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ON THE THEORY OF HYPERONS AND K-MESONS*

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Recent experimental results have firmly established the law of conservation of "strangeness"[1] for strong interactions. The concept of strangeness has been already most useful for the analysis and predictions of many reactions and selection rules. This property, or the equivalent ones such as the "atribute" and the "dionic number"[2], which was introduced semi-empirically results as a natural consequence from the scheme for attribution of isotopic spin to the hyperons and K-mesons due to Nishijima[3] and Gell-Mann[4]. For this reason it has been frequently assumed that the success of the application of the conservation of strangeness principle and of Nishijima-Gell-Mann scheme imply in the solid establishment of the validi-

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ty of the referred scheme. In the present paper we wish to show that the N - G scheme is not unique and to propose a different scheme which **is** also in excelent agreement with the known experimental results.

I. THE GELL-MANN-NISHIJIMA SCHEME AND THE STRANGENESS

In this scheme isotopic spin 1/2 is attributed to the nucleons, cascade particles and K-mesons (in the present paper we shall not be concerned with the difference between Θ and $\mathcal{T}$ forms of K-mesons and with parity properties — which we think are to be explained in the lines of such ideas as non conservation of parity in weak interactions[5]). Thus the neutral and charged forms of these particles are grouped in two component wave operators, the upper and lower ones corresponding to the eigenvalues +1/2 and -1/2 of the third component I_3 of the isotopic spin operator, respectively

$$N = \begin{pmatrix} P \\ n \end{pmatrix} ; \quad \Xi = \begin{pmatrix} \Xi_o \\ \Xi_- \end{pmatrix} ; \quad K = \begin{pmatrix} K_+ \\ K_o \end{pmatrix} ; \quad K' = i\,\mathcal{T}_2 K^+ = \begin{pmatrix} -K_o^+ \\ K_- \end{pmatrix} \qquad (1)$$

On the other hand Λ particle is assumed to have isotopic spin 0 and Σ particles are grouped in a triplet corresponding to isotopic spin 1 :

$$\Sigma = \begin{pmatrix} \Sigma_1 \\ \Sigma_2 \\ \Sigma_3 \end{pmatrix} ; \quad \Sigma_+ = \frac{1}{\sqrt{2}} (\Sigma_1 + i\Sigma_2) ; \quad \Sigma_- = \frac{1}{\sqrt{2}} (\Sigma_1 - i\Sigma_2)$$

$$\Sigma_o = \Sigma_3 \qquad (2)$$

Besides these, except the K particles which have ordinary spin integer, there are the antiparticles corresponding to all the other ones which have ordinary spin half integer. We shall assume that all half integer spin particles (hyperons) have spin 1/2 and that the K-particle has spin zero.

The strong interactions between these particles is to be described by a term in the Hamiltonian which is invariant under rotations in the isotopic spin space (also in the 4-dimensional space, of course). This term should lead to conservation of strangeness and of the number of particles.

The concept of strangeness has been incorporated to the theory by d'Espagnat and Prentki(6) who have made use of the complete group of orthogonal transformations in the isotopic spin space, for which there are two kinds of spinors transforming oppositely regarding to reflexions, and introduced an operator U , the isonumber which has eigenvalues +1 and -1 corresponding to spinors of the first and second kind, respectively; finally, assuming that N and K are first kind spinors and that Ξ , K' are of the second kind they were able to show that the "strangeness" is given by

$$S = U - \mathcal{N} \tag{3}$$

where $\mathcal{N}$ is the "number of particles" operator, whith eigenvalues +1 for spinor particles, -1 for antiparticles and 0 for bosons. For integer isotopic spin U comes out to be zero. It should be mentioned that the isonumber operator has also been introduced independently, by Schwinger(7) who called it the hypercharge (Y) .

The established values of S as well as the values of U (or Y) are given in table I.

Table I - Strangeness and isonumber (or hypercharge) for hyperons, K- and Π-mesons.

Particle	S	$U \equiv Y$
N	0	1
$\overline{\Xi}$	- 2	- 1
$\overline{\Sigma}$	- 1	0
Λ	- 1	0
K	1	1
K'	- 1	- 1
Π	0	0

The expression for the charge (in e units) comes out to be, in thei N - G theory:

$$Q = \frac{U}{2} + I_3$$

Now, if the interaction Hamiltonian is invariant under the full 3-dimensional orthogonal group in isotopic spin space the isonumber is conserved; if $\mathcal{N}$ is also con erved then the conservation of S is assured.

We should mention at this stage that the only features of the G - N theory which are responsible for its success in the interpretation of the experimental results are the following ones:

a - The particles N, Ξ, Σ and Λ have half integer ordinary spin, but K, K' and Π have integer spin, conservation of ordinary spin being assumed as usual.

b - The number of spinor particles (hyperons) is conserved.

c - The isotopic spin of N and Π are respectively 1/2 and 1; the sum of the isotopic spin of Λ or $\vec{\Sigma}$ and of K or K' is half integer; isotopic spin is conserved in strong interactions but not necessarily in weak ones (actually only "$\Delta I_3 \neq$ half integer" has been proved).

d - Strangeness as given by table I is conserved (or the quantity $U = \mathcal{N} + S$ is conserved).

e - The charge operator Q has only the eigenvalues 0, ± 1.

Thus we see that it is only when we wish to keep the definition of the charge as given by (4) that we are led to the N - G attribution of integer isotopic spin for $\vec{\Sigma}$, Λ and half integer for K, K' (in view of the fact that Q has to be 0, ± 1, as U is allways integer). Therefore there is place, in principle, for different attributions to these isospins.

II. GENERALIZED 4-DIMENSIONAL ISOTOPIC SPIN SPACE

In a recente paper(7) Schwinger has proposed a further extension of the isotopic spin space to a 4-dimensional one. In this way he unifies N and Ξ in one 4-isospinor, Λ and Σ in a 4-isovector and K, K' in a 4-isospinor. In this way, by imposing invariance of strong interactions with respect to rotations in a 4-dimensional isotopic spin space he unifies the four possible terms of the interactions between these particles which are invariant under rotations in the 3-dimensional isotopic spin space.

Thus he generalizes the group of rotations in isotopic spin space in such a way that besides the scalar and 3-vectors quantities there are also 4-vectors and 4-spinors*. So, if Ψ, ϕ_K (4- spinors) and Σ_μ are given by

$$\Psi = \begin{pmatrix} P \\ n \\ \Xi_o \\ \Xi_- \end{pmatrix} ; \quad \phi_K = \begin{pmatrix} K_+ \\ K_o \\ -K_o^+ \\ K_- \end{pmatrix} ; \quad \Sigma_\mu = (\vec{\Sigma}, \Lambda) \tag{5}$$

a 4-vector B_μ exists, besides the scalars $\bar{\Psi}\Psi$, and $\phi_K^+ \phi_K$ and the 3-vector $\vec{\Psi}\vec{\mathcal{T}}\Psi$ and $\phi_K^+ \vec{\mathcal{T}} \phi_K$, which is given by

$$B_r = i\bar{\Psi}\mathcal{T}_r \phi_K , \quad B_4 = \bar{\Psi}\phi_K , \quad (r = 1,2,3) \tag{6}$$

The scalar quantity which can be formed with Ψ, ϕ_K and Σ_μ is then $B_\mu \Sigma^\mu$, so the interaction Hamiltonian density is taken as:

$$\mathcal{H}_{int} = gB_\mu \Sigma^\mu + h.c = g\Psi(\Lambda + i\vec{\Sigma}.\vec{\mathcal{T}})\phi_K + h.c. \tag{7}$$

The theory is also invariant at this stage with respect to the transformation

$$N \rightleftarrows \Xi ; \qquad K \rightleftarrows K' \tag{8}$$

which assures the complete symmetry between N and Ξ.

It should be mentioned that according to Schwinger's philoso-

* The formulation of the 4-dimensional theory here attributed to Schwinger is a reconstitution of what we could infer from his paper and from indirect informations, so it may not coincide with Schwinger's formulation.

phy the masses of <u>all hyperons</u> are identical at this stage: not only those of N and Ξ, whose identity is assured by transformation (8), but also those of Λ and $\vec{\Sigma}$, although they are essentially different because they have isotopic spin different form those of N and Ξ. The introduction of the further interaction of hyperons and K-mesons with π-mesons would break the existing symmetry and results in the splitting of the mass multiplet in the same way as the mass difference between proton and neutron results from the asymmetrical interaction of these particles with the electromagnetic field. We shall mantain this assumption and disregard the mass differences among the hyperons, unless they are explicitely mentioned.

III. ALTERNATIVE SCHEME OF QUANTUM-NUMBERS FOR HYPERONS AND K-MESONS

If we wish to adhere to Schwinger's idea that Λ and $\vec{\Sigma}$ belong to the same mass multiplet as N and Ξ and that their mass differences result from asymmetric interactions with π-mesons then it seems that they should have similar properties or, in a sense, that they should be different forms of the same particle. Otherwise it would be difficult to explain the coincidence of these masses. This mass degeneracy could even result from invariance of the theory under some transformations. So we should assume, for instance, that they have the same isotopic spin (1/2) as well as the same ordinary spin (it is well established, at least, that they are all half integers).

In this section we shall try to formulate a scheme in which these conditions are fulfilled and, consequently, in which K- and K'-mesons are assumed to have <u>integer</u> isotopic spin. Thus without violating condition (c) of the list given at the end of section I. All the other conditions should also be satisfied.

For this we first group the wave functions of the hyperons in the following <u>isospinors</u>, instead of as in (1), (2):

$$N = \begin{pmatrix} P \\ n \end{pmatrix}, \quad \Xi = \begin{pmatrix} \Xi_0 \\ \Xi_- \end{pmatrix}, \Sigma_a = \begin{pmatrix} \Sigma_+ \\ \Sigma_n \end{pmatrix}, \quad \Sigma_b = \begin{pmatrix} \Sigma_{n'} \\ \Sigma_- \end{pmatrix} \tag{9}$$

Here $\Sigma_\pm$ represents the positive or negative Σ and Σ_n, $\Sigma_{n'}$, are linear combinations of the wave operators for the observed Σ_0 and Λ (please, forget their mass difference). It will be shown in section (V) that we should have indeed:

$$\Sigma_n = \frac{1}{\sqrt{2}} (\Lambda - i \Sigma_0); \quad \Sigma_{n'} = \frac{1}{\sqrt{2}} (\Lambda + i \Sigma_0) \tag{10}$$

In (9) the upper components correspond to isotopic spin 1/2 and the lower ones to -1/2.

It is clear from what was said before that the quantities U in (3) and in (4) cannot be anymore the same because U should now be integer in (3) and half integer in (4) for Σ and Λ particles. We chose to keep (5) and to modify the definition (4) of Q. In order to avoid confusion with the previous formulae we shall change notation and write, instead of (3)

$$S \qquad Y - \mathcal{N} \tag{3a}$$

the hypercharge Y being given in table I .

For Q we write:

$$Q = J_3 + I_3 \tag{11}$$

where J_3 is a new quantum number which has to be integer or half integer according to which I_3 is integer or half integer. It is clear that for hyperons, which have $I_3 = \pm 1/2$, the possible values of J_3 are also $\pm 1/2$ as Q can have only the values 0, ± 1; for K-mesons J_3 is necessarily integer (as I_3).

Now as Y and J_3 are nota anymore identical we should find the connection between them. So we introduce a new quantum number J_3' such as:

$$Y = J_3 + J_3' \tag{12}$$

J_3' is integer (or half integer) if J_3 is integer (or half integer) because Y is allways integer (see table I). One finds indeed that, in the same way as I_3 and J_3 , J_3' can have also only the values $\pm 1/2$ for the hyperons and is integer for K-mesons.

Let us now determine the values of the new quantum numbers J_3 and J_3' for π- and K-mesons.

For π-mesons we take as well established the values one for the isotopic spin ($I_3 = 0, \pm 1$). Thus we find, in view of: $Q = I_3$; $Y = 0$, that:

$$J_3 = J_3' = 0 \quad (\text{for } \pi\text{-mesons}) \tag{13}$$

For K-mesons we have, at this stage, a large arbitrarity in assigning values to I_3 , J_3 and J_3' . Only if we assume isotopic spin zero:

$$I_3 = 0 \quad (\text{for K-mesons}) \tag{14}$$

we have a unique possibility for J_3 and J_3' because now we have:

$$Q = J_3 \ ; \quad Y = Q + J_3'$$

In table II the values of I_3 , J_3 and J_3' , as well as of Q and Y , are given for all those particles. In the following section we shall find that (14) is indeed the correct choice if a physical meaning is given to J_3 and J_3' which assures conservation of the hypercharge as a consequence of invariant properties of the interaction Hamiltonian under rotations in a generalized isotopic spin space.

In the following sections we shall develope a theory in which we have, in opposition to the G - N scheme, the scheme of quantum numbers given in table II and which satisfies all conditions enumerated in section I , necessary for a good agreement with the experimental results, except the conservation laws which still have to be worked out.

IV. ISOTOPIC SPIN SPACE AND HYPERCHARGE SPACE

We wish to define the operators J_3 and J_3' with the eigenvalues considered in the previous section and to build the theory in such a way that the conservation of charge, isotopic spin and hypercharge in fast reactions would result form invariance properties of the interaction Hamiltonian. Alternatively, we should assure the conservation of I_3 , J_3 and J_3' .

For the interaction of π-mesons with nucleons this can be assured as usual, by imposing that the Hamiltonian is invariant under rotations in the isotopic spin space. The fact that iI_3 is the infinitesimal operator for rotation in the (1,2) plane assures the conservation of I_3 . In this case the conservation of J_3 and J_3' results from the facts that $J_3 = J_3' = \mathcal{N}$ both for nucleons and π-mesons and that*

$$\Delta \mathcal{N} = 0 \tag{15}$$

*The conservation of N is assured, as it is well known, by imposing invariance of the Hamiltonian under the transformation $\Psi \rightarrow e^{i\varphi}$, where φ is a real phase, the same for all hyperons particles, having opposite sign for the antiparticles and vanishing for bosons. We shall impose this invariance for our theory, in agreement with condition b of section I.

Table II - Quantum numbers I_3 , J_3 and J_3' for hyperons, K- and π-mesons.

Particles	I_3	J_3	J_3'	$Q = I_3 + J_3$	$Y = J_3 + J_3'$
P	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	1	1
n	$-\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	0	1
Ξ_o	$+\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	- 1
Ξ_-	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	- 1	- 1
Σ_+	$+\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	1	0
Σ_n	$-\frac{1}{2}$	$+\frac{1}{2}$	$-\frac{1}{2}$	0	0
$\Sigma_{n'}$	$+\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	0	0
Σ_-	$-\frac{1}{2}$	$-\frac{1}{2}$	$+\frac{1}{2}$	- 1	0
π_+	1	0	0	1	0
π_o	0	0	0	0	0
K_+	0	1	0	1	1
K_o	0	0	1	0	1
K_o'	0	0	- 1	0	- 1
K_-	0	- 1	0	- 1	- 1
π_-	-1	[illegible]	[illegible]	-1	0

In general this invariance will not be enough and we have to impose invariance under a more general rotation group. Now as we have for the hyperons three dichotomic variables (I_3 , J_3 and J'_3) which are independent the corresponding operators should commute with each other and their irreducible representation should be given by 8 x 8 matrices. Thus the wave function should have (at least) eight components (except those for the ordinary spin) which is exactly the number of the already discovered hyperons: N, Ξ , Σ_a and Σ_b . Therefore we could impose invariance under rotations in a 7-dimensional space*, which is the widest rotation group with an 8 x 8 spinor representation.

This 7-dimensional space can be built as the direct sum of the 3-dimensional <u>isotopic spin space</u> and a 4-dimensional <u>hypercharge space</u>; iI_3 being one of the three infinitesimal operators for rotations in the isotopic spin space we should take for iJ_3 and iJ'_3 two (commuting) of the six infinitesimal operators $M_{\mu\nu}$ of the hypercharge space, so that they all commute:

$$J_3 = \frac{1}{i} M_{12} \quad ; \qquad J'_3 = \frac{1}{i} M_{34} \tag{16}$$

In the spinor representation (8 x 8) each of the operators I_3 , J_3

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*A similar theory which starts directly from 7-dimensional space has been worked out by Dr. Maurice Neumar from the University of California. We are thankful to him for some information on his work.

and J'_3 have four eigenvalues $+1/2$ and four $-1/2$ as it is necessary for our purpose (this is really true only if the space is cartesian and so we assume this nature of the space). We shall call the hypervector $\vec{J} = (M_{23}, M_{31}, M_{12})$ the <u>hyperspin</u>.

We should mention that from now on we shall not assume invariance of the theory under the whole group of totations in 7-dimensional space because we would have to introduce three more particles, partners of the K , K' mesons, which have not been observed — if such particles will be found they can be included in the theory by a natural extension (these particles should have, however, hypercharge zero). We shall indeed assume only invariance under the restricted group of independent rotations in the 3-dimensional isotopic spin space and in the 4-dimensional hypercharge space. So a general tensor quantity will be of the form:

$$B^{r_1 \ldots r_m}_{\lambda_1 \ldots \lambda_n} \quad (r_i = 1,2,3 \ ; \quad \lambda_j = 1,2,3,4) \tag{17}$$

being a tensor of rank <u>m</u> in isotopic spin space and <u>n</u> in hypercharge space (we have omitted the ordinary tensor indices).

Now we shall see that there is a very strong restriction on such tensors that can be used to describe bosons if the charge and hypercharge can assume only the values 0, ± 1. Indeed it is well known from the theory of representations of the rotation group that the highest eigenvalue of I_3 is m and the highest eigenvalue of J_3 (and J_3') given by (16) is n . Thus the state with $I_3 = m$, $J_3 = n$ has the charge $m + n$ and from the condition that the highest value of the charge be 1 we have

$$m + n = 1$$

Thus we see that we are left with only two possibilities, viz

a) $m = 1$; $n = 0$ (isospin 1)

b) $m = 0$; $n = 1$ (hyperspin 1)

The first case corresponds to the π-mesons and is characterized by the fact that there are three states of the particles.

The second case corresponds to a particle with four states and corresponds to the K-mesons. Indeed, it is known that if the K-mesons are described by the hermitian hypervector

$$K_\mu = \begin{pmatrix} K_1 \\ K_2 \\ K_3 \\ K_4 \end{pmatrix} \tag{18}$$

then the operators M_{12} and M_{34} have the form:

$$iM_{12} = \begin{pmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} ; \quad iM_{34} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \end{pmatrix} . \tag{19}$$

So if make the transformation

$$K_\mu \longrightarrow \tilde{K}_\mu = \begin{pmatrix} K_+ \\ -iK_- \\ K_o \\ -iK_o^+ \end{pmatrix} \tag{20}$$

with

$$K_+ = \frac{1}{\sqrt{2}}(K_1 - iK_2) \;;\quad K_o = \frac{1}{\sqrt{2}}(K_3 - iK_4) \;;\quad K_- = K_+^+ \;. \tag{21}$$

we find

$$iM_{12} \longrightarrow \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \;;\quad iM_{34} \longrightarrow \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & -1 \end{pmatrix}$$

so that Q and Y are diagonalized:

$$Q = iM_{12} \longrightarrow \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \;;$$

$$Y = iM_{12} + iM_{34} \longrightarrow \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} \;.$$

Thus we see that we can identify the particles described by the linear combinations (21) with the observed K-particles, as their quantum numbers coincide with those given in table II.

V. HAMILTONIAN FOR THE INTERACTION OF K-MESONS WITH HYPERONS

As the K-mesons are represented by the hermitian 4-vector (K_μ) in the hypercharge space which is an scalar in the isotopic spin space we should form the invariant interaction hamiltonian density $\mathcal{H}_{int}$ by contracting K_μ with a 4-hypervector (scalar in isotopic spin space) formed with hermitian products of the components of the hyperon wave operator Ψ . Now if Γ_μ with $\mu = 1,2,3,4$ are four operators acting on the hypercharge components of Ψ (and so commuting with $\vec{I} = \frac{1}{2}\vec{\mathcal{T}}$) and having the anticommutation relations

$$\Gamma_\mu \Gamma_\nu + \Gamma_\nu \Gamma_\mu = 2\,\delta_{\mu\nu}.1 \tag{22}$$

then $\bar{\Psi}\Gamma_\mu\Psi$ is a 4-vector in the hypercharge space and a scalar in the isotopic spin space (here $\bar{\Psi} = \Psi^* \beta$) if the infinitesimal operators for rotation in the first space are taken as

$$M_{\mu\nu} = \frac{1}{4}\left(\Gamma_\mu \Gamma_\nu - \Gamma_\nu \Gamma_\mu\right) \tag{23}$$

and the invariant interaction Hamiltonian (density) will be given uniquely (if K_μ is scalar in ordinary space) by

$$\mathcal{H}_{int} = g\,K^\mu\,\bar{\Psi}\,\Gamma_\mu\,\Psi \quad , \tag{24}$$

g being a real coupling constant.

We should notice here that we would not gain any new possibility by considering the alternative expression for $\mathcal{H}_{int}$ where Γ_μ

is substituted by the "pseudo-vector" quantity

$$\Gamma_\mu \longrightarrow \Gamma_\mu' = i\,\Gamma_\mu\,\Gamma_1\,\Gamma_2\,\Gamma_3\,\Gamma_4\,, \tag{25}$$

because this new expression of $\mathcal{H}$ differe from (24) only by a unitary transformation which do not alter the Lagrangean density for free hyperons because no Γ matrices appear in it:

$$\mathcal{L}_o = \left(\frac{1}{2}\ \bar{\Psi}\gamma^\mu \frac{\partial \Psi}{\partial x^\mu} \cdot M\bar{\Psi}\Psi \right) + h.c. \tag{26}$$

Also we should not mix these two types of interaction terms because we wish to have all hyperons interacting with K-mesons with the same strength in order to assure that the identity of their masses in (26) will be mantained at this stage of the theory. Such mixture is indeed excluded if we impose also invariance of the theory in relation to inversions in hypercharge space. Such additional invariance requirement will be analized in the next section. Finally, it is superfluous to mention that if K_μ is pseudo-scalar (in ordinary space) an $i\gamma_5$ matrix should be inserted between $\bar{\Psi}$ and Ψ in (24).

Now we wish to choose a definite representation of the Γ_μ matrices in order to find how the Hamiltonian (24) compares with previous ones. It is convenient to take the representation:

$$\vec{\Gamma} = \begin{pmatrix} 0 & \vec{\eta} \\ \vec{\eta} & 0 \end{pmatrix} \qquad \Gamma_4 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{27}$$

so that

$$K_\mu\,\Gamma^\mu = \begin{pmatrix} 0 & \vec{K}\cdot\vec{\eta} - i\vec{K}_4 \\ \vec{K}\cdot\vec{\eta} + i\vec{K}_4 & 0 \end{pmatrix}$$

where $\vec{\eta}$ are 2 x 2 matrices identical in form to the Pauli matrices $\vec{\sigma}$. Thus, if $\Psi_1 = \begin{pmatrix} N \\ \Xi \end{pmatrix}$, $\Psi_2 = \begin{pmatrix} \Sigma_a \\ \Sigma_b \end{pmatrix}$ expression (24) for $\mathcal{H}_{int}$ becomes:

$$\mathcal{H}_{int} = g \ \bar{\Psi}_1 (\vec{K} \cdot \vec{\eta} - iK_4) \Psi_2 + h.c. \tag{28}$$

We have also, in view of the relations (21):

$$\vec{K} \cdot \vec{\eta} - iK_4 = \sqrt{2} \begin{pmatrix} K_o & K_o \\ K_- & -K_o^+ \end{pmatrix}$$

so that

$$\mathcal{H}_{int} = \sqrt{2}\, g \left[\bar{N} (K_o \Sigma_a + K_+ \Sigma_b) + \bar{\Xi} (K_- \Sigma_a - K_o^+ \Sigma_b) \right] + h.c. \tag{29}$$

if we introduce the isospinors which have the components:

$$K = \begin{pmatrix} K_+ \\ K_o \end{pmatrix} ; \quad K' = \begin{pmatrix} -K_o^+ \\ K_- \end{pmatrix} \tag{30}$$

and the matrix

$$\Sigma = \begin{pmatrix} \Sigma_{n'} & \Sigma_+ \\ \Sigma_- & \Sigma_n \end{pmatrix} \tag{31}$$

then (29) takes the form

$$\mathcal{H}_{int} = \sqrt{2}\ g \left(\bar{N} \Sigma K + \bar{\Xi} \Sigma K' \right) + h.c. \tag{32}$$

and finally, as we can always write

$$\sqrt{2}\ \Sigma = \Lambda + i\vec{\Sigma}\cdot\vec{\mathcal{T}}$$

our Hamiltonian takes the form:

$$\mathcal{H}_{int} = g\,[\bar{N}(\Lambda + i\vec{\Sigma}\cdot\vec{\mathcal{T}})K + \bar{\Xi}(\Lambda + i\vec{\Sigma}\cdot\vec{\mathcal{T}})K'] + h.c. \tag{33}$$

where

$$\Lambda = \frac{1}{\sqrt{2}}(\Sigma_n + \Sigma_{n'}) , \quad \Sigma_3 = \frac{1}{i\sqrt{2}}(\Sigma_{n'} - \Sigma_n)$$

$$\Sigma_1 = \frac{1}{i\sqrt{2}}(\Sigma_+ + \Sigma_-) , \quad \Sigma_2 = \frac{1}{\sqrt{2}}(\Sigma_- - \Sigma_+) \tag{34}$$

We see that $\mathcal{H}_{int}$ has not only taken the usual form (6),(8) but also has become, quite unexpectedly, formally identical to the interaction Hamiltonian (7) of Schwinger's theory. It should be mentioned here that we could modify (34) so that the factor i would not appear* in the term $i\vec{\Sigma}\,\vec{\mathcal{T}}$, as in reference (8). It should be mentioned that one of the reasons of this identify results from the fact that the group of transformations which leaves invariant the Hamiltonian is, in both cases, much wider than the group which was used in the establishment of the theory. It is indeed easy to formulate in the presente theory the transformation relation used in Schwinger's theory and to prove the invariance of our Hamiltonian, and therefore, the existence of quantities which correspond to the isotopic spin of Schwinger's (and N - G) theory, with eigenvalues

* Salam's result(9) that his coefficients g_5 to g_8 are all real, on grounds of invariance under charge conjugation, is based on the assumption that Λ and Σ transform in the same way under charge conjugation. If they transform with different signs g_5 could be real and g_6 pure imaginary, for instance, as in (33).

$\pm 1/2$ for the quadruplet (N, Ξ) 1 for Σ and 0 for Λ.

VI. INTERACTIONS OF THE HYPERONS WITH π-MESONS AND THE SPLITTING OF THE HYPERON MASS MULTIPLET

Our theory is completely symmetrical between all hyperons at this stage (before electromagnetic and π-mesons interactions are introduced). The complete degeneracy of the mass multiplet is assured by general principles, and this is true not only for the bare mass M but also for the self-mass ΔM resulting from the interaction (24). Thus the identity of the masses of the particles of isospin 1/2 and - 1/2 in the doublets (9) is assured by the invariance under rotations in the isotopic spin space. The identity of the masses of N and Ξ as well as that of Σ_a and Σ_b is assured by the invariance under rotations in the hypercharge space — here Schwinger's transformation (8) and the corresponding one for Σ_a and Σ_b are nothing more than rotation by an angle π in the (2,3) plane of the hypercharge space times a fase (i):

$$\psi \dashrightarrow \eta_1 \psi \quad ; \quad K_1 \longrightarrow K_1 \quad ; \quad K_2 \dashrightarrow -K_2 \quad ; \quad K_3 \longrightarrow -K_3 \quad ; \quad K_4 \longrightarrow K_4 \; . \tag{35}$$

Finally the identity of the masses of the (N, Ξ) to that of (Σ_a, Σ_b) quadruplet* is assured by the invariance of $\mathcal{H}_{int}$ under "space" reflexions in the hypercharge space:

$$\psi \longrightarrow \Gamma_4 \psi \quad ; \quad \vec{K} \dashrightarrow -\vec{K} \quad ; \quad K_4 \longrightarrow K_4 \; . \tag{36}$$

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* We should point out that Schwinger's inference about a possible separation of these quadruplets as a consequence of the interaction with K-mesons (Seattle conference, 1956) is not correct in view of the equivalence of his theory with the present one at this stage.

The introduction of the electromagnetic interaction breake the symmetry in the isotopic spin space and so leads to the splitting of the charge multiplets. The introduction of the interaction with π-meson in a way which break the symmetrics in the hypercharge space will lead to a further splitting. In a forthcoming paper we shall analize the ways in which such interactions may be introduced such that a splitting of the multiplet N, Ξ, Λ and $\vec{\Sigma}$ results.

We should antecipate that if this splitting is to be attributed to interactions with π-mesons then a lack of symmetry which lead to non conservation of isotopic spin in some of these interactions will be necessary in order to reproduce simultaneously the different structures of the (N, Ξ) and (Σ, Λ) quadruplets.

Indeed, in order to reproduce this structure the self-mass ΔM should have the form:

$$\Delta M = A + (B + C\eta_3)(1 + \Gamma_5) + (D + E\vec{\tau}\cdot\vec{\eta})(1 - \Gamma_5) \qquad (37)$$

Here $\Gamma_5 = -\Gamma_1 \Gamma_2 \Gamma_3 \Gamma_4$ has the eigenvalues +1 or -1 for the quadruplets (N, Ξ) and (Σ, Λ), respectively.

It is clear that ΔM being not invariant under rotations in the isotopic spin (3-dimensional) space some of the interactions of the π-mesons (with Σ, Λ, for instance) should not have this invariance.

This fact might be taken as an argument against the present formulation in view of the already accepted philosophy about the connection of the strength of the interaction with the lack of symmetry. On the other hand as the interaction with the electromagnetic is not invariant under such transformations we may expect that it is only the non conservation of parity (and of the hypercharge) that characterizes the weak interactions. On this connection it should also be mentioned that the Σ decay (and Ξ decay) in era tion which is an <u>isospinor</u> in the ordinary formulation(9) takes here the form

$$\bar{\Psi} \, \Gamma_4 \, \vec{\tau} \, \Psi . \vec{\bar{\pi}} \tag{38}$$

which is invariant under rotations in the isotopic spin space. Thus the fact that it is a weak interaction should result from the insertion in (38) of an operator which determines non conservation of parity.

VII. CONCLUDING REMARKS

In the preceeding sections we have succeded in formulating a theory of the interactions of hyperons with K-mesons which differs from the N - G theory not only on the attribution of isotopic spin to the particles but also on the transformation properties of the fields and so in the form of the interaction. One should then expect that although both theories coincide on the general properties of these particles which have up to now experimentally determined they would differ in detailed previsions of reactions which could be verified experimentally; thus one might expect that improved experiments could lead to a decision in favour of one of them. We have shown, however, that this is not the case and that the present theory is formally equivalent to the N - G theory in the generalized form of section II. Thus it will be impossible to decide between the two theories at least solely on the basis of reactions involving only hyperons and K-mesons. It is possible that even when the interactions with π-mesons and the weak interactions are introduced in the present theory this equivalence will be mantained. Even so the present formulation may be still convenient for the understanding of some symmetry properties and for computational purposes as it treats the hyperons more symmetrically.

1. Gell-Mann and A. Pais, Proc. Glasgow Conf. on Nuclear and Meson Phys. (1954) p. 324

2. R.G. Sachs, Phys. Rev. 99, 1576 (1955)

   M. Goldhaber, Phys. Rev. 101, 433 (1956)

3. Nakano and K. Nishijima, Prog. Theor. Phys. (Japan), 10, 581 (1953)
   K. Nishijima, Prog. Theor. Phys. (Japan), 12, 107 (1954)

4. Gell-Mann, Phys. Rev. 92, 833 (1953)

5. T.D. Lee and C.N. Yang, Phys. Rev. 104, 254 (1956)

6. B. d'Espagnat and J. Prentki, Nucl. Phys. 1, 33(1956)

7. J. Schwinger, Phys. Rev. 104, 1164 (1956)

8. A. Salam, Nucl. Phys. 2, 173 (1956)

9. G. Gatto, Nuovo Cimento 3, 318 (1956)
   C. Iso and M. Kawaguchi, Prog. Theor. Phys. 16, 177 (1956)