NOTAS DE FÍSICA

VOLUME VIII

Nº 9

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RIO DE JANEIRO

1961

Notas de Física - Volume VIII - Nº 9

THE $K^{+}_{\mu 3}$ AND K^{+}_{e3} DECAY THROUGH A CURRENT OF DEFINITE ISOTOPIC RANK

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(Received October 4, 1961)

ABSTRACT: The energy distribution and polarization of muons and electrons are calculated assuming an interaction through a vector current of definite isospin character, and using form factors obtained by means of dispersion relations. The effect of the K^{*} resonance is taken into account. The spectra, and to a less extent the polarization, are quite insensitive to this effect.

INTRODUCTION.

Theoretical investigations on the three body leptonic decay modes of K-mesons have been carried out by several authors[1, 2, 3] on the assumption that the lepton pair is locally produced. Within the concept of universality in weak interactions these decay processes would proceed through the coupling of vector currents. The matrix element is then given in terms of two form factors, which depend on the pion energy.

These form factors have been investigated by means of dispersion relation techniques[4]. The expressions obtained involve S and P-wave phase shifts for $K\pi$-scattering and depend on only one coupling parameter provided that the current has a definite isospin character $T = \frac{1}{2}$ or $T = 3/2$[5]. The distinguishing feature of such isospin currents is to predict a definite relation between the matrix elements for K^+ and K^0 decays giving the same lepton pair.

We shall assume that in the channel where the K^*-resonance occurs ($T = \frac{1}{2}$, $J = 0$ or 1)[6], the phase shift for $K\pi$-scattering is dominated by the resonance. In the other channels we shall take them zero. The energy spectra of muons and electrons calculated under these hypotheses are totally insensitive to the structure of the form factors. This comes about essentially because of the large mass (885 MeV)[6] of the K^* resonance. A unique spectrum is thus predicted. Brene et al.[7] have also calculated electron and muon spectra under several different assumptions. Some of their curves coincide with those in this paper.

The value obtained for the ratio between the total transition rates for the muon and electron modes is 0.65. An accurate determination of this branching ratio will be a crucial test for the present model.

The longitudinal polarization of the muons has also been calculated. The influence of the form factor structure is here slightly more pronounced. The average polarization predicted for the different possibilities were: for T = 3/2 current, 69%; for $T = \frac{1}{2}$, 63% if K^* has J = 0 and 73% if K^* has J = 1.

I. The form factors.

We assume that the lepton pair in $K^+_{\mu 3}$ and K^+_{e3} decay is produced under the following conditions:

1 - Via a local interaction with a vector current:

$$\langle \pi | J_\alpha | K \rangle = (4\, E_\pi\, E_k)^{-\frac{1}{2}} \left[\tfrac{1}{2}(p_k + p_\pi)_\alpha \;\; f_+ + \tfrac{1}{2}\,(p_k - p_\pi)_\alpha \;\; f_- \right]. \quad (1)$$

2 - In isospin space the current has a definite rank $T = \frac{1}{2}$ or T = = 3/2. A pure $T = \frac{1}{2}$ current implies:

$$\text{i)} \quad \langle \pi^- | J |\, K^0 \rangle = \sqrt{2}\, \langle \pi^0 | J |\, K^+ \rangle$$

$$\text{ii)} \quad \langle \pi^+ | J^\dagger |\, K^0 \rangle = 0.$$

The first restriction refers to processes in which $\Delta S = \Delta Q$. To check this condition one has to determine the relation between the transition rates for

$$K^+ \longrightarrow \pi^0 + \ell^+ + \nu$$

$$K^0 \longrightarrow \pi^- + \ell^+ + \nu$$

where ℓ is either muon or electron. In order to make sure that a process is due to pure K^o-mesons without contamination of $\bar{K}^o$ (or vice-versa) one has to select events of decay at very short time. The second restriction leads to the selection rule $\Delta S = \Delta Q$. Any violation of this rule in the decay of neutral K-mesons implies in the existence of a $T = 3/2$ current. However it may happen that the $T = 3/2$ current is coupled to the lepton current only when $\Delta S = - \Delta Q$. If so, it has no effect in the K^+-decay. Thus for the discussion of K^+-decay we shall require only the weak selection rule $\Delta T = \frac{1}{2}$ associated with $\Delta S = \Delta Q$ and expressed by the condition i). If the condition ii) is not satisfied the spectra of charged and neutral K-mesons could differ substantially except for those events selected as indicated before. The comparison between these spectra becomes then, a question of foremost importance.

We shall take for the form factors f_+ and f_- the expressions obtained in ref. (4) with the additional consideration that the $K\pi$ - interaction is dominated by the resonance at 885 MeV in the isospin state $T = \frac{1}{2}$ and angular momentum $J = 0$ or $J = 1$. The phase shift near the resonance can be represented by an expression:

$$\sin^2 \delta = \Gamma^2 M^2 / \left[(W^2 - M^2)^2 + \Gamma^2 M^2 \right] \qquad (2)$$

where W is the center of mass energy of the $K\pi$ - system, M is the resonant energy and Γ the full width. The form factors depend on the following integral over the phase shift:

$$I(W^2) = \frac{1}{\pi} \int_{(m_k + m_\pi)^2}^{\infty} \frac{W^2}{W'^2} \frac{\delta(W'^2) \, d\, W'^2}{W'^2 - W^2} .$$

Owing to lack of detailed information on the phase shift we shall proceed in the following way to calculate the real part of this integral. First perform a partial integration, imposing the condition $\delta = 0$ at threshold. The derivative of the phase shift is taken from (2) and the integration is extended below the threshold down to $-\infty$. This extension contributes a negligible amount to the integral. The following result is obtained in the physical region:

$$\exp.\ I(W^2) = e^{i\delta}\, M(M^2+\Gamma^2)^{\frac{1}{2}} / \left[(W^2-M^2)^2+\Gamma^2 M^2\right]^{\frac{1}{2}}$$

$$= M(M^2+\Gamma^2)^{\frac{1}{2}}/(M^2-W^2-i\Gamma M). \qquad (4)$$

In the last relation we have used (2) for δ. Actually we are interested in the value of exp. $I(W^2)$ for values of W^2 in the unphysical region below the threshold. In this region $I(W^2)$ is real and one should take the modulus of (4). Since the width of the resonance is very small the following approximation is valid in that region:

$$\exp.\ I(W^2) = M^2/(M^2-W^2). \qquad (5)$$

This result corresponds to taking a step function for the phase shift:

$$\delta = 0 \qquad \text{for} \qquad W^2 < M^2$$

$$\delta = \pi \qquad \text{for} \qquad W^2 > M^2.$$

About the non-resonant phase shifts nothing is as yet known. We shall take them equal to zero in this analysis. Thus we obtain the following expressions for the form factors (eqs. 19 and 20 of ref. 4) in the cases of $T = \frac{1}{2}$ and $T = 3/2$ currents and for either alternative of the angular momentum of K^*:

Case 1: $T = \frac{1}{2}$; $J = 0$

$$f_+(W^2) = f_+(0) \qquad (6)$$
$$f_-(W^2) = f_+(0)\,(m_k^2 - m_\pi^2)/(M^2 - W^2).$$

Case 2: $T = \frac{1}{2}$; $J = 1$

$$f_+(W^2) = f_+(0)\, M^2 / (M^2 - W^2) \qquad (7)$$
$$f_-(W^2) = - f_+(0)(m_k^2 - m_\pi^2)/(M^2 - W^2).$$

Case 3: $T = 3/2$

$$f_+(W^2) = f_+(0) \qquad (8)$$
$$f_-(W^2) = 0\ .$$

The relation between W^2 and the pion energy for decay at rest is:

$$W^2 = m_k^2 + m_\pi^2 - 2m_k\, E_\pi\ . \qquad (9)$$

The form factors for case 1 coincide with the results of Bernstein and Weinberg[8].

II. Energy spectrum and polarization.

The energy spectrum of muons and electrons for vector inter - action is given by:

$$\frac{dT}{dE} = \frac{2}{(4\pi)^3 m_k^4} \Big\{ 2(W_\ell - E)\Big[(E\, m_k - m^2)\, A_1 + m^2\, B_1\Big] - (m_k^2 + m^2 - 2\, m_k\, E)\, A_2$$

$$+ m^2 B_2 \Big\} \qquad (10)$$

where the A's and B's are the following functions of the lepton energy E:

$$A_1 = \int_{W_1}^{W_2} f_+^2 \, d E_\pi \qquad B_1 = \int_{W_1}^{W_2} \tfrac{1}{2} (f_+ + f_-)(f_+ \, d E_\pi$$

$$(11)$$

$$A_2 = \int_{W_1}^{W_2} f_+^2 (W_\pi - E_\pi) \, d E_\pi \qquad B_2 = \int_{W_1}^{W_2} \frac{1}{4} (f_+ + f_-)^2 (W_\pi - E_\pi) d E_\pi$$

with limits W_1 and W_2 given by:

$$W_\pi - \frac{W_\ell - E}{m_k^2 + m^2 - 2 m_k E} (E m_k - m^2 \mp p m_k) . \qquad (12)$$

In these expressions $W_\pi = (m_k^2 + m_\pi^2 - m^2)/2 m_k$ is the maximum pion energy and $W_\ell = (m_k^2 - m_\pi^2 + m^2)/2 m_k$ is the maximum energy of the muon or electron.

The energy spectra of muons and electrons are shown in figs. 1 and 2 respectively. The curves are pratically identical for all the three cases.

We know that present experimental data[9, 10] do not favour vector coupling[11]. However one has to wait for better statistics. The first thing one should look at, is the electron spectrum which depends only on one form factor. If the electron curve (Fig. 2) does not fit the data then pure vector interaction is ruled out. On the other hand a good fit will give some evidence of pure vector interaction. Next one must look at the muon spectrum which depends on both form factors. Here according to the goodness of fitting one can draw conclusions about:

a) The hypotheses of pure vector interaction.

b) The relation between the form factors.

If the hypotheses on the pure isospin character of the current is false the form factors contain terms from $T = \frac{1}{2}$ and $T = 3/2$ contributing differently for f_+ and f_-. The relation between them is then broken and it will be much more difficult to check on the validity of the analysis of ref. 1.

If instead of a direct coupling between weak currents the interaction proceeds through an intermediate vector boson of mass m_B, the form factors will be replaced by:

$$f_+ \longrightarrow \frac{m_B^2}{m_B^2 - W^2} f_+ , \quad f_- \longrightarrow f_- - \frac{m_K^2 - m_\pi^2}{m_B^2 - W^2} f_+ .$$

For $m_B \gtrsim M$ the influence of this change in the spectrum is negligible.

The experimental value for the branching ratio of $K^+_{\mu 3}$ to K^+_{e3} is quite uncertain. We quote the value 0.8 ± 0.2 obtained by Bruin et al.[9] compiling data from several sources. The prediction of the model is 0.65 which is consistent with the experimental value. An accurate determination of the branching ratio will be a crucial test of the model.

Expressions for the longitudinal polarization of muons were given in ref. 2. The curves for the polarization as function of energy using the form factors (6) and (7), are shown in fig. 3, and compared with the muon velocity. The polarization is more sensitive to the structure of the form factors and could perhaps distinguish between an S or P-wave resonance. The average polarization is in either case 63% and 73% respectively. The effect of an intermediate vector boson is a small rise in the polarization.

Finally we give in fig. 4 the neutrino spectrum for our form factors. As pointed out by Furuichi et al.[12], from the determination of the pion and lepton momenta in the decay in flight of the K^o one can deduce the neutrino energy for decay at rest. The neutrino spectrum can be used as a test of the identity of weak interactions for K^o and K^+ decays, should this model be satisfactory for the charged mesons. If, in addition to measurements on the charged particles produced in the decay, one is able to establish the complete kinematics of the events, then, of course, more powerful tests will be available.

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REFERENCES

1 - S. FURUICHI, T. KODAMA, S. OGAWA, Y. SUGAHARA, A. WAKASA and M. YONESAWA: Prog. Theor. Phys. (Japan), 17, 89 (1957).

2 - S. W. MAC DOWELL: Nuovo Cimento 6, 1445 (1957).

3 - L. OKUN': Ann. Rev. of Nuclear Science, Vol. 9, 61 (1959), This review article contains a complete list of references.

4 - S. W. MAC DOWELL: Phys. Rev. 116, 1047 (1959).

5 - The possibility of pure T = 3/2 strangeness violating vector current has been discussed by R. E. BEHRENDS and A. SIRLIN: Phys. Rev. Letters, 5, 476 (1960).

6 - M. ALSTON, L. W. ALVAREZ, P. EBERHARD, M. L. GOOD, W. GRAZIANO, H. K. TICHO and S. G. WOJCICKI: Phys. Rev. Letters 6, 300 (1961).

7 - N. BRENE, L. EGARDT and B. QVIST (Nuclear Physics 22, 553 (1961).

8 - We remark that the relation between the form factors obtained in ref. 4 can also be justified if one assumes that the strangeness violating vector current is "almost conserved". (Y. NAMBU, Phys. Rev. Letters 4, 380 (1960); J. BERNSTEIN and S. WEINBERG, Phys. Rev. Letters 15, 481 (1961)). It seems to us that the condition

$$\lim._{W^2 \to \infty} \left[(4\, E_\pi\, E_k)^{\frac{1}{2}} \langle \pi | \partial_\alpha\, J_\alpha(X) | K \right] = \lim._{W^2 \to \infty} (W^2\, f_o) = 0$$

imposed in the latter paper is too strong and not really required by the hypotheses invoqued. Indeed a dimensional argu

ment shows that $W^2 f_o$ can go to a limit proportional to the square of a mass (actually $m_k^2 - m_\pi^2$) which would become zero if the meson masses were zero. On the other hand following NAMBU's argument one is led to impose on the dimensionless function f_o (we are excluding the weak coupling constant which has dimension $[M]^{-2}$) the condition $\lim_{W^2 \to \infty} f_o = 0$, which was the assumption made in ref. 4. The form factors obtained in the present paper in the case $T = \frac{1}{2}$, with $J = 1$ for K^*, or in the case $T = 3/2$, are consistent with that assumption but incompatible with the conditions of Bernstein and Weinberg.

9 - M. BRUIN, D. J. HOLTHUIZEN and B. JONGEJANS, Nuovo Cimento, 9 423 (1958). This paper gives further references to experimental work.

10 - G. ALEXANDER, R. H. W. JOHNSTON and C. O'CEALLAIGH: Nuovo Cimento 6, 478 (1957).

11 - S. FURUICHI, Y. SUGAHARA, A. WAKASA, and YONEZAWA: Nuovo Cimento, 5, 285 (1957).

12 - S. FURUICHI, Nuovo Cimento, 7, 269 (1958).

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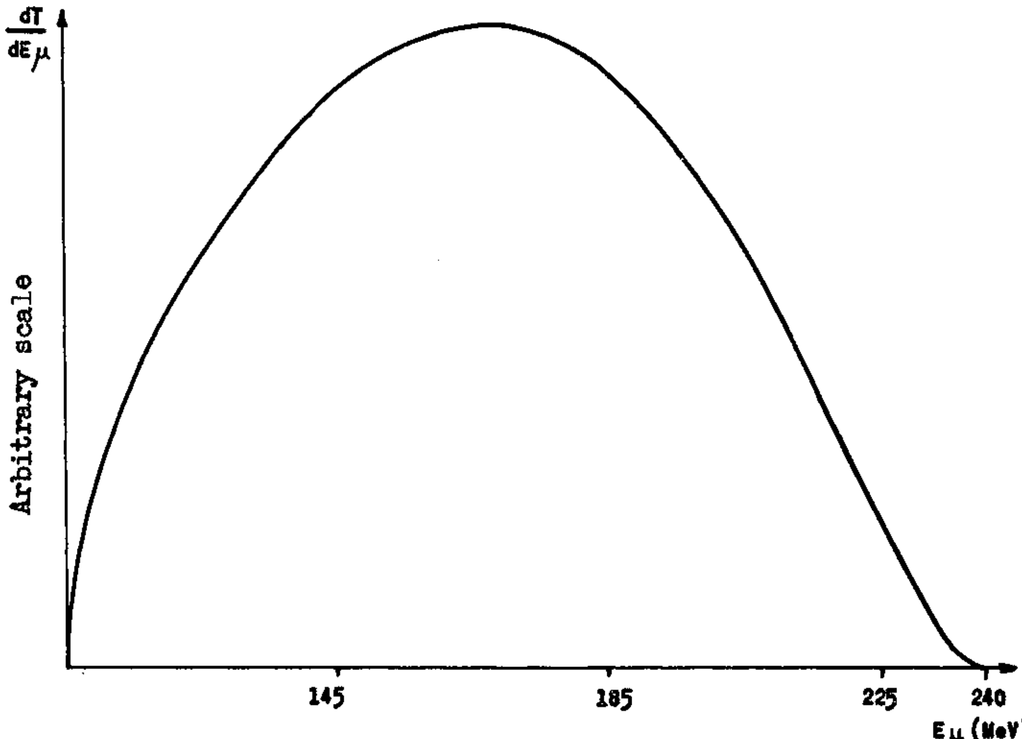

Fig. 1 - Muon spectrum with form factors for Case 2 ($T = \frac{1}{2}$, $J = 1$). The curves for the other cases are pratically identical to this one.

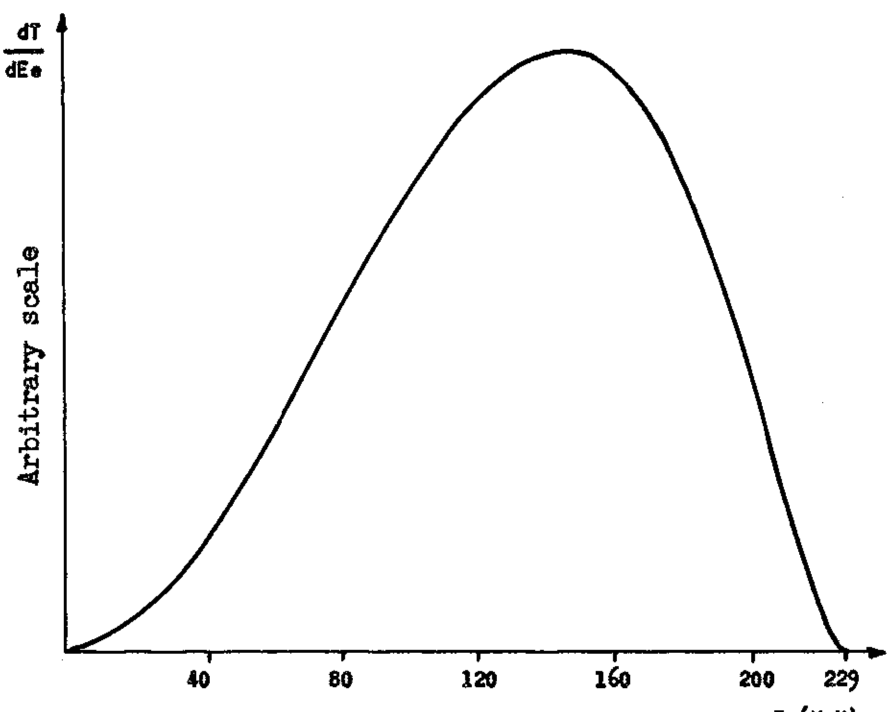

Fig. 2 - Electron spectrum with form factors for Case 2 ($T = \frac{1}{2}$, $J = 1$). The curves for the other cases are pratically identical to this one.

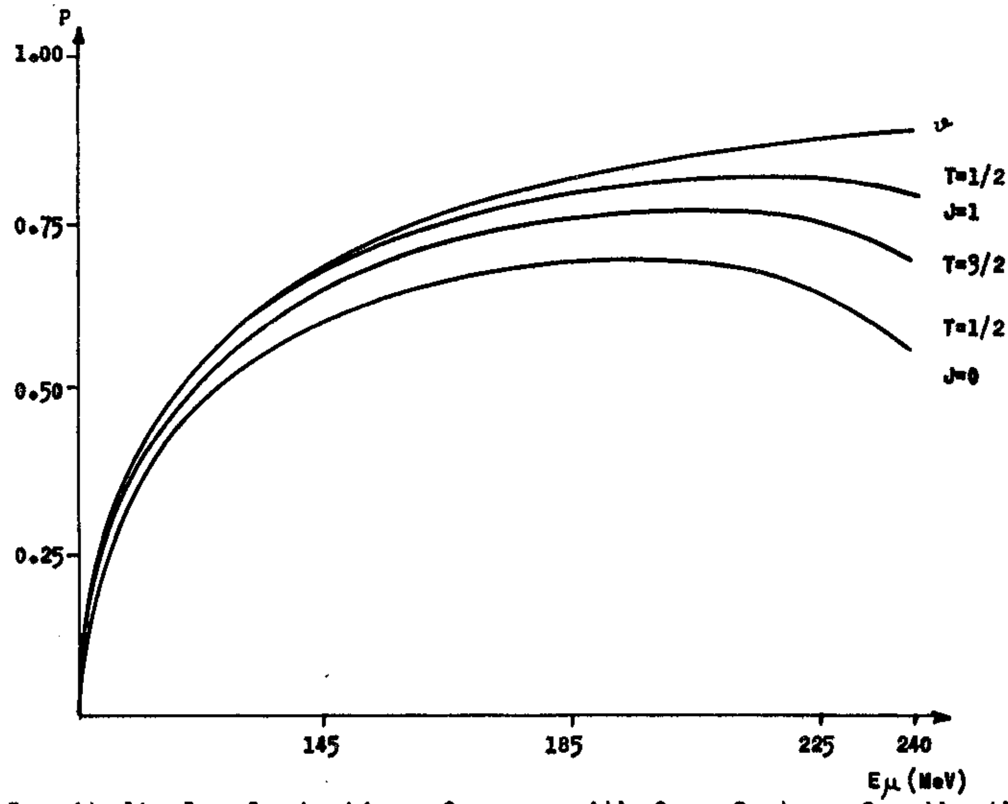

Fig. 3 - Longitudinal polarization of muons with form factors for the three cases considered. The velocity of the muons is drawn for comparison

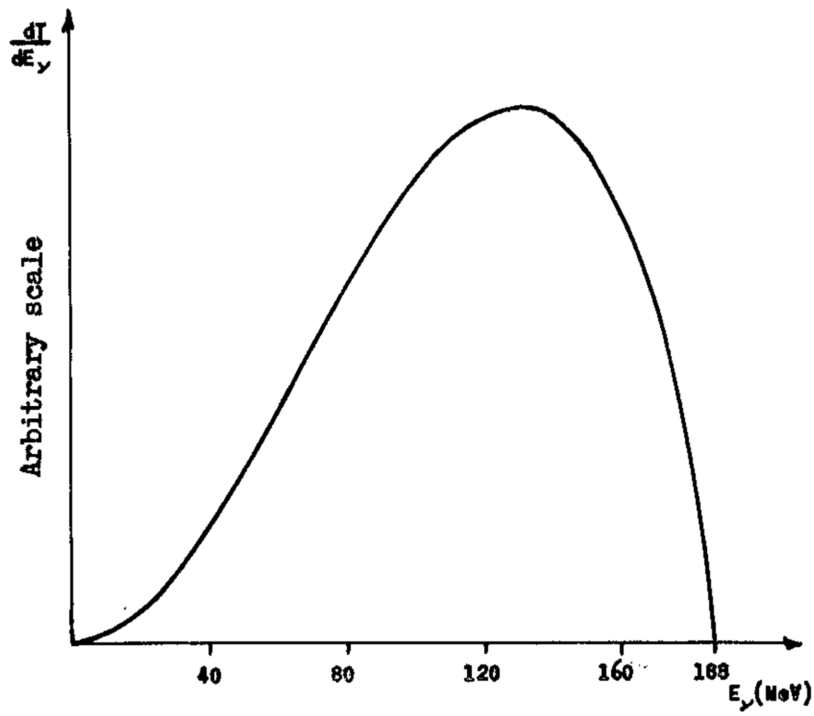

Fig. 4 - Neutrino spectrum for decay at rest, of $K_{\mu 3}$, with the form factors for Case 2 ($T = \frac{1}{2}$, $J = 1$). The curves for the other cases are pratically identical to this one.