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ABSTRACT

A static, nonsingular, plane-symmetric scalar field of long range is considered under the general relativity, and a one-parametric class of exact solutions with cosmological time is obtained, in harmonic coordinates. In the absence of any material source, the gravitation originated by the pure scalar field can be studied in detail. A velocity-dependent acceleration field is found, acting attractively on the component of the velocity normal to the plane of symmetry, and repulsively on the component parallel to that plane. Particles at rest are insensitive to the gravitation, although the time component of the energy momentum tensor is point dependent and positive definite.

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1. INTRODUCTION

In the fifties, some interest was focussed on the minimum coupling of a long range scalar field with gravitation [1]. Other couplings and different types of scalar fields were later studied, like the conformal coupling [2], the field of Brans and Dicke [3], the complex scalar fields [4] and the short range fields [5]. For its simplicity, however, the field minimally coupled to gravitation still deserves special consideration [6].

In this paper, a one-parametric class of solutions of Einstein's field equations for a diffused source of the long range scalar field is obtained, with cosmological time. The solution is static in harmonic coordinates, with planar symmetry, so will not be found appropriate in the discussions of observational cosmology; but its existence may give reasons to hope that there also exist non-static solutions which might play important roles to avoid singular epochs found in the cosmological models. The solution presented here has further merit that a velocity-dependent acceleration field is found, acting attractively on a test particle with a velocity normal to the plane of symmetry and repulsively on the particle moving parallel to that plane. Particles at rest are found to be insensitive to the gravitational field, although the time component of the energy-momentum tensor is non-uniform and positive definite.

In Sec. 2 the solution is obtained in harmonic coordinates. The timelike and null geodesics are studied in Sec. 3. Finally, an analysis of the gravitational field is presented in Sec. 4, based on these geodesics.

2. FIELD EQUATIONS AND RESULTS

The Einstein-scalar equations are [1]

$$R_{ij} = -2 \quad \partial_i S \ \partial_j S \quad , \tag{1}$$

where latin indices vary from 0 to 3. For static, plane symmetric systems with cosmological time we use the line element

$$ds^2 = dt^2 - (dx^2 + dy^2)\, e^{2A} - dz^2\, e^{2B} \quad , \tag{2}$$

where the potentials A, B and S only depend on the coordinate $z = x_{(3)}$. We impose the harmonic coordinates condition [7]

$$\partial_i \left[(-g)^{1/2}\, g^{ij} \right] = 0 \quad , \tag{3}$$

and obtain the field equations (a prime means d/dz)

$$A'' = 0 \quad , \quad B' = 2A' \quad , \quad S'^2 = A'^2 \quad . \tag{4}$$

For systems presenting symmetry with respect to the plane z = 0 we find the exact solution

$$A = kZ \quad , \quad B = 2kZ \quad , \quad S = \mp kZ \quad , \tag{5}$$

$$Z = |z| \quad , \quad k = \text{const} \quad . \tag{6}$$

The potentials A, B and S are continuous through z = 0; however, their normal derivatives are not, due to a source of scalar field [8] uniformly distributed on that plane. When k = 0 the scalar field vanishes, and the space-time becomes Minkowskian.

Some gravitational invariants of the system are

$$R = 2\,k^2\,e^{-4kZ}\quad , \tag{7}$$

$$R^{ij}\,R_{ij} = \frac{1}{3}\,R^{ijkl}\,R_{ijkl} = R^2\quad , \tag{8}$$

and the nonvanishing components of the energy-momentum tensor are

$$T^0_0 = T^1_1 = T^2_2 = -\,T^3_3 = R/(16\pi) > 0\quad . \tag{9}$$

For negative k, all the quantities (7) to (9) diverge when $Z \to \infty$, while for positive k these quantities are regular in the finite regions and tend to zero in the regions far from the sources. We then only consider the case $k > 0$.

3. GEODESICS FOR $k > 0$

The geodetic differential equations arising from the line element (2), (5) are

$$\ddot{t} = 0\quad , \quad \ddot{x} = -2k\dot{x}\dot{Z}\quad , \quad \ddot{y} = -2k\dot{y}\dot{Z}\quad , \tag{10}$$

$$\ddot{Z} = k\left[(\dot{x}^2 + \dot{y}^2)\,e^{-2kZ} - 2\dot{Z}^2\right]\quad , \tag{11}$$

where a dot means d/ds. A first integral is

$$\dot{t}^2 - (\dot{x}^2 + \dot{y}^2)\,e^{2kZ} - \dot{Z}^2\,e^{4kZ} = 1\quad , \tag{12}$$

corresponding to timelike geodesics.

A seven-parametric solution of (10) and (12) is

$$t + a_1 = a_2 s \quad , \quad x + a_3 = a_4 C/k \quad , \quad y + a_5 = a_6 C/k \ , \tag{13}$$

$$s + a_7 = (a_2^2 - 1)^{-1/2}(a_4^2 + a_6^2)\left[C + \tfrac{1}{2}\sinh 2C\right]/(2k) \quad , \tag{14}$$

where the a's are constants of integration, and where the function C(Z) is given by

$$e^{kZ} = (a_4^2 + a_6^2)^{1/2} \cosh C \quad . \tag{15}$$

Considering the symmetries of the system, one finds from (13) to (15) two essentially different classes of geodesics:

1) Geodesics not crossing the plane of symmetry. One typical example is

$$s = (1 - v^2)^{1/2}\, t \quad , \tag{16}$$

$$x = 0 \quad , \quad e^{k(Z-d)} = \cosh(ky\, e^{-kd}) \quad , \tag{17}$$

$$2kvt = ky\, e^{kd} + \tfrac{1}{2}\, e^{2kd} \sinh(2ky\, e^{-kd}) \quad , \tag{18}$$

where v and d are parameters satisfying $0 \leq v < 1$, $d \geq 0$. This example represents a motion in the plane $x = 0$ (see dashed lines in Fig. 1). For $t < 0$ the particle is approaching the plane of symmetry, in an oblique direction. When $t = 0$ the particle is in the position $y = 0$, $Z = d$, and is moving parallel to the y-axis. With increasing t, the particle travels away from the plane of symmetry. The whole trajectory is symmetric under reflexion through the z-axis.

2) Geodesics crossing the plane of symmetry. Consider the example

$$s = (1 - v^2)^{1/2}\, t \quad , \tag{19}$$

$$x = 0 \ , \ \cosh(ky \sec\alpha) = e^{kZ} \sec^2\alpha - (e^{2kZ}\sec^2\alpha - 1)^{1/2} \tan\alpha \ , \tag{20}$$

$$2kvt = ky \cos\alpha + (e^{2kZ} + \sin^2\alpha)\tanh(ky \sec\alpha) \quad , \tag{21}$$

where α and v are parameters satisfying $0 < \alpha < \pi/2$, $0 \leq v < 1$. This example also represents a motion in the plane $x = 0$ (see heavy lines in Fig. 1). For $t < 0$ the particle obliquely approaches the plane of symmetry. When $t = 0$ the particle crosses the plane $z = 0$, with its velocity making an angle α with the y-axis. With increasing t, the particle moves away from the plane of symmetry. The whole trajectory is symmetric under reflexion through the origin.

In both examples the parameter v represents the modulus of velocity of the test particle, as is seen from (16) and (19). Since v is absent in (17) and (20), one concludes that the path followed by the particle does not depend on the modulus of its velocity, in this gravitational field. The equations (16) to (21) then also represent null geodesics, if one makes $v = 1$.

Finally, making $v = 0$ in the solution (16) to (18) one obtains

$$s = t \quad , \quad x = 0 \quad , \quad y = 0 \quad , \quad Z = d \ . \tag{22}$$

This solution represents a test particle at rest, at an arbitrary distance d from the plane of symmetry. This is an interesting result, since the existence of gravitation is ensured by (8). To understand this result on nonrelativistic grounds, one remarks (Tolman [9]) that the relativistic quantity which represents the diffuse source of the velocity-independent, nonrelativistic gravitation is $T^0_0 - T^1_1 - T^2_2 - T^3_3$, which vanishes due to (9).

4. DISCUSSIONS

The expression $S = \mp k z$ resembles the nonrelativistic Coulomb potential Φ associated to a uniform, planar distribution of electric charge σ on z-plane: $\Phi = \frac{1}{2}\sigma|z|$. We then interprete $\pm 2k$ as a surface density of scalar source.

Our system has T^0_0 positive definite, and an anisotropic state of stresses $(T^1_1 = T^2_2 > 0 \; , \; T^3_3 < 0)$. This anisotropy produces an interesting, velocity-dependent acceleration field. We found, in (11) with $k > 0$, that particles in motion parallel to the plane of symmetry are pushed away from it, while particles in motion perpendicular to the plane of symmetry are accelerated towards it. In spite of this acceleration, particles receding from the plane of symmetry in the normal directions always escape to infinity. This is easily seen from the timelike geodesic

$$s = (1 - v^2)^{1/2} \quad , \quad x = y = 0 \quad , \quad e^{2kZ} = 1 + 2kv|t| \quad , \qquad (23)$$

obtained from (19) to (21) in the limit $\alpha \to \pi/2$, and where one finds that Z monotonically increases with $|t|$.

Finally, one should remark that both shapes of orbits

(17) and (20) are analogous to those obtained in nonrelativistic mechanics for a plane symmetric potential of the form $V(Z) \propto \exp(-2kZ)$; however, the velocity of motion in these orbits is not the same as in the nonrelativistic analogue.

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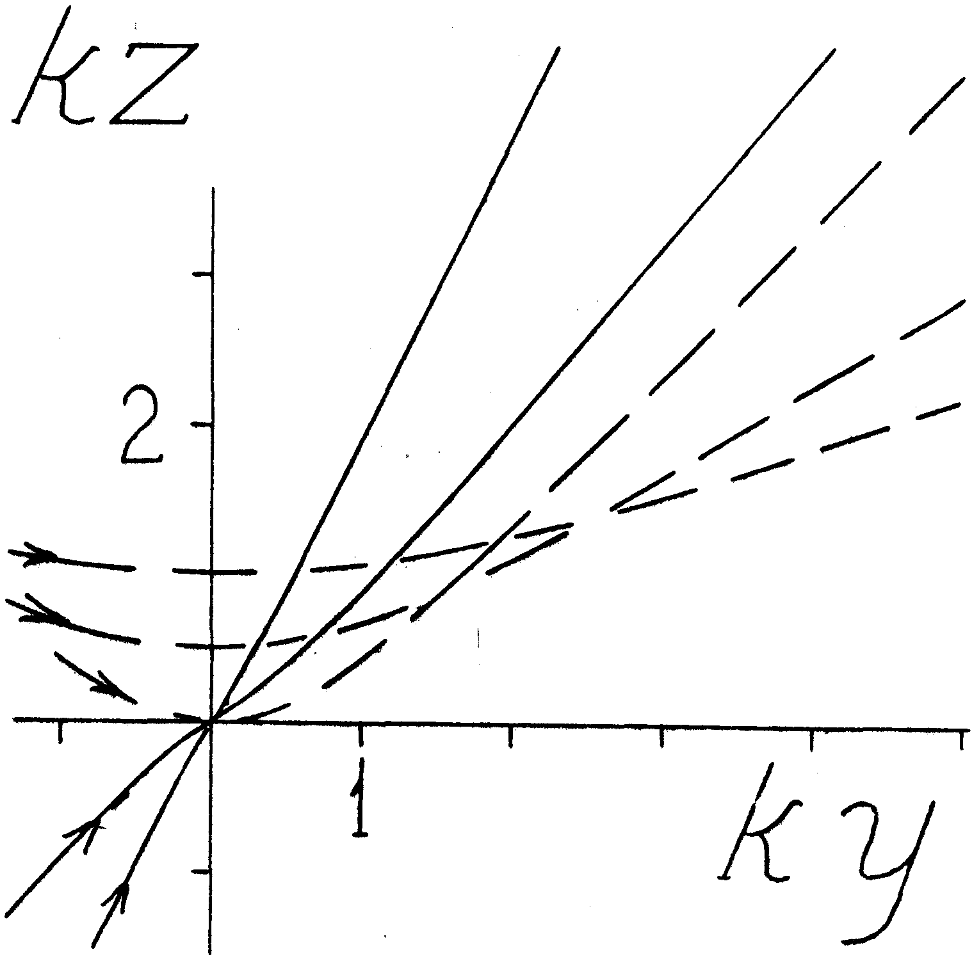

Fig. 1. Paths followed by test particles on the plane x = 0. Broken lines represent geodesics which do not cross the plane z = 0; the paths whose minimum distance d to that plane is given by kd = 0, 0.5 and 1 are presented. Heavy lines represent geodesics which cross the plane of symmetry (z = 0); the paths corresponding to angles of incidence $\alpha = 30°$ and $60°$ with the y-axis are presented.