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GAUGE AND NON-GAUGE CURVATURE TENSOR COPIES

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ABSTRACT

The space-time connections giving rise to the same curvature tensor are constructed and the corresponding geometries compared. The notion of gauge and non-gauge copies in the context of tetrad formulation is elucidated and illustrated by an explicit calculation. Some comments are made on the copies in Einstein-Cartan and Weyl-Cartan geometries.

1. INTRODUCTION.

In gauge theories gauge equivalent potentials (gauge copies) are very useful in handling certain problems. In connection with non-abelian Yang-Mills theory[1], for example, the calculation[2] of topological number of the multi-instanton solution can be carried out as an application of Gauss' theorem if we use t'Hooft's solution[3] together with another compact and gauge equivalent solution[2]. For the abelian case an important application was found by Wu and Yang[4] in their formulation of Dirac monopole[5] theory. Each monopole divides the space in two overlapping sections. Regular potentials in each of the sections can be defined such that in the intersection region the two are gauge equivalent.

In the non-abelian case we may also obtain two or more potentials not related by any gauge transformations associated to the same gauge covariant field strength. These 'field strength copies' have been studied in several recent papers[6].

For an affine space-time manifold, likewise, the curvature tensor does not determine the (non-tensorial) space-time connections uniquely. The gauge transformations or local coordinate dependent transformations corresponding to the tangent space group arise naturally in the tetrad formmulation[7] used in gravitation. We now also have gauge or spin connections corresponding to the local Lorentz (gauge) rotations of the tetrad frame. A gauge covariant field strength, spin curvature tensor, with two local indices and two coordinate indices may be defined. The two curvature tensors are related by eq. (20) in Sec.3.

We discuss in this work the construction of curvature ten sor copies in the context of the geometry of affine space-time manifold. The notation is defined in Sec.2. In Sec.3 the curvature tensor copies are constructed and the corresponding geometries compared. The notion of gauge copies is also elucidated. Finally, Sec.4 contains explicit calculation and describes briefly the procedure to follow in the case of Weyl-Cartan geometry.

2. NOTATION

A local frame of reference is specified by giving a set of four linearly independent tetrad fields [7] $e^{\ell}_{\mu}(x)$ and their inverses $e^{\mu}_{\ell}(x)$, e.g., $e^{\mu}_{\ell}\, e^{m}_{\mu} = \delta^{\ell}_{m}$. This leads to $e^{\mu}_{\ell}\, e^{\ell}_{\nu} = \delta^{\mu}_{\nu}$. Here the Greek letters $\mu,\nu,.. = 0,1,2,3$, label the curved space (world) vector indices while the Latin letters $l,m,.. = 0,1,2,3$ indicate the vector indices in the local tangent space. The local tangent space group will be taken to be the Lorentz transformation group $\mathcal{L}$. In order to define covariant derivatives we introduce as usual space-time affine connections $\Gamma^{\mu}_{\nu\lambda}$ and local gauge spin connections $\omega^{\ell}_{\lambda}\,m$ corresponding to the Lorentz gauge transformations. The index λ in ω is tensorial while $\omega_{\lambda} \equiv (\omega^{\ell}_{\lambda\; m})$ as a matrix transforms under $\mathcal{L}$ as a gauge connection, The covariant derivative of a local four-vector $A^{\ell}(x)$ (w.r.t. $\mathcal{L}$) is defined by

$$A^{\ell}{}_{;\lambda} = \partial_{\lambda} A^{\ell} + \omega^{\ell}_{\lambda m}\, A^{m} \tag{1}$$

The transformation of ω_{λ} then follows to be

$$\omega_{\mu} \xrightarrow{\mathcal{L}} \Lambda\omega_{\mu}\, \Lambda^{-1} - (\partial_{\mu}\Lambda)\Lambda^{-1} \tag{2}$$

where $\Lambda(x) \equiv (\Lambda^{\ell}{}_{m})$ is Lorentz transformation matrix, e.g., $\Lambda^{p}{}_{\ell}\,\Lambda^{q}{}_{m}\,\eta_{pq} = \eta_{\ell m}$ where $\eta_{\ell m}$ is constant metric tensor and $\Lambda^{\ell}{}_{m}$ are real. Requiring that $(A^{\ell}B_{\ell})_{;\lambda} = \partial_{\lambda}(A^{\ell}B_{\ell})$ we derive

$$A_{\ell;\lambda} = \partial_{\lambda}A_{\ell} - \omega^{m}_{\lambda\ \ell}A_{m} \tag{3}$$

To, expose our convention we have, for example

$$e^{\mu}_{\ell;\lambda} = \partial_{\lambda}e^{\mu}_{\ell} + \Gamma^{\mu}_{\alpha\lambda}\,e^{\alpha}_{\ell} - \omega^{m}_{\lambda\ \ell}e^{\mu}_{m}$$

$$e^{\ell}_{\mu;\lambda} = -e^{m}_{\mu}\,e^{\ell}_{\nu}\,e^{\nu}_{m;\lambda} = \partial_{\lambda}e^{\ell}_{\mu} - \Gamma^{\alpha}_{\mu\lambda}e^{\ell}_{\alpha} + \omega^{\ell}_{\lambda\ m}e^{m}_{\mu} \tag{4}$$

These definitions are consistent with the definitions $A^{\mu} = e^{\mu}_{\ell}\,A^{\ell}$, $A_{\mu} = e^{\ell}_{\mu}\,A_{\ell}$ etc... The symmetric space-time metric tensor $g_{\mu\nu}$ is given as $g_{\mu\nu} = \eta_{\ell m}\,e^{\ell}_{\mu}\,e^{m}_{\nu}$ and we check easily $A_{\mu} = g_{\mu\nu}\,A^{\nu}$. The $g^{\mu\nu}$ is defined to the inverse of $g_{\mu\nu}$. We also note that

$$\eta_{\ell m}(\omega)_{;\lambda} = -2\omega_{\lambda(\ell m)} \quad ; \quad \eta^{\ell m}(\omega)_{;\lambda} = 2\omega_{\lambda}{}^{(\ell m)} \tag{5}$$

vanish only if $\omega^{\ell m}_{\lambda} = -\omega^{m\ell}_{\lambda}$. For completeness sake we also consider the covariant derivative of (local) Dirac spinor field written as

$$\psi_{;\lambda} = (\partial_{\lambda} + \Gamma_{\lambda})\psi(x) \tag{6}$$

We find that for $\psi \xrightarrow{\mathcal{L}} S(\Lambda(x))\psi(x)$

$$\Gamma_{\mu} \xrightarrow{\mathcal{L}} S(\Lambda)\,\Gamma_{\mu}S^{-1}(\Lambda) - (\partial_{\mu}S(\Lambda))\,S^{-1}(\Lambda) \tag{7}$$

For infinitesimal transformations $\Lambda^{\ell}_{m} = \delta^{\ell}_{m} + \lambda^{\ell}_{m}$

$$\delta\Gamma_{\mu} = \frac{1}{2}\lambda^{\ell m}\left[\sigma_{\ell m},\Gamma_{\mu}\right] - \frac{1}{2}\sigma_{\ell m}(\partial_{\mu}\lambda^{\ell m}) \qquad (8)$$

where $\sigma_{\ell m} = \frac{1}{4}\left[\gamma_{\ell},\gamma_{m}\right]$. On the other hand

$$\delta\omega_{\mu}^{[\ell m]} = \lambda^{\ell}{}_{n}\,\omega_{\mu}^{[nm]} - \omega_{\mu}^{[\ell n]}\lambda_{n}{}^{m} - \partial_{\mu}\lambda^{\ell m} \qquad (9)$$

It is then easily shown that we may write $\Gamma_{\mu} = \frac{1}{2}\sigma_{\ell m}\omega_{\mu}^{[\ell m]} = \frac{1}{2}\sigma_{\ell m}\omega_{\mu}^{\ell m}$. The constant Dirac matrices γ^{ℓ} remain invariant under local gauge transformations

$$\gamma^{\ell} \xrightarrow{\mathcal{L}} \gamma^{\ell} = \Lambda^{\ell}_{m}(x)\; S(\Lambda)\; \gamma^{m}\; S^{-1}(\Lambda) \qquad (10)$$

It is also clear that the coloumn (spinor) matrix index in γ^{ℓ} transforms contragradiently to the row (spinor) index. Hence it follows $(\partial_{\lambda}\gamma^{\ell}=0)$

$$\gamma^{\ell}{}_{;\lambda} = \omega_{\lambda}^{\ell}{}_{m}\gamma^{m} + \left[\Gamma_{\lambda},\gamma^{\ell}\right] \qquad (11)$$

From $\gamma^{\mu}(x) = e^{\mu}_{\ell}\,\gamma^{\ell}$ and $\gamma^{\mu}(x) \xrightarrow{\mathcal{L}} S(\Lambda)\gamma^{\mu}(x)\; S^{-1}(\Lambda)$ we obtain

$$\gamma^{\mu}{}_{;\lambda} = \partial_{\lambda}\gamma^{\mu} + \Gamma^{\mu}_{\rho\lambda}\gamma^{\rho} + \left[\Gamma_{\lambda},\gamma^{\mu}\right] \qquad (12)$$

Using the identity

$$\left[\Gamma_{\mu},\gamma^{\ell}\right] = \frac{1}{2}(\omega_{\mu m}{}^{\ell} - \omega^{\ell}_{\mu m})\gamma^{m} \qquad (13)$$

we find

$$\gamma^{\ell}{}_{;\lambda} = \omega_\lambda{}^{(\ell m)} \gamma_m \tag{14}$$

It follows also that

$$\{\gamma^\mu(x), \gamma^\nu(x)\} = 2e^\mu_\ell \; e^\nu_m \eta^{\ell m} I = 2g^{\mu\nu}(x) I \tag{15}$$

For the covariant derivative of metric tensor we have

$$\begin{aligned} g_{\mu\nu;\lambda}(\Gamma) &= \partial_\lambda \; g_{\mu\nu} - \Gamma^\alpha_{\mu\lambda} \; g_{\alpha\nu} - \Gamma^\alpha_{\nu\lambda} \; g_{\mu\alpha} \\ &= \eta_{\ell m;\lambda}(\omega) \, e^\ell_\mu e^m_\nu + \eta_{\ell m} \, \{e^\ell_{\mu;\lambda}(\Gamma,\omega) e^m_\nu + e^\ell_\mu e^m_{\nu;\lambda}(\Gamma,\omega)\} \end{aligned} \tag{16}$$

We note in passing that the usual metricity condition of Einstein-Cartan geometry, $g_{\mu\nu;\lambda}(\Gamma) = 0$, may be obtained by suitable adjustments in the values of $\eta_{\ell m;\lambda}(\omega)$ and $e^\ell_{\mu;\lambda}(\Gamma,\omega)$. We have rather elaborated on notation to make clear that no restrictions on the symmetry properties of $\omega_\lambda^{\ell m}$ are imposed by the definitions used above. For example, in the case of Weyl-Cartan geometry (see Sec.4) it has both symmetric and antisymmetric parts.

3. CURVATURE TENSOR COPIES: GAUGE COPIES.

The space-time curvature tensor is given by

$$R^{\mu}{}_{\nu\lambda\rho}(\Gamma) = \partial_{\lambda}\Gamma^{\mu}_{\nu\rho} + \Gamma^{\mu}_{\beta\lambda}\,\Gamma^{\beta}_{\nu\rho} - (\lambda\leftrightarrow\rho) \tag{17}$$

We may also define field strength $P_{\lambda\rho}(\omega)$ from the local connection ω_{λ} which transforms covariantly w.r.t. the local gauge transformations

$$P_{\lambda\rho}(\omega) = \partial_{\lambda}\omega_{\rho} - \partial_{\rho}\omega_{\lambda} + [\omega_{\lambda},\omega_{\rho}]$$

$$P_{\lambda\rho}(\omega) \longrightarrow \Lambda P_{\lambda\rho}(\omega)\Lambda^{-1} \tag{18}$$

Then the indices in local or spin curvature [7] tensor $R^{\ell}{}_{m\lambda\rho}(\omega) \equiv (P_{\lambda\rho})^{\ell}_{m}$ are all tensorial and

$$[A^{\ell}{}_{;\lambda\rho} - A^{\ell}{}_{;\rho\lambda}] = -R^{\ell}_{m\lambda\rho}(\omega)A^{m} - (\Gamma^{\alpha}_{\lambda\rho}-\Gamma^{\alpha}_{\rho\lambda})A^{\ell}{}_{;\alpha} \tag{19}$$

The two curvature tensors are connected by the following equation

$$e^{\ell}_{\mu;\lambda\rho}(\Gamma,\omega) - e^{\ell}_{\mu;\rho\lambda}(\Gamma,\omega) + (\Gamma^{\alpha}_{\lambda\rho} - \Gamma^{\alpha}_{\rho\lambda})\, e^{\ell}_{\mu;\alpha}(\Gamma,\omega)$$

$$= R^{\alpha}_{\mu\lambda\rho}(\Gamma)e^{\ell}_{\alpha} - R^{\ell}_{m\lambda\rho}(\omega)e^{m}_{\mu} \tag{20}$$

Consider the set of affinities (Γ,ω) satisfying $e^{\ell}_{\mu;\lambda}(\Gamma,\omega)=0$ so that

$$R^{\alpha}_{\mu\lambda\rho}(\Gamma) = R^{\ell}_{m\lambda\rho}(\omega)e^{m}_{\mu}e^{\alpha}_{\ell} \tag{21}$$

and the set $(\bar{\Gamma},\omega)$ satisfying

$$e^{\ell}_{\mu;\lambda}(\bar{\Gamma},\omega) = -K^{\ell}_{\lambda\ m} e^{m}_{\mu} \tag{22}$$

or

$$e^{\ell}_{\mu;\lambda}(\bar{\Gamma},\bar{\omega}) = 0 \tag{23}$$

where $\bar{\omega}_{\lambda} = \omega_{\lambda} + K_{\lambda}$. We then have

$$R^{\alpha}_{\mu\lambda\rho}(\bar{\Gamma}) = R^{\ell}_{m\lambda\rho}(\bar{\omega}) e^{m}_{\mu}\, e^{\alpha}_{\ell} \tag{24}$$

Substituting eq.(22) in eq.(20) we find that the term involving torsion $(\bar{\Gamma}^{\mu}_{\lambda\rho} - \bar{\Gamma}^{\mu}_{\rho\lambda})$ cancels out obtaining the result

$$\{P_{\lambda\rho}(K) + [\omega_{\lambda}, K_{\rho}] - [\omega_{\rho}, K_{\lambda}]\}^{\ell}_{m} = R^{\alpha}_{\mu\lambda\rho}(\bar{\Gamma}) e^{\ell}_{\alpha}\, e^{\mu}_{m} - R^{\ell}_{m\lambda\rho}(\omega) \tag{25}$$

It follows that if K_{λ} satisfies (independent of $\bar{\Gamma}$)

$$P_{\lambda\rho}(K) + [\omega_{\lambda}, K_{\rho}] - [\omega_{\rho}, K_{\lambda}] = 0 \tag{26}$$

we will have

$$R^{\alpha}_{\mu\lambda\rho}(\bar{\Gamma}) = R^{\ell}_{m\lambda\rho}(\omega) e^{m}_{\mu}\, e^{\alpha}_{\ell}$$

$$= R^{\alpha}_{\mu\lambda\rho}(\Gamma) \tag{27}$$

while the eq.(26) is equivalent to $P_{\lambda\rho}(\bar{\omega}) = P_{\lambda\rho}(\omega)$. We may find $\bar{\Gamma}$ from eq.(23)

$$\bar{\Gamma}^{\alpha}_{\mu\lambda} = \Gamma^{\alpha}_{\mu\lambda} + L^{\alpha}_{\mu\lambda},$$

$$L^{\alpha}_{\mu\lambda} = K^{\ell}_{\lambda m} \; e^{m}_{\mu} \; e^{\alpha}_{\ell} \tag{28}$$

Moreover, we have

$$\eta_{\ell m;\lambda}(\overline{\omega}) = -2(\omega_\lambda + K_\lambda)_{(\ell m)}$$

$$g_{\mu\nu;\lambda}(\overline{\Gamma}) = \eta_{\ell m;\lambda}(\overline{\omega}) e^{\ell}_{\mu} \; e^{m}_{\nu} \tag{29}$$

The corresponding expressions for $g_{\mu\nu}(\Gamma)_{;\lambda}$etc. are obtained by setting $K_\lambda=0$ and the two space-time geometries corresponding to Γ and $\overline{\Gamma}$ may be compared. Thus we have constructed curvature tensor copies obtainable from eqns. (26)-(29). A solution of eq.(26) gives rise to a'gauge copy' if $\overline{\omega}_\lambda$ and ω_λ are connected by a Lorentz gauge transformation, e.g.,

$$K_\lambda = \Lambda\omega_\lambda\Lambda^{-1} - \omega_\lambda - (\partial_\lambda\Lambda)\Lambda^{-1} \tag{30}$$

As expected, eq.(26) for this case implies $\Lambda P_{\lambda\rho}(\omega)\Lambda^{-1} = P_{\lambda\rho}(\omega)$. It is also worth pointing out that the last term in eq.(30) is always antisymmetric and that ω_λ may be decomposed into its irreducible symmetric and antisymmetric components. Thus, for example, if ω_λ is antisymmetric, say, corresponding to Einstein-Cartan geometry, a symmetric K_λ cannot correspond to a gauge copy and a copy with antisymmetric K_λ corresponds again to E.-C. geometry.

4. ILLUSTRATIONS OF SOME CURVATURE TENSOR COPIES

An obvious symmetric solution of eq.(26) is $K_\lambda = -I(\partial_\lambda \chi)$, $\mathrm{Tr}\, K_\lambda = -4(\partial_\lambda \chi)$, where $\chi(x)$ is an arbitrary scalar field and $K_\lambda^{\ell m} = K_\lambda^{m \ell}$. It corresponds to

$$\overline{\Gamma}^{\alpha}_{\mu\lambda} = \Gamma^{\alpha}_{\mu\lambda} - \delta^{\alpha}_{\mu}(\partial_\lambda \chi)$$

$$g_{\mu\nu;\lambda}(\overline{\Gamma}) = 2g_{\mu\nu}(\partial_\lambda \chi) - 2\omega_{\lambda(\ell m)} e^{\ell}_{\mu}\, e^{m}_{\nu} \qquad (31)$$

A more general solution may be obtained with the ansatz

$$K_\lambda = a(x)\partial_\lambda \chi$$

$$\partial_\lambda a(x) + [\omega_\lambda, a(x)] = 0 \qquad (32)$$

where $a(x)$ may correspond to a symmetric or antisymmetric solution.

For an illustration we will consider a metric space defined by the following line element [8]

$$ds^2 = dt^2 - 2A(t)dz\,dt - C^2(t)(dx^2 + dy^2) \qquad (33)$$

We have $g_{00} = 1$, $g_{11} = g_{22} = -C^2$, $g_{03} = g_{30} = -A$, $g^{03} = -A^{-1}$, $g^{11} = g^{22} = -C^{-2}$, $g^{33} = -A^{-2}$ and $\sqrt{-g} = AC^2$ for the non-vanishing elements. A set of tetrad fields is easily found with the non-vanishing elements given by

$$e_o^{(0)} = 1, \quad e_1^{(1)} = e_2^{(2)} = C, \quad e_3^{(3)} = e_3^{(0)} = -A,$$
$$e^0_{(o)} = -e^0_{(3)} = 1, \quad e^3_{(3)} = -A^{-1}, \quad e^1_{(1)} = e^2_{(2)} = C^{-1} \tag{34}$$

where the indices enclosed inside brackets are the tangent space indices. We also assume, for definiteness sake, Γ's to be Christofell connections, $\Gamma^\lambda_{\mu\nu} = \{{\lambda \atop \mu\nu}\}$. The internal spin connections determined from, $e^\ell_{\mu;\lambda}(\{\},\overset{0}{\omega}) = 0$, are antisymmetric and found to be $(\omega_\lambda \equiv \overset{0}{\omega}_\lambda)$,

$$\overset{0}{\omega}_3 = 0, \quad \overset{0}{\omega}_1 = \dot{C}\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad \overset{0}{\omega}_2 = \dot{C}\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$\overset{0}{\omega}_0 = \dot{A}\,A^{-1}\begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \quad \overset{0}{\omega}_1\overset{0}{\omega}_2 = \overset{0}{\omega}_2\overset{0}{\omega}_1 = 0 \tag{35}$$

A traceless symmetric solution is found to be $a(t) = A^2 M$ where the non-vanishing elements of M are $M^0{}_0 = M^3{}_0 = -M^0{}_3 = -M^3{}_3 = -1$. Since ω_λ are antisymmetric this cannot correspond to the case of a gauge copy. We find

$$\bar{\Gamma}^\mu_{\nu\lambda} = \{{\mu \atop \nu\lambda}\} + A(\partial_\lambda \chi)\,\delta^\mu_3\,\delta^0_\nu$$

$$g_{\mu\nu;\lambda}\ (\bar{\Gamma}) = 2A^2(\partial_\lambda \chi)\delta^0_\mu\ \delta^0_\nu \tag{36}$$

and verify by direct calculation $R^\alpha_{\mu\lambda\rho}(\bar{\Gamma}) = R^\alpha_{\mu\lambda\rho}(\Gamma)$.

An antisymmetric solution is found to be $a(t)=(\frac{A}{\dot{C}})\overset{0}{\omega}_1$ and corresponds to

$$\bar{\Gamma}^\mu_{\nu\lambda} = \{^{\ \mu}_{\nu\lambda}\} + \left[\frac{A}{C}\ \delta^\mu_1\ \delta^0_\nu - C\delta^\mu_3\ \delta^1_\nu\right](\partial_\lambda \chi)$$

$$g_{\mu\nu;\lambda}(\bar{\Gamma}) = 0 \tag{37}$$

However, this case can be shown to correspond to a gauge copy. We find $P_{\lambda\rho} = (-\frac{\ddot{C}}{\dot{C}} + \frac{\dot{A}}{A})\left[\delta^0_\rho(\overset{0}{\omega}_1\delta^1_\lambda + \overset{0}{\omega}_2\delta^2_\lambda) - (\lambda \leftrightarrow \rho)\right]$ so that in order to satisfy $\Lambda P_{\lambda\rho}(\overset{0}{\omega})\Lambda^{-1} = P_{\lambda\rho}(\overset{0}{\omega})$ we require $\Lambda\overset{0}{\omega}_{1,2}\Lambda^{-1} = \overset{0}{\omega}_{1,2}$ apart from the restrictions that Λ be a Lorentz matrix. Adding to these the restrictions arising from eq.(30) a tedious calculation shows that the Lorentz gauge matrix $\Lambda(x)$ is given by

$$\Lambda(x) = \begin{pmatrix} 1+\frac{1}{2}\ \psi^2 & -\psi & 0 & -\frac{1}{2}\ \psi^2 \\ -\psi & 1 & 0 & \psi \\ 0 & 0 & 1 & 0 \\ \frac{1}{2}\ \psi^2 & -\psi & 0 & 1-\frac{1}{2}\psi^2 \end{pmatrix}$$

where $\psi = A(t)\chi(x)$.

The case of Weyl-Cartan geometry may also be discussed. The geometry is characterized by $g_{\mu\nu;\lambda}(\Gamma) = 2g_{\mu\nu}\phi_\lambda$ where ϕ_λ is Weyl field. Since we require $e^\ell_{\mu;\lambda} = 0$ it follows that $\eta_{\ell m;\lambda}(\omega) =$

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