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ON A PARITY VIOLATING NUCLEON NUCLEON POTENTIAL *

R. Lacaze **
Centro Brasileiro de Pesquisas Físicas
Rio de Janeiro - Brazil

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ABSTRACT

A static parity violating nucleon-nucleon potential due to the two pion exchange is constructed, assuming the current-current and CVC hypotheses for weak interactions. Rescattering terms as well as the $\pi\pi$ P - wave in the strong π- N vertex are taken into account.

Our results differ from those obtained by Blin Stoyle in a perturbation calculation for the strong interaction; namely, the range of our potentials is longer and this might modify Partovi's results for the asymmetry coefficients in photodisintegration of the deuteron.

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I - INTRODUCTION

As suggested by a certain number of experiments [1,2], there is some evidence for a parity violation in nuclear interaction, the rate of the parity non conserving parts of the interaction being of the order 10^{-7}.

Such a violation is allowed when one assumes the current-current hypothesis [3] in the weak processes by introducing a weak nucleon-nucleon interaction. Such an interaction may be obtained by considering either a direct interaction or a π-meson exchange between the nucleons.

The first type of interaction (called contact interaction) has been studied by F. C. Michel [4] but it gives rise to very short range forces. The second type (π-meson exchange) has been studied by R. J. Blin-Stoyle [5] in second order in perturbation theory for the strong interaction vertex (see fig. 1). This calculation does not take into account the rescattering terms which are known to be important in the π-N scattering, so that it seems interesting to treat more accurately this type of mechanism giving rise to the parity violating nucleon-nucleon forces.

The effects of parity violating potentials have been studied by different authors. Thus static potential of the form $(\vec{\sigma}^1 \wedge \vec{\sigma}^2).\vec{r}$ has been used for computing the effects of parity violation in the deuteron photodisintegration and also in the radiative capture of neutrons by hydrogen and by deuteron [6,7].

With the help of this model Partovi [7] has shown that such effects are not easily measurable at the present. For example, in the photodisintegration of the deuteron, the asymmetry coefficients of the cross sections and the polarizations are expected to be of the order of 10^{-8} to 10^{-7}.

In the same way, a velocity dependent potential $(\vec{\sigma}^1 - \vec{\sigma}^2).\vec{p}$ has been used for evaluating the non regular γ transition amplitudes of the Ta^{181}, Lu^{175}, Fe^{57} and Tl^{203} nuclei [8,9]. The circular polarizations found experimentally (of the order 10^{-4}) are compatible with a rate F taken between 9×10^{-7} and 110×10^{-7}, where F stands [10] for the rate of the parity non conserving to the parity conserving forces.

A precise determination of the parity violating nucleon-nucleon potential may favour more important effects in light nuclei, and on the other hand, may provide more accurate predictions in heavy nuclei.

We have thus computed the contribution to the parity non conserving N - N potential due to the two pion exchange, taking into account the rescattering terms, as well as the $\pi - \pi$ p-wave term in the π - N scattering. Furthermore, the effects of form factors in the weak currents are also included.

In the strong interaction case, Amati, Leader and Vitale [11] have computed the two pion exchange contribution (TPEC) to the elastic scattering amplitude, using an analytic continuation of the pion-nucleon amplitude at low energy. Cottingham and Vinh Mau [12] have shown that, with a certain approximation, one can derive from the TPEC an equivalent energy independent potential.

We shall see that the current hypothesis for weak processes will allow a parity violating part in the pion-nucleon amplitude and, consequently, a parity non conserning nucleon-nucleon interaction.

In the following we shall use the method and the notations of references (11) and (12). In section II, we expand the nucleon-nucleon scattering amplitude in a set of scalar and pseudo scalar perturbative operators and we generalize for the coefficients of these invariants the dispersion relation postulated in ref. (11).

Unitarity is used for the computation of the spectral functions in terms of the elastic pion-nucleon scattering amplitude. The latter is calculated in section III to the first order in the weak interaction.

From the scattering amplitude we construct, in section IV, the potentials in the way given by Cottingham and Vinh Mau.

The results are presented and discussed in section V.

II - THE SCATTERING AMPLITUDE IN FIELD THEORY

We consider the elastic scattering of two nucleons from a state with four momenta n_1, p_1, to a state with four momenta n_2, p_2 (fig. 2a).

We choose the three linearly independent four-vectors

$$N = \frac{1}{2}(n_1 + n_2) \qquad P = \frac{1}{2}(p_1 + p_2) \qquad \Delta = n_1 - n_2 = p_2 - p_1 \qquad \text{(II.1)}$$

and the three scalars

$$s = -(p_1 + n_1)^2 \qquad t = -(n_1 - n_2)^2 \qquad \bar{t} = -(p_1 - n_2)^2 \qquad \text{(II.2)}$$

with the constraint

$$s + t + \bar{t} = 4\, m^2 .$$

The scattering amplitude is defined from the S matrix elements, by

$$\begin{aligned} S_{fi} &= S_{fi} + i\,\delta^4(n_1 + p_1 - n_2 - p_2)(m/2\pi E)^2 \\ &\times \bar{u}(p_2)\chi_{p_2}^{+}\,\bar{u}(n_2)\chi_{n_2}^{+}\, M\, u(n_1)\chi_{n_1}\, u(p_1)\chi_{p_1} \end{aligned} \qquad \text{(II.3)}$$

where E is the nucleon energy in the center of mass system and $u(\alpha)$ and χ_α are respectively the space-time spinor and isospinor of the nucleon with four momentum α.

M may be expanded in a set of spin and isospin invariant operators:

$$M = \sum_{i,j,k} P_{ij}{}^{k}(w,t,\bar{t})\, S_i^k\, T_j{}^k \qquad \text{(II.4)}$$

where k stands for the symbols + or -, with $S_i{}^{+}$ ($S_i{}^{-}$) and $T_j{}^{+}$ ($T_j{}^{-}$) symmetric (antisymmetric) with respect to the exchange of the nucleons n and p.

If one only requires charge conservation, the operators $T_j^{\pm}$ take the form [5 and 13]

$$T_j^{\,+} = 1^n 1^p,\ \vec{\tau}^{\,n}.\vec{\tau}^{\,p},\ \tau_3{}^n\tau_3{}^p,\ \tau_3{}^n 1^p + 1^n\tau_3{}^p;\quad j = 1,4 \qquad \text{(II.5)}$$

$$T_j^{\,-} = \tau_3{}^n 1^p - 1^n\tau_3{}^p,\ (\vec{\tau}^{\,n}\wedge\vec{\tau}^{\,p})_3 \qquad ;\ j = 1,2$$

where $(\vec{\tau}^{\,n}\wedge\vec{\tau}^{\,p})_3$ changes sign under time reversal.

The $S_j^{\pm}$ operators are chosen to be:

$$S_1^{\,+} = 1^n\, 1^p \qquad S_6^{\pm} = \gamma_5{}^n\gamma_5{}^p(i\gamma^n.P \pm i\gamma^p.N)$$

$$S_2^{\pm} = i\gamma^n.P 1^p \pm 1^n i\gamma^p.N \qquad S_7^{\pm} = \gamma_5{}^n\, i\gamma^n.P 1^p \pm 1^n\gamma_5{}^p i\gamma^p.N)$$

$$S_3^{\,+} = i\gamma^n\ .\ P\ i\gamma^p.N \qquad S_8^{\pm} = (\gamma_5{}^n \pm \gamma_5{}^p)\gamma^n.\gamma^p \qquad \text{(II.6)}$$

$$S_4^{\,+} = \gamma^n.\gamma^p \qquad S_9^{\pm} = i\gamma_5{}^n 1^p \pm 1^n i\gamma_5{}^p$$

$$S_5^{\,+} = \gamma_5{}^n\gamma_5{}^p \qquad S_{10}^{\pm} = i\gamma_5{}^n i\gamma^p.N \pm i\gamma^n.P\, i\gamma_5{}^p$$

It is possible to show that all other possible spin operators may be expressed in terms of these ten operators, by developping γ^n and γ^p in terms of four orthogonal four vectors constructed with N, P and Δ and by using the Dirac equation. According to the transformations properties given in Table I, these operators $S_i^{\pm}$ may be classified [14] as proper scalars for i = 1,5, time pseudoscalars for i = 6, space pseudoscalars for i = 7,8, and proper pseudo-scalars for i = 9, 10.

i	P	T
1 - 5	+	+
6	+	-
7 - 8	-	+
9 - 10	-	-

Table I - Transformations propesties of the $S_i^{\pm}$.

These invariant operators are a generalization of those defined by Amati, Leader and Vitale; they appear in the computation of fourth order Feynman graphs (an invariant of the form $(\gamma_5^n \pm \gamma_5^p)\ i\gamma^n \cdot P\ i\ ^p . N$ also appears but one can express it in terms of $S_7^{\pm}$ and $S_8^{\pm}$ without introducing any new singularity).

The Mandelstam representation for the scalar functions $P^k_{i,j}$ may then be considered as valid to the fourth order. If it is assumed to be valid for any order, then one can write

$$P_{ij}^{\pm}(w,t,\bar{t}) = \frac{\alpha_{ij}^{\pm}}{t-\mu^2} + \frac{1}{\pi}\int_{4\mu^2}^{\infty} \frac{\rho_{ij}^{\pm}(w,t')}{t'-t}\,dt' + \frac{1}{\pi^2}\int_{9\mu^2}^{\infty}\frac{dt'}{t'-t}\int_{4m^2}^{\infty}\frac{\chi_{ij}^{\pm}(w',t')}{w'-w}\,dw' \qquad \text{(II.7)}$$

As we are interested in the long range forces, we shall neglect the contribution of the last integral in (II.7) the t-dependence of which corresponds to a potential due to an exchange of to at least three pions [12].

In fact we are only interested in the parity violating effects and then we restrict ourselves to the contributions $P^{\pm}_{i,j}$ with i = 6,10.

The pole term in (II.7) corresponds to the one pion exchange (fig. 2b). It was pointed out, some time ago [15], that a parity violation in the π-N vertex implies violation under time reversal, but only if charge independence is assumed. In fact, if we define the form factors by:

$$\langle n_2|J^V|n_1\rangle = i\bar{u}(n_2)\Big\{F_1(t)\gamma_\mu - \frac{g}{m} F_2(t)\sigma_{\mu\nu}(n_1-n_2)^\nu + i\,\frac{a}{m_\mu} F_3(t)(n_1-n_2)_\mu\Big\}u(n_1)\tau^- \qquad (II.8)$$

$$\langle 0|J_\mu|\pi\rangle = -\mu F_\pi(t)q_\mu\phi^-$$

where m, μ, m_μ are the nucleon, pion and lepton masses respectively, g is the gyromagnetic ratio, a some constant and $F_1(0) = F_2(0) = F_3(0) = 1$, we get, with the current-current hypothesis, a non vanishing pole term contribution to the invariants $S_9^+T_2^+$, $S_9^+ T_3^+$, $S_9^- T_2^-$ defined by (II.5) and (II.6). This pole contribution to (II.7) is given by:

$$\alpha_{92}^+ = \frac{1}{2}\, g_{\pi N}\, G_V\, \frac{\mu}{m_\mu}\, a\, t\, \mathrm{Im}\Big\{F_\pi^*(t)\, F_3(t)\Big\}$$

$$\alpha_{93}^+ = -\alpha_{92}^+ \qquad (II.9)$$

$$\alpha_{92}^- = \frac{1}{2}\, g_{\pi N}\, G_V\, \frac{\mu}{m_\mu}\, a\, t\, \mathrm{Re}\Big\{F_\pi^*(t)\, F_3(t)\Big\}$$

where $g_{\pi N}$ is the π - N - coupling constant.

We note that such a pole term corresponds to a second kind current as defined by Weinberg [16] but, if the form factor $F_i(t)$ are real, this contribution, as opposed to the lepton case [17], does not imply a time reversal violation on account of particular isotopic spin dependence (contribution to $(\vec{\tau}^n \wedge \vec{\tau}^p)_3$). However, this term implies in non conservation of a vector current and is disregarded in the actual computation.

The spectral functions $\rho^{\pm}_{ij}(w,t')$ are obtained by writing the unitarity relation in the t channel for the M matrix. We obtain:

$$\sum_{ijk} \rho^{k}_{ij}(w,\ t')\ S_i\ {}^{k}T_j{}^{k} = \frac{1}{2(2\pi)^2 t}\,\mathcal{M} \tag{II.10}$$

with $\mathcal{M} = \sum_{2\pi} \langle \bar{p}_1\ p_2 | \tau^{+} | 2\pi \rangle \langle 2\pi | \tau | n_1 \bar{n}_2 \rangle$

where τ stands for the annihilation $N + \bar{N} \rightarrow 2\pi$ amplitude.

The summation is to be extended to the isospin indices and to the four momenta of the pions in the intermediate state (the latter is essentially an angular integration [11]).

If we put τ into the form $\tau = \tau^{S} + \tau^{W}$ where τ^{S} stands for the strong interaction amplitude and τ^{W} for the weak interaction amplitude, the terms $\tau^{S+}\tau^{W}$ and $\tau^{W+}\tau^{S}$ will give us the contribution we are looking for. These annihilation amplitudes are obtained by analytic continuation of the elastic scattering $\pi - N$ amplitude.

Such an amplitude is written in the more general form as:

$$F = F_1 - F_2\vec{t}\cdot\vec{\tau} + F_3 t_3 \tau_3 - \frac{1}{2} F_4 t_3 + \frac{1}{3} F_5 \tau_3 + \frac{3}{2} F_6 t_3{}^{2} + \frac{1}{2} F_7 \tau_3 t_3{}^{2} \tag{II.11}$$
$$+ F_8(\vec{t}_\wedge \vec{\tau})_3 + \frac{1}{4} F_9(\vec{t}\cdot\vec{\tau} t_3 + t_3 \vec{t}\cdot\vec{\tau}) + F_{10}((\vec{t}_\wedge \vec{\tau})_3 t_3 - t_3(\vec{t}_\wedge \vec{\tau})_3)$$

with [18]:

$$F_i = - A_i - A_{i5} i\gamma_5 + i\gamma.QB_i + i\gamma.Q\gamma_5 B_{i5} \qquad \text{(II.12)}$$

where Q is one half of the sum of the pion quadrimomenta, A and B are scalar functions of the usual scalars s, $\bar{s}$, t and $\vec{t}$ and $\vec{\tau}$ are the usual isotopic spin matrices.

We take this general form for the weak interaction amplitude F^W but the restrictions upon the strong interaction amplitude reduce F^S to:

$$F^S = F_1^S - F_2^S \, \vec{t} \cdot \vec{\tau} \qquad \text{(II.11')}$$

with

$$F_i^S = - A_i^S + i\gamma.Q \, B_i^S \qquad \text{(II.12')}$$

If α and β are the isotopic spin indices of the two pions, we must consider the product of the matrix elements $F_{\alpha\beta}^{S*} F_{\beta\alpha}^{W}$ and take the sum over the isotopic spin states of the pions.

We get

$$\begin{aligned}
\sum_{\alpha\beta} F_{\alpha\beta}^{S*} F_{\beta\alpha}^{W} = {} & 3 F_1^{S*}(F_1^W + F_6^W) \, 1^n 1^p \\
& + 2 F_2^{S*} F_2^W \vec{\tau}^n \cdot \vec{\tau}^p \\
& - 2 F_2^{S*} F_3^W \tau_3^{\,n} \tau_3^{\,p} \qquad \text{(II.13)} \\
& + \frac{1}{2} \left[F_1^{S*}(F_5^W + F_7^W + F_9^W) + F_2^{S*} F_4^W \right] (\tau_3^n + \tau_3^p) \\
& + \frac{1}{2} \left[F_1^{S*}(F_5^W + F_7^W + F_9^W) - F_2^{S*} F_4^W \right] (\tau_3^n - \tau_3^p) \\
& + 2 F_2^{S*} F_8^W (\vec{\tau}^n . \vec{\tau}^p)_3
\end{aligned}$$

Taking into account the definitions (II.5), we can write the above formula in the form

$$\sum_{\alpha\beta} F_{\alpha\beta}^{s*} F_{\beta\alpha}^{w} = \sum_{ijk\ell} C^{\ell}_{ijk} F_i^{s*} F_j^{w} T_k^{\ell} \qquad \text{(II.13)}$$

where T_k^{ℓ} are the isotopic spin operators and the coefficients C^{ℓ}_{ijk} can be obtained from (II.13).

With the appropriate phase space factors, we can then write:

$$\mathcal{H} = \frac{1}{16} (t(t-4\mu^2))^{\frac{1}{2}} \sum_{ijk\ell} C^{\ell}_{ijk} \mathcal{H}_{ij} T_k^{\ell} + S_y \qquad \text{(II.14)}$$

where the term S_y comes from the product $F^{w*} F^{s}$ and can be obtained directly by symmetrisation.

If we restrict ourselves to the part of the $\mathcal{H}$ which violates the spacial parity conservation, $\mathcal{H}_{ij}$ takes the form:

$$\mathcal{H}_{ij} = \int \left[-A_i^{s*} + i\gamma^p.Q\, B_i^{s*}\right]\left[-A^w_{j5}\, i\gamma_5{}^n + i\gamma^n.Q\gamma_5{}^n B_{j5}{}^w\right] d\Omega_{\vec{Q}} \qquad \text{(II.15)}$$

$$= a_{ij} i\gamma_5{}^n + b_{ij} i\gamma^p.N i\gamma_5{}^n + c_{ij} i\gamma^n.P\gamma_5{}^n + d_{ij}\gamma^n.\gamma^p\gamma_5{}^n + e_{ij} i\gamma^n.P\, i\gamma^p.N\gamma_5{}^n$$

where

$$a_{ij} = \int \left[A_i^{s*} A_j^{w} + m\alpha B_i^{s*} A_j^{w}\right] d\Omega_{\vec{Q}}$$

$$b_{ij} = -\int \beta B_i^{s*} A_j^{w}\, d\Omega_{\vec{Q}}$$

$$c_{ij} = -\int \left[\alpha A_i^{s*} B_j^{w} + m\left(\alpha^2 - \frac{P^2}{D}\delta\right) B_i^{s*} B_j^{w}\right] d\Omega_{\vec{Q}} \qquad \text{(II.16)}$$

$$d_{ij} = -\int \delta B_i^{s*} B_j^{w}\, d\Omega_{\vec{Q}}$$

$$e_{ij} = \int \left(\alpha\beta + \frac{N.P}{D}\delta\right) B_i^{s*} B_j^{w}\, d\Omega_{\vec{Q}}$$

with

$$D = N^2P^2 - (N.P)^2$$

$$\alpha = \frac{1}{D}\,(N^2 Q.P - P.N\; Q.N)$$

$$\beta = \frac{1}{D}\,(P^2 Q.N - P.N\; Q.P)$$

$$\delta = Q^2 - (\alpha P + \beta N)^2$$

After symmetrization, the operators $(i\gamma^n.P\gamma_5^n i\gamma^p.N \pm$ $\pm\, i\gamma^n.P\; i\gamma^p.N\gamma_5^p)$ can be expressed in terms of the others:

$$i\gamma^n.P\; i\gamma^p.N(\gamma_5^n+\gamma_5^p) = \frac{\bar{t}}{2}\,(\gamma_5^n+\gamma_5^p)\gamma^n.\gamma^p + m(\gamma_5^n i\gamma^n.P+\gamma_5^p i\gamma^p.N)$$

$$i\gamma^n.P\; i\gamma^p.N(\gamma_5^n-\gamma_5^p) = -\frac{w}{2}\,(\gamma_5^n-\gamma_5^p)\gamma^n.\gamma^p - m(\gamma_5^n i\gamma^n.P-\gamma_5^p i\gamma^p.N)$$

The we obtain the expansion of $\mathcal{M}$ in terms of the invariant operators and the knowledge of the pion nucleon amplitude in the appropriate region of integration allows us to compute the spectral functions of (II.10).

III - THE π - N AMPLITUDE

We are mainly interested on the nucleon scattering near the threshold, i.e. for $w \simeq 4\,m^2$, and then for small t and $\bar{t}$. The most important contribution to the integral of the spectral function $\rho_{ij}^{\pm}(w,t')$ in equation (II.7) occur for not too large values of the variable t'. These spectral functions can then be obtained by extrapolation of the pion nucleon amplitude at low energy [12].

For the strong interaction part such an amplitude has been constructed by Bowcock, Cottingham and Lurié [19] and used in references (11) and (12). We make use here of an equivalent amplitude which we have built up in a previous work [20].

This amplitude takes into account the pole term due to the nucleon exchange, a rescattering term approximated by the pole of the N^*_{33} resonance and the $\pi - \pi$ interaction in the S and P waves also approximated by poles (ρ pole for the latter).

The $\pi - N$ amplitude to the first order of weak interaction can be constructed in a similar way in the framework of the current-current hypothesis. However the terms corresponding to the exchange of the nucleon and of the isobar N^*_{33} (fig. 3) do not contribute if the vector current is conserved. Indeed the product of the π current with another current is nothing else than the divergence, as can be seen in the computation of the pole term. Thus these contributions are ruled out in practical computation.

The term equivalent to the $\pi - \pi$ p wave contribution may be obtained either by including directly the ρ, or by introducing the ρ in the form factor of the 2 pion current; this last procedure was adopted here. We make use of the following currents:

$$\langle \pi_2 | J_\mu{}^V | \pi_1 \rangle = - 2F_{\pi\pi}(t) Q_\mu t^+$$

$$\langle n_2 | J_\mu{}^A | n_1 \rangle = -\lambda \; i \, \bar{u}(n_2) \left\{ F_A(t)\gamma_\mu - i \frac{b}{m_\mu} F_p(t)\Delta_\mu + \frac{c}{m} F_{MI}(t)\sigma_{\mu\nu}\Delta_\nu \right\} \gamma_5 u(n_1) \tau^- \qquad \text{(III.1)}$$

where $\lambda = G_A / G$

and obtain by comparing with (II.11) and (II.12):

$$
\begin{aligned}
A_{25}^{W} &= 4\, G_A \frac{c}{m} N.Q \,\mathrm{Im}\left\{F_{\pi\pi}^{*}(t)\, F_{MI}(t)\right\} \\
A_{35}^{W} &= A_{25}^{W} \\
A_{85}^{W} &= -\,4\, G_A \frac{c}{m} N.Q \,\mathrm{Re}\left\{F_{\pi\pi}^{*}(t)\, F_{MI}(t)\right\} \qquad \text{(III.2)} \\
B_{25}^{W} &= 2\, G_A \,\mathrm{Re}\left\{F_{\pi\pi}^{*}(t)\, F_{A}(t)\right\} \\
B_{35}^{W} &= B_{25}^{W} \\
B_{85}^{W} &= -\,2\, G_A \,\mathrm{Im}\left\{F_{\pi\pi}^{*}(t)\, F_{A}(t)\right\}
\end{aligned}
$$

The form factors $F_{\pi\pi}(t)$, $F_A(t)$ and $F_{MI}(t)$ are normalized to 1 for $t = 0$ and in the physical region for t, they may become complex [21].

The explicit expression for A_i^S, B_i^S, A_{j5}^W, B_{j5}^W in terms of $\vec{Q}$ allows us to perform the integration (II.13). The spectral functions ρ_{ij}^{k} are then determined by (II.10) bur their t - dependence for $t \geqslant 4\mu^2$ requires the knowledge of the form factors. We assume that the form factors $F_{\pi\pi}(t)$ and $F_A(t)$ are dominated by the exchange of the ρ and of the A_1 [22] resonance (1080 MeV), respectively (we assume for the latter $j^{PG} = 1^{+-}$). The $F_{MI}(t)$ form factor corresponds to the exchange of a 1^{++} system [21] and could be dominated by the B resonance (1210 MeV). However there is no evidence of such a pseudo-tensorial current and it has not been included in our numerical computations.

IV - DEFINITION OF THE POTENTIAL

A nucleon nucleon potential constructed so as to reproduce the scattering amplitude in field theory has been obtained by Cottingham and Vinh Mau [12] in the case where parity conservation is assumed. In a similar way, we can obtain a potential to the first order of weak interactions, the main difference being the introduction of supplementary invariants. Besides that, if we restrict ourselves to the determination of a parity violating potential, the absence of the pion exchange (because of the absence of a second kind vector current) simplifies this determination.

The scattering amplitude has been expanded in a set of linearly independent invariants (II.4). There will be then as many types of linearly independent potentials as these invariants:

$$V(\vec{r}) = \sum_{\alpha j k} V_{\alpha j}{}^{k}(r)\Omega_{\alpha}{}^{k}\, T_{j}{}^{k} \qquad \text{(IV.1)}$$

where $k = \pm 1$ and the $T_j{}^k$ are isospin invariants defined in (II.5). We have chosen the Ω_α^k to be:

$$\Omega_{ps}{}^{+} = (\vec{\sigma}^{n} \wedge \vec{\sigma}^{p}).\vec{r} \qquad \Omega_{ps}{}^{-} = \vec{\sigma}^{n}.\vec{P}\vec{\sigma}^{p}.\vec{L} + \vec{\sigma}^{p}.\vec{P}\vec{\sigma}^{n}.\vec{L}$$

$$\Omega_{pv}{}^{+} = (\vec{\sigma}^{n} - \vec{\sigma}^{p}).\vec{P} \qquad \Omega_{pv}{}^{-} = (\vec{\sigma}^{n} + \vec{\sigma}^{p}).\vec{P}$$

$$\Omega_{pTs}{}^{+} = (\vec{\sigma}^{n} - \vec{\sigma}^{p}).\vec{r} \qquad \Omega_{pTs}{}^{-} = (\vec{\sigma}^{n} + \vec{\sigma}^{p}).\vec{r} \qquad \text{(IV.2)}$$

$$\Omega_{pTv}{}^{+} = (\vec{\sigma}^{n} \wedge \vec{\sigma}^{p}).\vec{P} \qquad \Omega_{pTv}{}^{-} = -(\vec{\sigma}^{n}.\vec{r}\,\vec{\sigma}^{P}.\vec{L} + \vec{\sigma}^{p}.\vec{r}\vec{\sigma}^{n}.L)/r^{2}$$

where $\vec{L} = \vec{r}_{\wedge}\vec{P}$ and $\vec{\sigma}^{n}$, $\vec{\sigma}^{p}$ are the Pauli spin operatos acting on the nucleons n and p.

In the momentum space the potential is written:

$$V(\vec{\Delta}) = \sum_{\alpha jk} \tilde{V}_{\alpha j}^{\ k} \ \tilde{\Omega}_{\alpha}^{\ k} \ T_j^{\ k} \qquad (IV.3)$$

ith [23]

$$\tilde{\Omega}_{ps}^{\ +} = - i(\vec{\sigma}^n \wedge \vec{\sigma}^p).\vec{\Delta} \qquad \tilde{\Omega}_{ps}^{\ -} = i(\vec{\sigma}^n.\vec{P}\vec{\sigma}^p.\vec{P}\wedge\Delta + \vec{\sigma}^p.\vec{P}\vec{\sigma}^n.\vec{P}\wedge\vec{\Delta})$$

$$\tilde{\Omega}_{pv}^{\ +} = (\vec{\sigma}^n - \vec{\sigma}^p).\vec{P} \qquad \tilde{\Omega}_{pv}^{\ -} = (\vec{\sigma}^n + \vec{\sigma}^p).\vec{P} \qquad (IV.4)$$

$$\tilde{\Omega}_{pTs}^{\ +} = - i(\vec{\sigma}^n - \vec{\sigma}^p).\bar{\Delta} \qquad \tilde{\Omega}_{pTs}^{\ -} = - i(\vec{\sigma}^n + \vec{\sigma}^p).\bar{\Delta}$$

$$\tilde{\Omega}_{pTv}^{\ +} = (\vec{\sigma}^n \ \vec{\sigma}^p).\vec{P} \qquad \tilde{\Omega}_{pTv}^{\ -} = (\vec{\sigma}^n.\vec{\Delta}\vec{\sigma}^p.\vec{P}\wedge\vec{\Delta} + \vec{\sigma}^p.\vec{\Delta}\vec{\sigma}^n.\vec{P}\wedge\vec{\Delta})/\Delta^2$$

In order to identify the potential scattering amplitude given by the Lippman-Schwinger expansion and the field theory scattering amplitude, we expand the latter in terms of these invariants. By expressing the Dirac matrices and spinors with the help of the Pauli operators and spinors, we get

$$\bar{u}(n_2)\ \bar{u}(p_2)\ S_i^{\pm}\ u(p_1)\ u(n_1) = \sum_{\alpha} X_{i\alpha}^{\pm}\ \chi_{p_2}^{s+}\ \chi_{n_2}^{s+}\tilde{\Omega}_{\alpha}^{\pm}\ \chi_{n_1}^{s}\ \chi_{p_1}^{s} \qquad (IV.5)$$

In the adiabatic approximation [12 and 24], the transformation matrix X concerning the parity violating invariants is

$X_{i\alpha}^{\pm}$	PS^+	PS^-	$PV^{\pm}$	$PTS^{\pm}$	$PTV^{\pm}$
7	$+ t/8m^2$	$1/2\ m^2$	2	0	0
8	$+ 1/m$	$1/2\ m^3$	$- 2/m$	0	0
9	0	0	0	$1/2\ m$	$- t/8\ m^3$
10	0	0	0	$1/2\ m$	$- 3t/8\ m^2$

This transformation is energy independent and introduces no new singularity.

If we then consider the potential

$$V(\vec{\Delta}) = -\frac{1}{(2\pi)^3} \sum_{\alpha j k} \left\{ \sum_i X_{i\alpha}{}^k(t) \frac{1}{\pi} \int_{4\mu^2}^{\infty} \frac{\rho_{ij}{}^k(t')}{t' - t} dt' \right\} \tilde{\Omega}_\alpha^k \, T_j{}^k \qquad \text{(IV.7)}$$

It will reproduce the scattering amplitude defined in (II.7) for the longest range, since the one pion exchange term does not contribute.

The potential (IV.1) is obtained by an inverse Fourier transformation. If the interchange of the order of integration is assumed to be valid, one obtains:

$$V_{\alpha j}{}^k(r) = -\frac{1}{(2\pi)^3} \sum_i \int_{4\mu^2}^{\infty} X_{i\alpha}{}^k(t) \, \rho_{ij}{}^k(t) \, O_\alpha{}^k(r,t) \frac{e^{-rt^{\frac{1}{2}}}}{r} dt \qquad \text{(IV.8)}$$

where the $O_\alpha(r,t)$ depend on the potential type (scalar, vector, tensor)

$$\begin{aligned} O_\alpha^k(r,t) &= 1 && \text{for } (\alpha, k) = PV^{\pm}, PTV^{+} \\ &= \frac{t^{\frac{1}{2}}}{r}\left(1 + \frac{1}{rt^{\frac{1}{2}}}\right) && \text{for } (\alpha, k) = PS^{\pm}, PTS^{\pm} \qquad \text{(IV.9)} \\ &= \frac{1}{3}\left(1 + \frac{3}{rt^{\frac{1}{2}}} + \frac{3}{r^2 t}\right) && \text{for } (\alpha, k) = PTV^{-} \end{aligned}$$

The polynomial form of $X_{i\alpha}{}^k(t)$ makes the formula (IV.8) valid as long as we are only interested to the longest range forces.

V - RESULTS

The amplitudes A_{15}^{W} and B_{15}^{W} obtained in (III.2) in the same way as the A^{S} and B^{S} [20] amplitudes enable us to obtain trough (II.13) the contributions to the spectral functions; the integral (II.16) exhibits no special difficulty: the integrals (IV.8) are computed numerically, the form factors (section III) being chosen as Breit-Wigner formulae and normalized to unity at the origin.

In the limit where the resonances ρ and A_1 are narrow enough (this corresponds to take the principal value of the integral with extrapolation of the phenomenological form factors defined for $t < 0$ [22 and 25]), one obtains

$$V(\vec{r}) = \left(V_{ps}(r)(\vec{\sigma}^{n} \wedge \vec{\sigma}^{p}) \cdot \frac{\vec{r}}{r} + V_{pv}(r)(\vec{\sigma}^{n} - \vec{\sigma}^{p}) \cdot \vec{P}\right)(\tau_3^{\,n}\tau_3^{\,p} - \vec{\tau}^{n} \cdot \vec{\tau}^{p}) \tag{V.1}$$

where

$$V_{ps}(r) = - r\, V_{ps,2}^{+} = r\, V_{ps,3}$$
$$V_{pv}(r) = - V_{pv,2}^{+} = + V_{pv,3}^{+} \tag{V.2}$$

with $G = 1.25 \times 1.01 \times 10^{-5}/M^2$ we obtain the curves for $V_{ps}(r)$ and $V_{pv}(r)$ drawn in fig. 4 and 5. We have also drawn, in fig. 4, the potentials obtained by Blin Stoyle [5]:

$$\mathcal{V}_{ps} = \frac{Gf^2}{2\pi}\left(1 + \frac{2}{\mu r} + \frac{1}{\mu^2 r^2}\right)\frac{e^{-2\mu r}}{r^3}$$
$$\mathcal{V}_{pv} = \frac{G}{4m}\,\delta(r) \simeq 0.82 \cdot 10^{-8}\,\mu^{-3}\,\delta(r) \tag{V.3}$$

We notice on fig. 4 that the static potential that we have obtained (solid line) has a much longer range than the Blin-Stoyle potential. One might then think that the corresponding effects in the observed processes will be more important. Note also that the computations performed by Partovi [7] correspond to a weak coupling constant G smaller than the one used in Blin-Stoyle potential (1×10^{-5} erg cm^3 instead of 1.40×10^{-5} erg cm^3). Furthermore, we have obtained an important velocity depending potential, the corresponding effects of which may modify Partovi's results.

The potential (V.1) seems to us to be more reasonable, but it may be changed if we include large width for ρ and A_1, second kind axial current and vector current non-conservation.

With the experimental width of ρ and A_1, we obtain a contribution to $\Omega_{ps}^{-}(\tau^n \wedge \tau^p)_3$ and $\Omega_{pv}^{-}(\tau^n \wedge \tau^p)_3$ but it seems that a more careful definition of the analytic continuation rules out such contributions.

The real part of the second kind axial current gives rise, trough (III.2), to a contribution of the form $\Omega_{pTs}^{-}(\tau^n \wedge \tau^p)_3$ and $\Omega_{pTv}^{-}(\tau^n \wedge \tau^p)_3$. The numerical calculation with such a term of the order of the vector current shows that the contribution to $\Omega_{pTs}^{-}(\tau^n \wedge \tau^p)_3$ is important but such a result has no particular significance. Another contribution of this form may also be obtained with a second kind vector current (which violates the vector current conservation). The effect of such a contribution due to the pion pole has been studied [26], but it seems not possible to test the existence of second kind weak current with such an effect.

The effect of the non-conservation of the vector N-N^* current is more easily appreciable. Indeed if we include in the vector N-N^* current [27] a "maximum violation" [28], we obtain a contribution quite large but the main result is that this contribution takes the form:

$$V'(\vec{r}) = (V'_{ps}(r)(\vec{\sigma}^n \wedge \vec{\sigma}^p).\frac{\vec{r}}{r} +$$

$$+ V'_{pv}(r)(\vec{\sigma}^n - \vec{\sigma}^p).\vec{P})(1^n 1^p - \frac{1}{8}(\vec{\tau}^n . \vec{\tau}^p + \tau_3^n \tau_3^p))$$

and such a potential would give rise to effects in pp and nn interaction.

* * *

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FIGURE CAPTIONS

Fig. 1 - Diagram studied by Blin-Stoyle.

Fig. 2 - 2a - Nucleon-Nucleon scattering.

2b - One pion exchange.

2c - Two pion exchange.

Fig. 3 - π - N scattering exchange in the direct channel.

Fig. 4 - The potential $V_{ps}(r)$

In dotted, the corresponding potential obtained by Blin-Stoyle.

Fig. 5 - The potential $V_{pv}(r)$.

* * *

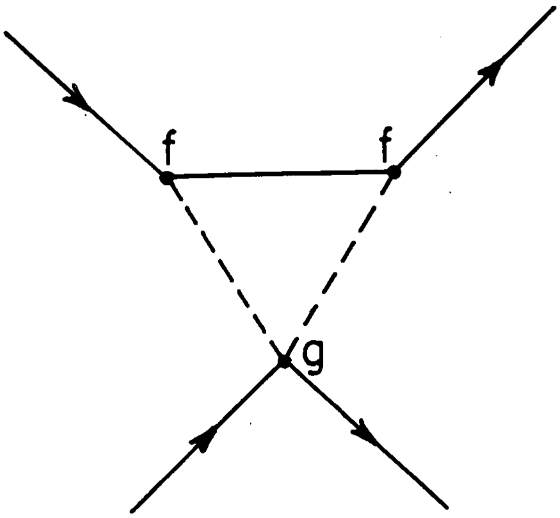

Fig. 1

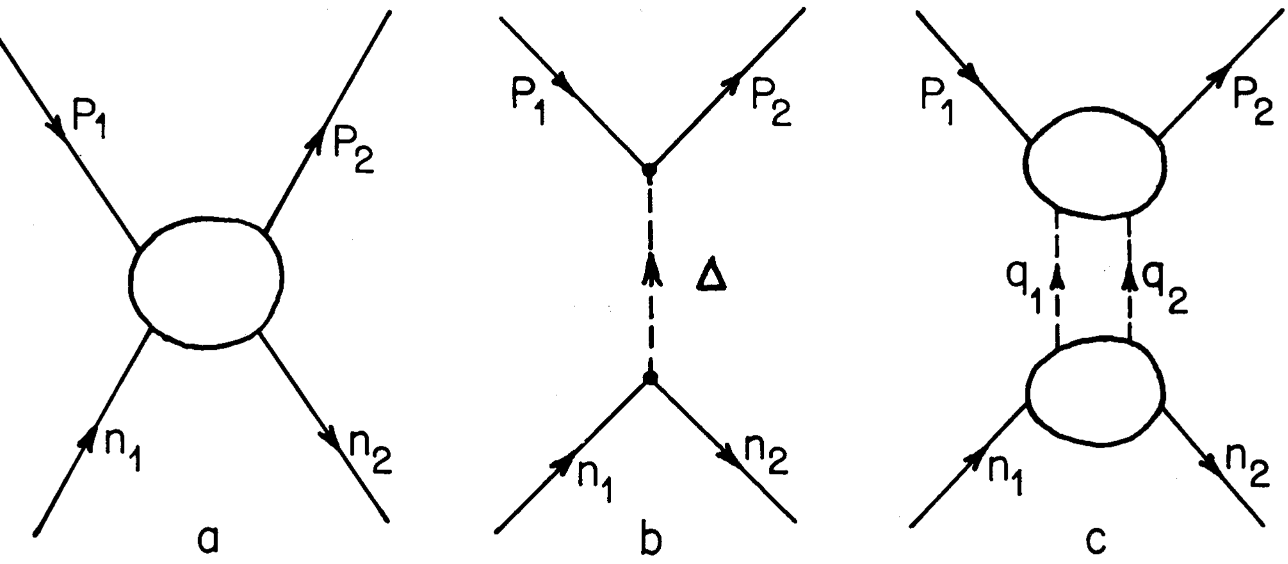

Fig. 2

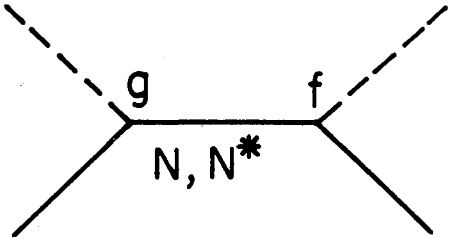

Fig. 3

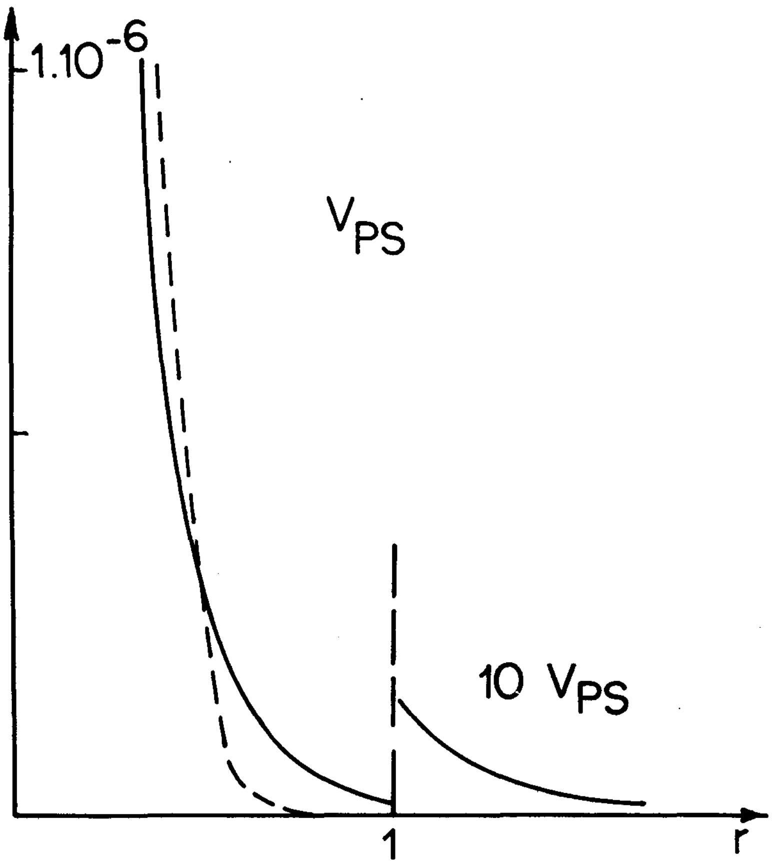

Fig. 4

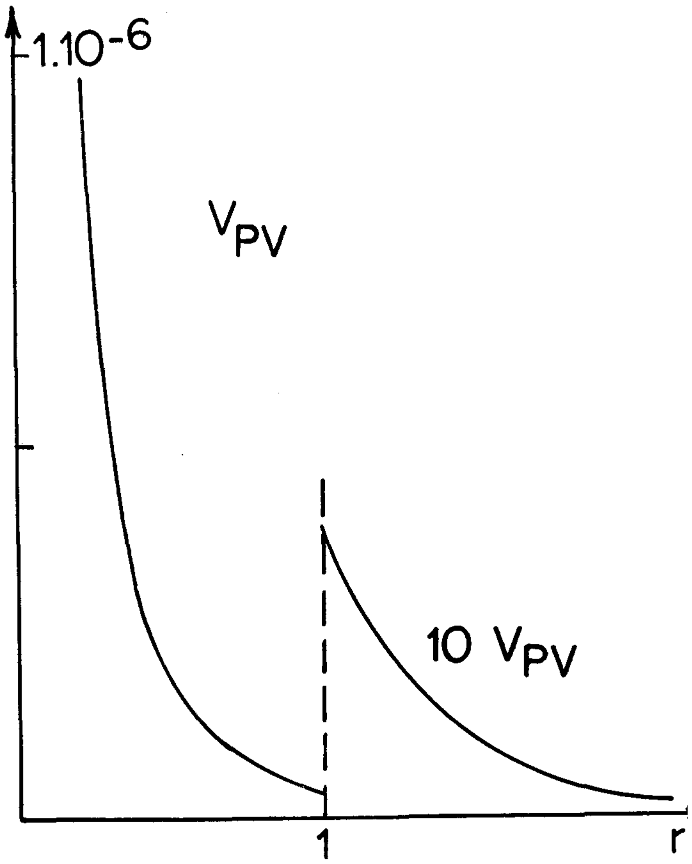

Fig. 5

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$$(m+m^*)^2\,(F_1^v + F_2^v) + (m^2 - m^{*2})\,F_3^v - tF_4^v = 0$$

From the photo production analysis (M. Gourdin, Ph. Salin; Nuovo Cimento 27, 193, 309 (1963)), $F_1^v(o) = -F_2^v(o) = 5{,}6$ and $F_3^v(o) = F_4^v(o) = 0$. Our "maximal violation" stands for $F_1^v(\mu^2) = +F_2^v(\mu^2) = 5{,}6$ and $F_3^v(\mu^2) = F_4^v(\mu^2) = 0$.

* * *