{\mu\nu}$ attributes to processes involving this propagator non renormalizable divergences similar to those of massive vector fields.

THE HEURISTIC SALAM-WEINBERG CURRENTS FOR SPIN-3/2 FIELDS

In order to find out possible forms of weak and electromagnetic couplings of R-S particles we will use, as a heuristic method, the standard SU(2) x U(1) model : this means that we postulate, besides the familiar Higgs and vector gauge fields, a left-handed isospinor of spin-3/2 fields $\nu_{3/2}$ and $e_{3/2}$ which we denote by ν^μ and e^μ respectively :

$$\mathscr{L}^\mu = \begin{pmatrix} L & \nu^\mu \\ L & e^\mu \end{pmatrix} \qquad (6a)$$

and a right-handed isoscalar :

$$\mathscr{R}^\mu = R\, e^\mu \qquad (6b)$$

L and R are the usual helicity projection operators defined by

$$L, R = \frac{1 \mp \gamma_5}{2} \qquad (6c)$$

Given the form (2) of the free matter field lagrangian, we can therefore write the fermionic part of the lagragian :

$$\mathbf{L}^F = - \varepsilon^{\mu\nu\rho\sigma} (\bar{\mathscr{L}}_\mu \gamma_5 \gamma_\nu D_\rho^L \mathscr{L}_\sigma + \bar{\mathscr{R}}_\mu \gamma_5 \gamma_\nu D_\rho^R \mathscr{R}_\sigma) \qquad (7a)$$

where the expressions for the coderivatives $D_\rho^{L,R}$ are :

$$\begin{cases} D_\rho^L = \partial_\rho + i\, g \frac{\vec{\tau}}{2} \vec{B}_\rho - \frac{ig'}{2} C_\rho \\ D_\rho^R = \partial_\rho - i g' C_\rho \end{cases} \qquad (7b)$$

g (resp. g') is the coupling constant for SU(2) (resp. U(1)) group having the gauge fields $\vec{B}_\rho$ (resp. C_ρ).

Apart from the usual terms corresponding to the Higgs (ϕ) and vector gauge fields, the lagrangian will contain a Yukawa-type-term for the spin-3/2 field in interaction with the Higgs field ϕ of the form :

$$\mathbf{L}_y = -\frac{i}{2} G \quad \varepsilon^{\mu\nu\rho\sigma} \bar{\mathscr{L}}_\mu \gamma_5 \gamma_\nu \gamma_\rho \mathscr{R}_\sigma \phi \; + \text{h.c} \qquad (8)$$

Each of these lagrangians is invariant in the local gauge group $G = SU(2) \times U(1)$, i.e under the transformations :

$$\begin{cases} \vec{B}_\mu \cdot \frac{\vec{\tau}}{2} \to \vec{B}'_\mu \cdot \frac{\vec{\tau}}{2} = U(x) \{ \vec{B}_\mu \cdot \frac{\vec{\tau}}{2} - \frac{i}{g} \partial_\mu \} U^{-1}(x) \\ C_\mu \to C'_\mu = V(x) \{ C_\mu - \frac{i}{g'} \partial_\mu \} V^{-1}(x) \\ \mathscr{L}^\mu \to \mathscr{L}'^\mu = V^{-1/2}(x)\, U(x)\, \mathscr{L}^\mu \\ \mathscr{R}^\mu \to \mathscr{R}'^\mu = V^{-1}(x) \quad \mathscr{R}^\mu \\ \phi \to \phi' = V^{1/2}(x)\, U(x)\, \phi \end{cases} \qquad (9)$$

where $U(x) = e^{ig\vec{\Lambda}(x)\cdot \frac{\vec{\tau}}{2}}$, $V(x) = e^{ig'\alpha(x)}$ are transformation of the groups SU(2) and U(1) respectively.

In analogy with the classical theory for spin-1/2, we obtain :

a) the mass term, derived from $\mathbf{L}_y$ (equ. (8)) for the matter spin-3/2 field :

$$\mathbf{L}_M = -\, i\, \frac{M}{2}(e)\, \varepsilon^{\mu\nu\rho\sigma} \quad \bar{e}_\mu \gamma_5 \gamma_\nu \gamma_\rho e_\sigma \qquad (10a)$$

where

$$M(e) = \lambda G \qquad (10b)$$

is the mass of the spin-3/2 charged field e^{μ} in terms of the Yukawa-Higgs coupling constant G, and λ the vacuum expectation value of the neutral Higgs field.

In this model the spin-3/2 particle of a Dirac neutrino-type remains massless, that is

$$M(\nu) = 0 \tag{10c}$$

b) From $\mathbf{L}^{F}$ (equa. (7a)) we obtain the EM and weak currents associated with these R-S fields, contained in the lagrangian piece $\mathbf{L}_c$:

$$\mathbf{L}_c = -\frac{g}{\sqrt{2}} (W^-_\rho J^{-\rho}_W + h.c) - \frac{1}{2}\sqrt{g^2 + g'^2}\ Z_\rho J^\rho_z - e A_\mu J^\mu_{em} \tag{11a}$$

where the value of the currents associated with the physical fields $W^{\pm}$, Z^0 and γ is :

$$J^{-\rho}_W = i\, \varepsilon^{\mu\nu\rho\sigma}\ \bar{e}_\mu \gamma_5\gamma_\nu (L\, \nu_\sigma) \tag{11b}$$

$$J^\rho_z = i\, \varepsilon^{\mu\nu\rho\sigma}\left[(\overline{L\,\nu})_\mu \gamma_5\gamma_\nu (L\,\nu)_\sigma + \bar{e}_\mu \gamma_5\gamma_\nu \left(\frac{3g'^2 - g^2}{2(g^2 + g'^2)} + \frac{1}{2}\gamma_5\right) e_\sigma\right] \tag{11c}$$

$$J^\rho_{em} = i\, \varepsilon^{\mu\nu\rho\sigma}\ \bar{e}_\mu \gamma_5\gamma_\nu e_\sigma \tag{11d}$$

and the electrique charge has the usual value :

$$e = \frac{g\, g'}{\sqrt{g^2 + g'^2}}$$

We notice that :

1) The electromagnetic current is conserved and this results from the identity :

$$\varepsilon^{\alpha\nu\beta\mu}\gamma_5\gamma_\mu = i(\gamma^\alpha\gamma^\nu\gamma^\beta - g^{\alpha\nu}\gamma^\beta + g^{\alpha\beta}\gamma^\nu - g^{\nu\beta}\gamma^\alpha) \qquad (12)$$

and the equations (1) :

$$\partial_\rho J^\rho_{em} = 0$$

2) The weak current has a vectorial and axial part with the standard linear combination for the charged R-S field, while only the left handed part of the neutral R-S field occurs.

c) Having the usual gauge group in the same representation, all that concerns the gauge fields and the Higgs field remains unchanged as compared to the classical spin-1/2 theory.

d) A Majorana mass may be attributed to the spin-3/2 neutrino $\nu_{3/2}$ by the introduction of a coupling with a Higgs triplet[6,7].

THE VERTEX $e_{3/2} - e_{3/2} - \gamma$

Let us come back to the Lagrangian for EM interaction and consider the vertex given in Fig. 1 for an interaction $\gamma - e_{3/2} - e_{3/2}$.

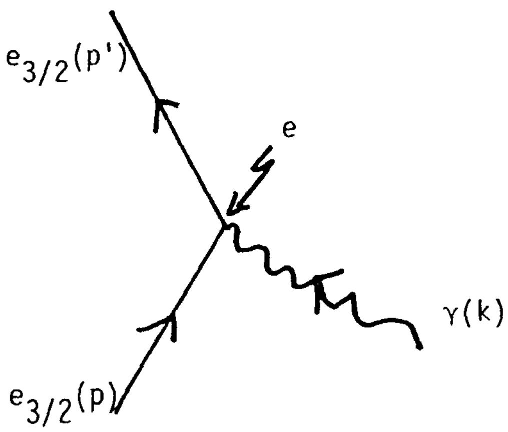


FIGURE 1 : The vertex $e_{3/2} - e_{3/2} - \gamma$

According to equation (11a), it is described by

$$\mathbf{L}_{\gamma\text{-}3/2\text{-}3/2} = - \, ie \, \varepsilon^{\mu\nu\rho\sigma} \, \bar{e}_{\mu} \, \gamma_5 \gamma_\nu \, e_\sigma \, A_\rho \tag{13}$$

For external fermionic lines, it can be rewritten in the momentum space (momenta are indicated in parenthesis), in a much more handling way as a result of the condition (1c) :

$$\mathbf{L}^{ext}_{\gamma\text{-}3/2\text{-}3/2} = e \;\bar{e}_{\mu}\,(p')\, \not{A}(k)\; e^{\mu}(p) \tag{14}$$

The form (13) is precisely what we obtain when applying minimal coupling to $\mathscr{L}_{RS}$ given by (2). We can however construct terms containing A_μ derivatives, analogous to those which give a contribution to the magnetic moment of the nucleon. In the usual spin-1/2 theory, this is given by the Pauli term :

$$\bar{\psi}\,\sigma_{\mu\nu}\, D^{\mu}D^{\nu}\,\psi \simeq e\,\bar{\psi}\,\sigma_{\mu\nu}\,\psi\, F^{\mu\nu}$$

We can therefore postulate for the R-S fields :

$$\mathbf{L}' = - \,\frac{i}{4M}\,\varepsilon^{\mu\nu\rho\sigma}\,\bar{\psi}_\alpha\,\gamma_5\gamma_\mu\gamma_\nu\, D_\rho D_\sigma\,\psi^\alpha \tag{15a}$$

which turns out to be, after some calculations :

$$\mathbf{L}' = \frac{e}{2M}\,\bar{\psi}_\alpha\,\sigma^{\mu\nu}\,\psi^\alpha\, F_{\mu\nu} \tag{15b}$$

Obviously, such a coupling to the EM field would be at the origin of an anomalous magnetic moment for the spin-3/2 lepton.

THE VERTEX $e_{3/2} - e_{1/2} - \gamma$

From Fig. 1 we see that another possible graph is obtained if one spin-3/2 line is replaced by a spin-1/2 line

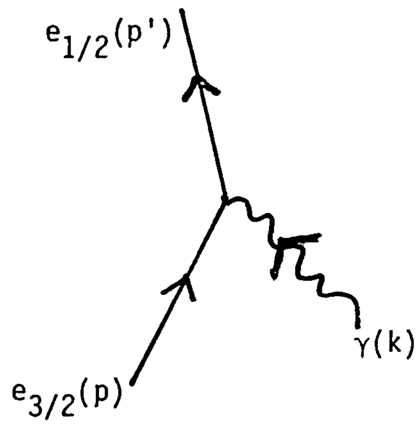


FIGURE 2 : The vertex $e_{1/2}$- $e_{3/2}$ - γ

Clearly, this coupling is not given by the previous model, and we have to make some hypothesis.

We postulate that the interaction between the charged R-S field, the corresponding spinor and the EM field is given by the lagrangian

$$\mathbf{L} = - \kappa\, \varepsilon^{\mu\nu\rho\sigma}\, \bar{e}_{3/2\,\mu}\, \gamma_5 \gamma_\nu\, D_\rho D_\sigma\, e + \text{h.c} \tag{16}$$

where κ is a coupling constant and D_μ is the coderivative associated with the electromagnetic field, $D_\mu = \partial_\mu + ie\, A_\mu$. Equ. (16) is nothing but the free RS lagrangian (2) in which we have made the substitution of one spin-3/2 field ψ_β by $D_\beta\, \psi$ where ψ is a spin-1/2 field :

$$\psi_\beta \rightarrow D_\beta\, \psi \tag{17}$$

Making use of the Riemann tensor in that simple case we have, after an integration by parts :

$$\mathbf{L} = - \kappa\, e\, j^\sigma_{em}\, A_\sigma \tag{18a}$$

where

$$j^\sigma_{em} = i\, \varepsilon^{\mu\nu\rho\sigma}\, \partial_\rho [e_{3/2\mu}\, \gamma_5 \gamma_\nu\, e_{1/2}] \tag{18b}$$

is a trivially conserved current.

In the case when the vertex of Fig. 2 is involved in such a manner that the fermionic lines are external, (18a) can be written in the following more suitable forme for calculations :

$$\mathbf{L}^{ext}_{\gamma-1/2-3/2} = + i \kappa e \bar{e}^{\rho}_{3/2}(p')(p_\rho \gamma^\sigma + M g^\sigma_\rho) e_{1/2}(p) \cdot A_\sigma(k) \qquad (19)$$

where we have neglected the mass of spin-1/2 lepton as compared to that of the corresponding spin-3/2 one[8]. Simple dimensional considerations show that κ behaves like the inverse of a mass.

This allows us, for instance, to write down the matrix element for the radiative decay of a spin-3/2 charged particle.

THE VERTEX $L_{3/2} - \ell_{1/2} - W, Z, \gamma$

After having determined a possible form for the $3/2 - 1/2 - \gamma$ interaction, we may ask what can be the coupling of the charged and/or neutral spin-1/2, spin-3/2 leptons with the allowed gauge fields $W^\pm$, Z^0 and γ.

For this purpose, we generalize the previous rule : assuming the local $G = SU(2) \times U(1)$ invariance, we postulate the above interaction is given by :

$$\mathbf{L} = - \kappa \varepsilon^{\mu\nu\rho\sigma} \left[\bar{\mathscr{L}}_\mu \gamma_5\gamma_\nu D^L_\rho D^L_\sigma \mathscr{L} + \bar{\mathscr{R}}_\mu \gamma_5\gamma_\nu D^R_\rho D^R_\sigma \mathscr{R} \right] \qquad (20)$$

where $\mathscr{L}$ and $\mathscr{R}$ are the corresponding spin-1/2 quantities of (6a) and (6b).

The Riemannian tensors for the coderivatives (7b) are :

$$[D^L_\rho, D^L_\sigma] = -ig\ \mathscr{B}_{\rho\sigma} + i\frac{g'}{2}\ \mathscr{C}_{\rho\sigma} \tag{21a}$$

$$[D^R_\rho, D^R_\sigma] = ig'\ \mathscr{C}_{\rho\sigma} \tag{21b}$$

where

$$\begin{cases} \mathscr{B}_{\rho\sigma} = \frac{\tau_i}{2}\,(\partial_\sigma B^i_\rho - \partial_\rho B^i_\sigma + g\,\varepsilon^{ijk}\,B^j_\rho\,B^k_\sigma) & (21c) \\ \mathscr{C}_{\rho\sigma} = \partial_\sigma C_\rho - \partial_\rho C_\sigma & (21d) \end{cases}$$

are the usual tensors associated with the gauge fields B^i_ρ and C_ρ .

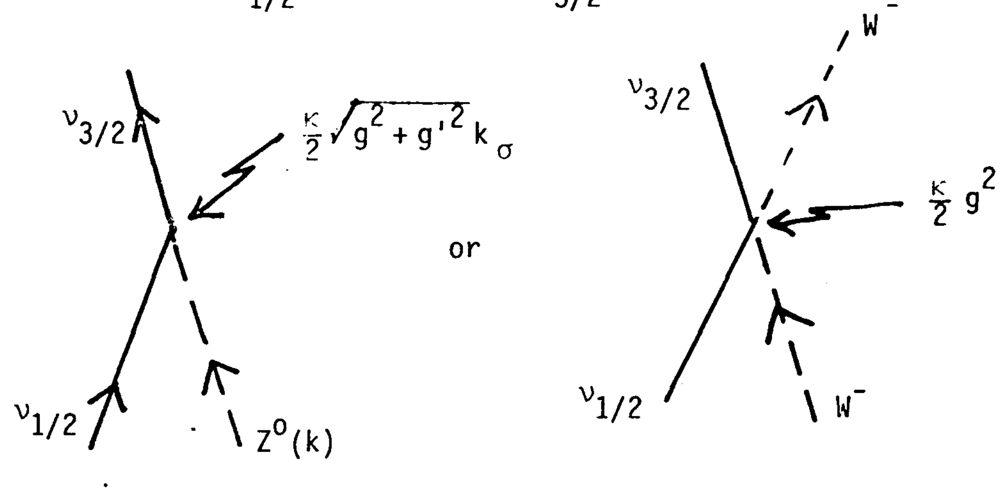
Reexpressed in term of the physical fields W, Z and γ, we have afer straightforward computations, in momentum space :

$$\begin{aligned} \mathbf{L} = \kappa\,\varepsilon^{\mu\nu\rho\sigma}\Bigg[& \frac{1}{2}\Big(\sqrt{g^2+g'^2}\,k^{(z)}_\sigma\,Z_\rho + g^2\,W^+_\rho\,W^-_\sigma\Big)(\overline{L\nu_\mu})\,\gamma_5\gamma_\nu(L\nu) \\ & + \frac{g}{\sqrt{2}}\,\Big(k^{(w)}_\sigma\,W^+_\rho - \frac{g^2}{\sqrt{g^2+g'^2}}\,W^+_\rho\,Z_\sigma - e\,W^+_\rho\,A_\sigma\Big)(\overline{L\nu_\mu})\,\gamma_5\gamma_\nu\,e \\ & + \frac{g}{\sqrt{2}}\Big(k^{(w)}_\sigma\,W^-_\rho + \frac{g^2}{\sqrt{g^2+g'^2}}\,W^-_\rho\,Z_\sigma + e\,W^-_\rho\,A_\sigma\Big)\,\bar{e}_\mu\,\gamma_5\gamma_\nu(L\nu) \\ & + \bar{e}_\mu\,\gamma_5\gamma_\nu\Bigg(\Bigg(\frac{3g'^2-g^2}{4\sqrt{g^2+g'^2}}\,k^{(z)}_\sigma\,Z_\rho + \frac{g^2}{4}\,W^+_\sigma\,W^-_\rho - e\,k^{(\gamma)}_\sigma\,A_\rho\Bigg) + \\ & \qquad + \frac{1}{4}\,\gamma_5\,\Big(\sqrt{g^2+g'^2}\,k^{(z)}_\sigma\,Z_\rho - g^2\,W^+_\sigma\,W^-_\rho\Big)\Bigg)e\Bigg] \end{aligned} \tag{22}$$

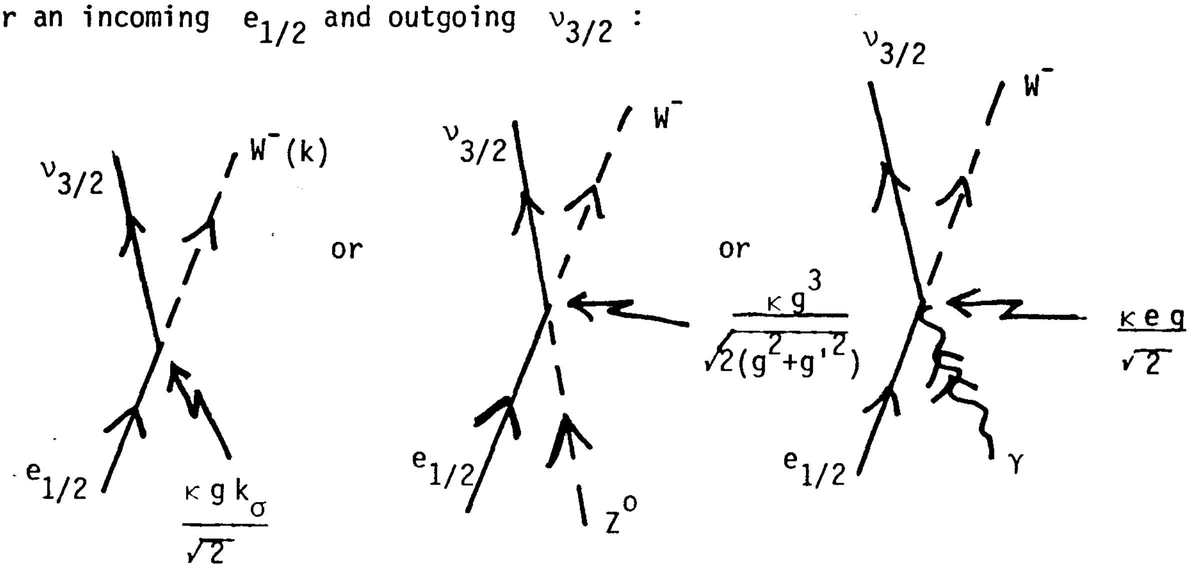
Here we have omitted the spin value label; the space-time index on the field indicates that is a R-S field.

All the possible couplings contained in equ. (22) can be more explicitely exihibited in graphs, we have :

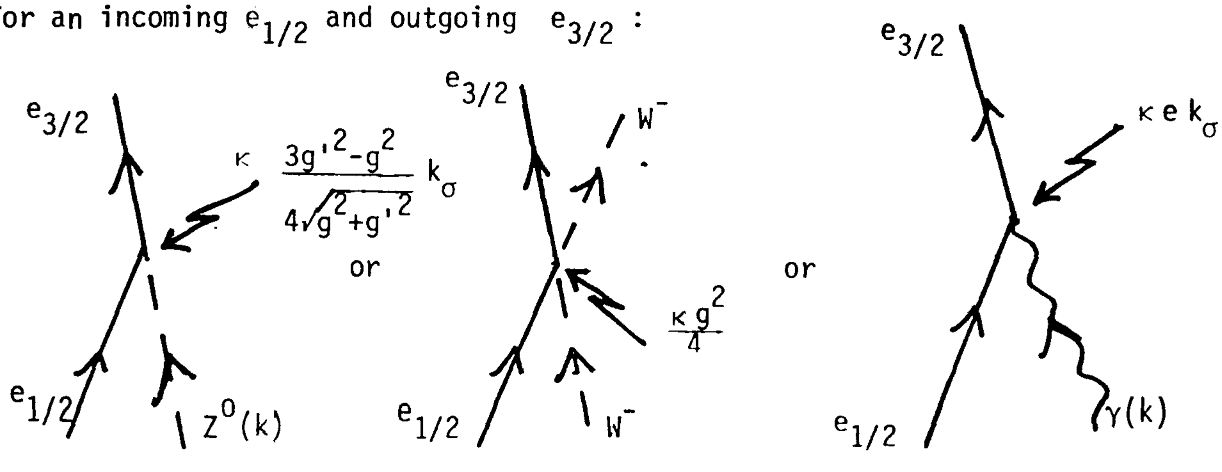
a) for an incoming $\nu_{1/2}$ and outgoing $\nu_{3/2}$:



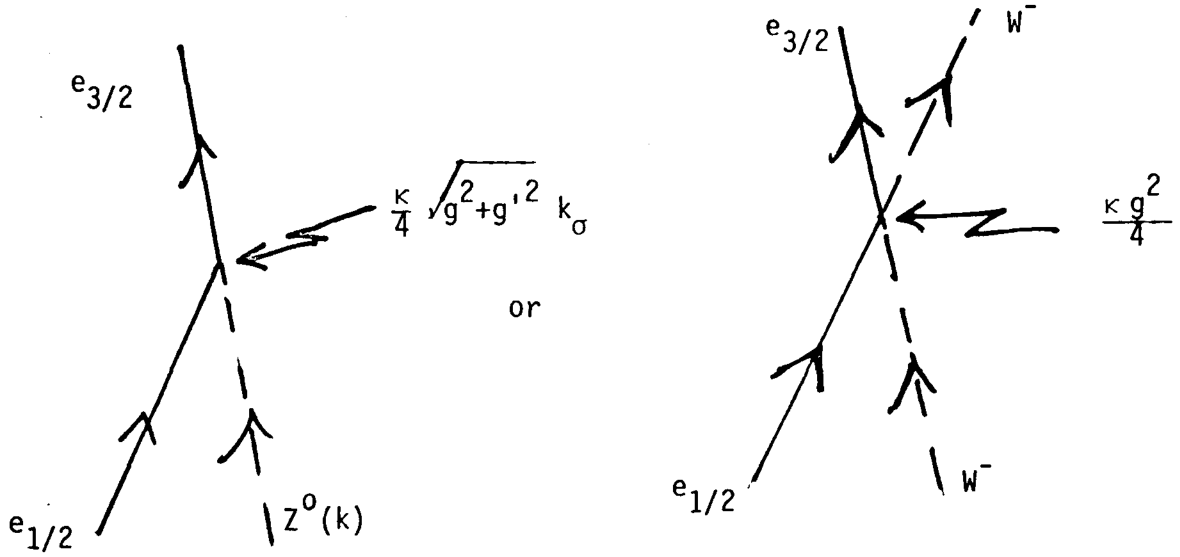
b) for an incoming $e_{1/2}$ and outgoing $\nu_{3/2}$:



c) for an incoming $e_{1/2}$ and outgoing $e_{3/2}$:



for vectorial couplings



for axial coupling

FIGURE 3

In writing (22) we have made no assumption on the reality of the 1/2 or 3/2 fermions. In that case, the use of the identity (12) and the equations (1) for the mouvement of the free R-S field allow us to rewrite this lagrangian in a more handling form for calculations (just in the same manner as for (19) as compared to (18)). We can verify that (18) is of course included in (22). The occurence of these new possible couplings gives rise to corrections to cross sections involving even spin-1/2 leptons only. For instance, in the usual Compton scattering, we have now to consider a new graph (Fig. 4) where the virtual electronic line is replaced by the propagator of the corresponding $e_{3/2}$ field. Suppose we are in the domain where :

$$\frac{q^2}{2M^2} << 1 \tag{23}$$

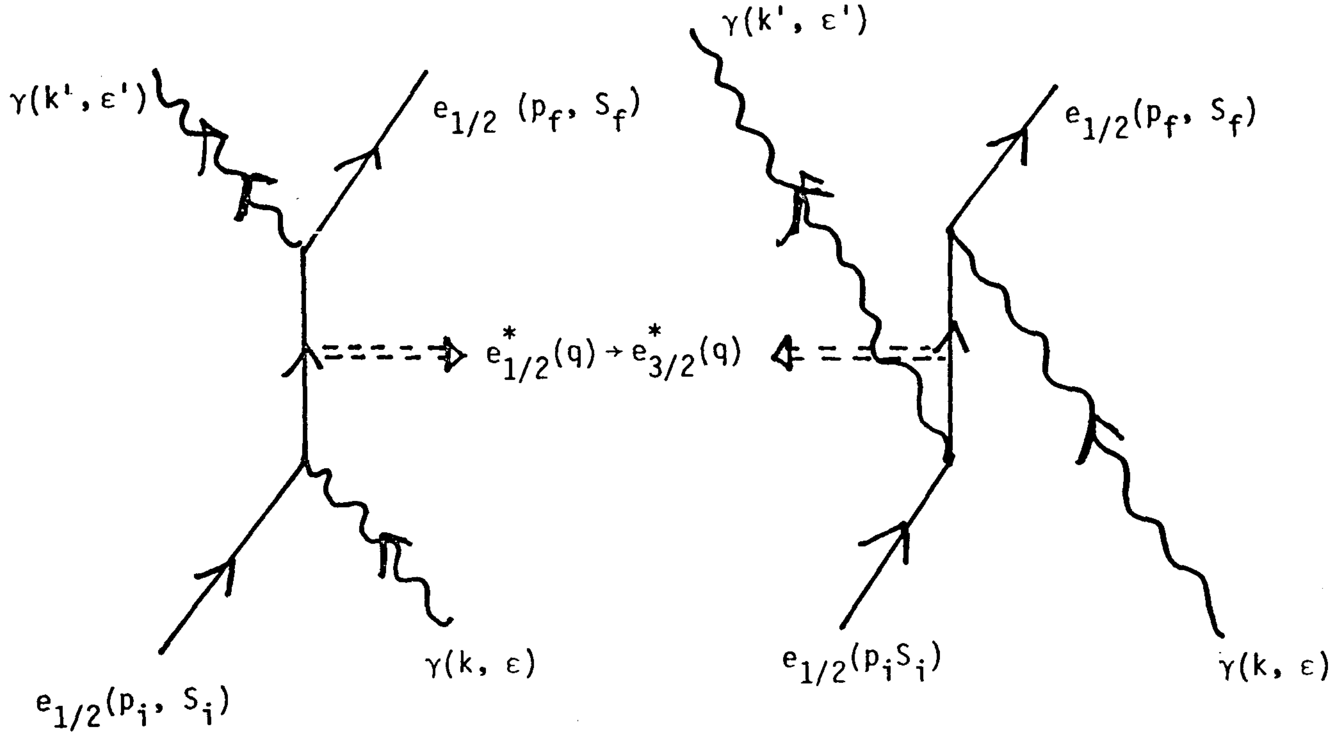


FIGURE 4 : contribution to Compton Scattering

Postulating $M \simeq 20$ GeV[10], this means that in the frame where the electron is at rest, the energy E_γ of the incident photon is less than an energy of the order of 10^6 GeV. But given the value[10] :

$$P^{(+)\mu\nu}(q) = \sum_s U^\mu_{3/2}(q,s)\bar{U}^\nu_{3/2}(q,s) = -\frac{\not{q} - M}{2M}\left(g^{\mu\nu} - \frac{1}{3}\gamma^\mu\gamma^\nu - \frac{q^\mu}{3M^2}\gamma^\nu\not{q} - \frac{q^\nu}{3M^2}\not{q}\gamma^\mu\right) \tag{23a}$$

for the positive energy projector and

$$P^{(-)\mu\nu}(q) = \sum_s V^\mu(q,s)\bar{V}^\nu(q,s) = -P^{(+)\mu\nu}(-q) \tag{23b}$$

for the negative energy projector, the approximation $\frac{q^2}{2M^2} \simeq 0$ in (23a) cancels the interference term given by the graphs of Fig. 5. In order words, this means that these contributions may become significantly non negligible for energies of the incident photon of the order of 10^6 GeV. Moreover, the coupling constant κ we had to introduce because of dimensional arguments must be adjusted to fit experimental features.

If we look a the production of spin-3/2 leptons, it can be made at lowest orders through e^+e^- annihilation in the direct and crossed channels of Fig. 5 (where we do not write the graph with intermediate Z^0) :

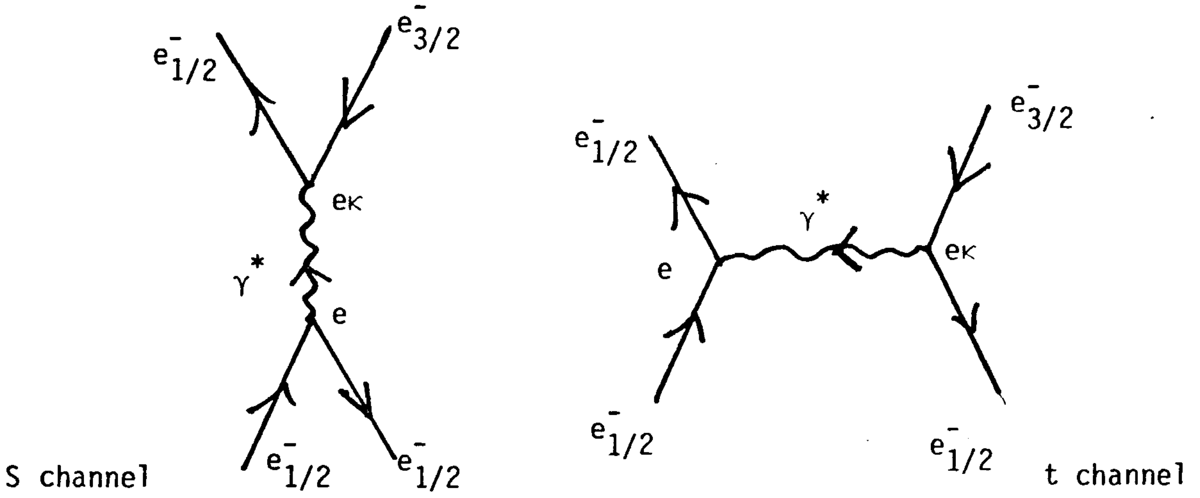


FIGURE 5 : $e^+_{1/2}\, e^-_{1/2} \to e^-_{1/2}\, e^+_{3/2}$

The $e_{3/2}$ decaying again in the favoured mode :

$$e^-_{3/2} \to e^- + \gamma \qquad (24)$$

Experimental investigations at high energies in search for this reaction are desirable.

If we consider all the possible graphs given by the Fig. 4, we can construct diagrams involving the two kinds of leptons and the physical gauge fields in different ways. Calculations on reactions based on the present formalism will be published elsewhere.

* * *

Acknowledgments : We should like to thank M. ABUD for stimulating discussions.

REFERENCES

[1] See for instance J. Wess and J. Bagger : "Supersymmetry and Supergravity", Princeton Univ. Press (1983).

[2] J. Leite Lopes et al., Phys. Rev. D25, 1854 (1982).

[3] G. Velo and D. Zwanziger, Phys. Rev. D186, 1337 (1969) ; A. Das and D. Freeman, Nucl. Phys. B144,271 (1976) ; J. Leite Lopes and D. Spehler, Lett. Nuovo Cimento 26, 576 (1979).

[4] T.D. Lee : "Particle Physics and Introduction to Field Theory", p. 658 and followings, Harwood Acad. Pub. (1981).

[5] J. Leite Lopes et al., Lett. Nuovo Cimento 35, 60 (1982).

[6] N. Fleury et al., Lett. Nuovo Cimento 36, 401 (1983).

[7] T.P. Cheng and Ling-Fang Li, Phys. Rev. D22, 2860 (1980).

[8] Mass limits for excited spin-1/2 leptons are given by E. Lohrmann : "Review of $e^+ e^-$ Physics with PETRA", DESY Report 83-102 . Masses lower than 61 GeV for an electron-type and lower than 18 GeV for muon-type excited leptons are excluded.

[9] This is just an optimistic order of magnitude (see Ref. 8)

[10] for a review of properties of spin-3/2 fields, see D. Spehler : Thèse de Docotorat d'Etat, CRN/HE 82-02, Université Louis Pasteur, Strasbourg (1982).