NOTAS DE FÍSICA

VOLUME V

Nº 8

THE IMAGINARY PART OF THE OPTICAL POTENTIAL

by

L. C. Gomes

CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Av. Wenceslau Braz, 71

RIO DE JANEIRO

1959

Notas de Física - Volume V - Nº 8

THE IMAGINARY PART OF THE OPTICAL POTENTIAL*

L. C. Gomes+

Centro Brasileiro de Pesquisas Físicas

(Received July 1st, 1959)

ABSTRACT

The imaginary part of the optical potential has been investigated for low energy incoming neutrons, by means of the nucleon-nucleon cross sections in nuclear matter. The cross sections have been calculated under the assumption that pair correlations for low excited states of nuclear matter are the same as those formed in the ground state. The dependence of the effective mass on the single particle momentum has been taken into consideration using an empirical solution which reproduces the present assumptions for the single particle spectrum. The results have been applied to the nuclear surface in the Thomas-Fermi approximation. The maximum in the imaginary potential was found to be at the surface outside of the half-density radius. For low incident energies it is about 1.5 fermis beyond this radius.

---

* This work is supported in part through AFC Contract AT(30-1) - 2098, by funds provided by the U. S. Atomic Energy Comission, the Office of Naval Research and the Air Force Office of Scientific Research.

+ Now at Laboratory for Nuclear Science, Massachusetts Institute of Technology, Cambridge, Massachusetts.

The reasonable success of the Independent-Pair approximation[1] for the calculation of the properties of nuclear matter suggests an application of the same method to the calculation of the imaginary part of the potential in the optical model of the nucleus. It is the magnitude which determines the "absorption" of a nuclear particle propagating within nuclear matter with an energy higher than the Fermi limit. This absorption is equivalent to the removal of the particle from the configuration space of the one-particle problem described by its motion in the optical potential.

In the approximation considered here it is equivalent to a collision with another particle within the nuclear matter.

The imaginary potential $-iW$ which would describe this absorption is given by

$$W = \frac{1}{2} v_\alpha \rho \langle \sigma \rangle \tag{1}$$

where v_α is the velocity of the particle absorbed, and $\langle \sigma \rangle$ is its average collision cross-section with the particles in the nuclear matter of density ρ. Hence the problem reduces to the calculation of $\langle \sigma \rangle$.

In this note we try to estimate the value of the imaginary potential with simple considerations which are not very accurate, but which are probably accurate enough to bring out the essential features. It must be born in mind that the approximations inherent in the fundamental assumptions do not warrant exact evaluations. Very similar considerations were carried out by Verlet and Gavoret[2]. Their

1. L. C. Gomes, J. D. Walecka, and V. F. Weisskopf, Annals of Physics 3, 341 (1958)
2. L. Verlet and J. Gavoret, Nuovo Cimento 3, 505, (1958).

approach differs from ours only in the treatment of the nucleon-nucleon forces. They make use of a separable potential which fits the scattering data at low energy. The separability of the potential makes it possible to calculate exactly the influence of nuclear matter on the scattering. It is questionable, however, whether this advantage out weighs the uncertainties introduced by the unphysical character of a separable potential. A treatment of the same problem has also been reported by Shaw[3], and Harada and Oda[4].

The collision cross-section is different from its value for a collision of two isolated particles because fo three reasons:

1. Certain final states of the collisions are excluded because of the Pauli-pronciple.

2. The effective mass of the particles is different from their actual mass.

3. The interaction acts differently for a pair within a Fermi gas then for an isolated pair.

The effect of point 1) has been calculated by Lane and Wandel[5] and Clementel and Villi[6]. We will refer to their results in which the points 2) and 3) have been left out, as the "final states" approach. Point 2) can be easily taken into account if the effective mass is known as a function of the momentum. We will use the following empirical dependence which reproduces present assumptions for a

3. G. L. Shaw, Bulletin American Physical Society 4, 49, (1959)

4. K. Harada and N. Oda, Progress Theoretical Physics 21, 260, (1959)

5. A. M. Lane and C. F. Wandel, Phys. Rev. 98, 1524, (1955)

6. E. Clementel and C. Villi, Nuovo Cimento 2, 176, (1955)

particle with momentum k_α (k_F is the Fermi momentum).

$$\frac{m}{m^*} = \frac{1}{\nu} = 1 + \frac{A\, k_F^3}{1 + B\, k_\alpha^2} \tag{2}$$

with $A = 0.48\ f^3$ $\quad B = 1.53\ f^2$ $\quad$ (f is a fermi).

Point 3) implies that the scattering is governed by the Bethe-Goldstone equation rather than by the ordinary two-particle Schroedinger equation. A collision of two particles with initial momentum $\vec{k}_\alpha$ and $\vec{k}_\beta$ is described by

$$(\nabla^2 + k^2)\, \psi_{\alpha\beta}(\vec{r}) = \frac{\nu}{2} \int F(\vec{r} - \vec{r}\,')\, v(r')\, \psi_{\alpha\beta}(\vec{r}\,')\, d\vec{r}\,' \tag{3}$$

Here $k = |\vec{k}_\alpha - \vec{k}_\beta|$ is the magnitude of the relative momentum, and $\vec{r} = \vec{r}_1 - \vec{r}_2$ is the vector between the particles, $v(r)$ is the interaction potential, $F(\vec{r})$ is given by

$$F(\vec{r}) = \int_\Gamma e^{i\vec{k}\cdot\vec{r}} \frac{d\vec{k}}{(2\pi)^3}$$

where the region of integration Γ includes only non-occupied momentum states for the pair:

$$|\vec{P} + \vec{k}| \geqslant \vec{k}_F\ , \quad |\vec{P} - \vec{k}| \geqslant \vec{k}_F\ , \quad \vec{P} = \tfrac{1}{2}(\vec{k}_\alpha + \vec{k}_\beta)$$

The solution of (3) can be written in the form

$$\psi_{\alpha\beta}(\vec{r}) = e^{i\vec{k}\cdot\vec{r}} + f_{\alpha\beta}(\theta, \varphi)\, e^{ikr}/r$$

and the scattering amplitude is given by

$$f_{\alpha\beta} = -\frac{1}{4\pi}\,\nu\,\eta(\vec{k}_f.\vec{P})\int e^{-i\vec{k}_f\cdot\vec{r}}\ \frac{v(r)}{2}\ \psi_{\alpha\beta}(\vec{r})d\vec{r} \tag{4}$$

where $\vec{k}_f$ is the final momentum of magnitude k and direction θ, φ ; furthermore we have

$$\eta(\vec{k}_f,\vec{P}) = \begin{cases} 1 & \text{for } |\vec{k}_f + \vec{P}| \geqslant k_F \text{ and } |\vec{k}_f - \vec{P}| \geqslant k_F \\ 0 & \text{otherwise} \end{cases}$$

Expression (4) differs from the scattering amplitude for the isolated case by the factor $\nu\eta$ and σ by the fact that $\psi_{\alpha\beta}(\vec{r})$ is different from the wave function in the isolated case.

Let us call

$$S_{\alpha\beta} = \left| \frac{1}{2}\int e^{-i\vec{k}_f\cdot\vec{r}}\ v(r)\,\psi_{\alpha\beta}(\vec{r})d\vec{r}\right|^2 \tag{5}$$

$S_{\alpha\beta}$ would be differential scattering cross-section $\sigma_{\alpha\beta}$ of an isolated pair, if $\psi_{\alpha\beta}$ were the solution of the ordinary two-particle Schroedinger equation. $\sigma_{\alpha\beta}$ is almost independent of the scattering angle at the momenta considered here. We therefore are justified to assume that, here also, $S_{\alpha\beta}$ is independent of θ, φ . We then can calculate the average cross-section $\langle\sigma\rangle$ appearing in (1):

$$\begin{aligned}\langle\sigma\rangle &= \frac{1}{N}\sum_{\beta}\int |f_{\alpha\beta}|^2\ \frac{|\vec{k}_\alpha - \vec{k}_\beta|}{k_\alpha}\sin\theta\, d\theta\, d\varphi \\ &= \frac{3}{4\pi k_F^3}\,\nu^2\int d\vec{k}_\beta\ \frac{|\vec{k}_\alpha - \vec{k}_\beta|}{k_\alpha}\,\xi(\vec{k}_\alpha,\vec{k}_\beta)\langle S_{\alpha\beta}\rangle_{spin}\end{aligned} \tag{6}$$

where

$$\xi(\vec{k}_\alpha,\vec{k}_\beta) = \frac{1}{4\pi}\int \eta\,(\vec{k}_f,\vec{P})\ \frac{\delta(k_f^2 - k^2)}{k^2}\ d\vec{k}_f$$

and $\langle S_{\alpha\beta}\rangle_{spin}$ is the average of $S_{\alpha\beta}$ over the different spin pairs, isotopic and ordinary. We make use of a well known approximate relation, $\sigma_{PP} = \sigma_{nn} = \frac{1}{4}\sigma_{nP}$ which gives us

$$\langle S_{\alpha\beta}\rangle_{spin} = \frac{5}{8}\,(S_{\alpha\beta})_{nP}$$

where $(S_{\alpha\beta})_{nP}$ is the cross-section given by (5) between a neutron and a proton of momentum $\vec{k}_\alpha$ and $\vec{k}_\beta$ respectively.

We assume that the contributions to the cross-section from angular momenta l higher than 0, are not different from the isolated case. We can calculate them by subtracting from the experimental cross-section the (l = 0) part as given by the phases of Christian, Gammel and Thaler[7]. The (l = 0) part is calculated by means of formula (3). The wave function $\psi_{\alpha\beta}$ is taken from the work of Gomes-Walecka and Weisskopf[1]. In that paper only collisions of pairs are considered for which both partners are within the Fermi-distribution. For the actual density of nuclear matter the s-part of this function can be approximated by

$$\varphi_0 = \sqrt{4\pi}\ \left(\frac{\sin kr}{kr} - \frac{\sin kc}{kc}\cdot\frac{\frac{\pi}{2} - \mathrm{Si}(1.10\ k_F r)}{\frac{\pi}{2} - \mathrm{Si}(1.10\ k_F c)}\right) \quad (7)$$

where c is the core radius. We use here the same function for pairs of which one partner is outside the Fermi-distribution. This

7. J.L. Gammel, R.S. Christian and R.M. Thaler, Phys. Rev. 105, 311 (1956)

will be a reasonable approximation if its momentum is not too far from . For the potential $v(r)$ we use a central potential with a core radius $C = 0.4$ and an attractive exponential well which reproduces the singlet scattering length and effective range. The effect of the tensor force and of the singlet-triplet difference is neglected. It probably plays a smaller role here then in the isolated case just as in nuclear matter[1]. An approximate evaluation of expression (5) is shown in Fig. 1 as function of $4k^2 = |\vec{k}_\alpha - \vec{k}_\beta|^2$ together with the scattering cross-section $\sigma_{\alpha\beta}$ for an isolated pair. This allows us now to calculate the imaginary potential with the help of (6), for a given incident momentum $k_\alpha > k_F$. The relation of k_α with the incident energy ϵ of the entering particle is as follows. The kinetic energy of the particle inside the nucleus will be $\epsilon + B + E_F$, where E_F is the Fermi energy and B is the binding energy. Hence we find

$$\frac{1}{\nu} k_\alpha^2 = \frac{1}{\nu} k_F^2 + B + \epsilon \tag{8}$$

where ν is the ratio of effective mass to normal mass. Equation (8) and (2) give a relation between ϵ and k_α.

Table I shows the result of this calculation for nuclear matter of normal density ($\rho = 1.94 \times 10^{38} \mathrm{cm}^{-3}$, $k_F = 1.42\ f^{-1}$).

The first column gives the energy ϵ, the second one the value of W as calculated by our method, the third one the value of ν for the corresponding k_α as given by (8). The fourth column gives W as calculated by the "final states" approach, and the fifth is the value calculated with our method but with $\nu = 1$. The values in the fifth column are larger than the ones in the fourth

Table I

ε	I. PA. M.		W by the "final states" approach	I. PA. M. W for $\nu = 1$
	W	ν		
1	1.06	0.77	1.88	2.11
7	2.34	0.78	4.03	4.59
14 Mev.	4.62 Mev.	0.80	6.88 Mev.	7.82 Mev.

one, because in the most important energy region $S_{\alpha\beta}$ is larger than the isolated cross-section $\sigma_{\alpha\beta}$, as shown in Fig. 1. The most important difference between the second and the fourth column comes mainly from the effective mass.

The small values of W reflect the fact that collisions are strongly repressed by the Pauli-Principle (5); this, in turn, is caused by the high Fermi momentum. We expect, therefore that the lower density at the nuclear surface will give rise to a higher absorption, in spite of the fact that the density enters as a factor in the expression of W. In order to get a first orientation of this effect, we have calculated W as a function of the nuclear radius by calculating first W as a function of density and then substituting the well known density distribution:

$$\rho(r) = \rho(0) \left[1 + \exp \ (r - C)/a \right]^{-1}$$

with a = 0.65 and C being the half-density radius (C = ... $1.07 \times 10^{-13} A^{1/3}$ cm). This method can only serve as a crude approximation since our calculation of $W(\rho)$ is correct only for constant ρ. Hence it is only applicable, if ρ does not change over distances d characteristic to the problem ($d \sim k_F^{-1}$). This is not fulfilled with the above $\rho(r)$.

The dependence of W on ρ can be found as follows: There is an explicit dependence of the integral (6) on k_F and eq. (2) gives the dependence of r on k_F. The integral (5) also depends implicitely on the density because of the fact that the approximative expression (7) for $\psi_{\alpha\beta}$ only holds for densities close to the nuclear matter density. For low ρ, $\psi_{\alpha\beta}$ goes over into the solution of the isolated problem. Hence $S_{\alpha\beta}$ should go over into $\sigma_{\alpha\beta}$ for $\rho \rightarrow 0$. In order to obtain a crude orientation, we have calculated $S_{\alpha\beta}$ with expression (7) for densities ρ from the central density down to that density ρ^* for which we get $S_{\alpha\beta}(\rho^*) = \sigma_{\alpha\beta}$. From ρ^* to $\rho = 0$, we simply have put $S_{\alpha\beta}(\rho) = \sigma_{\alpha\beta}$.

It is then simple to compute the imaginary potential as a function of the radius. The result is shown in Fig. 2. The curves show that there is a strong increase at the surface of the nucleus caused by an increase of the effective mass and a lessening of the effect of the Pauli-principle. It is perhaps significant that the maximum of absorption lies outside the nuclear radius C which is the point where the density drops to one-half.

It is highly doubtful, however, whether our method of calculating W is applicable to the region where classically no particle would be allowed. This is the region in which the real part of the potential is less than the binding energy of the last nuclear (8 Mev.). This region is outside a radius D, which is marked in Fig.2

and was obtained from the potential as given by Ross, Mark and Lawson[8]. The curves for W are entirely meaningless for $r > D$. In order to get some vague information about W in that region, we have also calculated W without taking into account the Pauli-principle, by simply using the scattering cross-sections for isolated pairs. The nucleons in the nucleus were assumed to be distributed with a Fermi distribution corresponding to the density ρ and the momentum of the incident particle was assumed to be given by (8). The resulting W is higher than the one calculated by the previous method, but it is of the same order for values $r > D$. We therefore believe that the fall-off of W resulting from the previous calculation is not an unreasonable estimate, even for $r > D$.

A preponderance of collisions outside the nuclear radius would have two consequences: It would mean that direct reactions are favored, since collisions in the surface would make compound nucleus formation less likely. Also, the Coulomb barrier for nuclear reactions is expected to correspond to a larger radius then C, in particular in respect to direct reactions.

Acknowledgement: The author wishes to express his gratitude to Professor V. F. Weisskopf for proposing and orienting the development of the work reported here and to CAPES and Conselho Nacional de Pesquisas (Brazil) for their partial support during his stay at M. I. T.

---

8. A.A. Ross, H. Mark, and R.D. Lawson, Phys. Rev. 102, 1613 (1956).

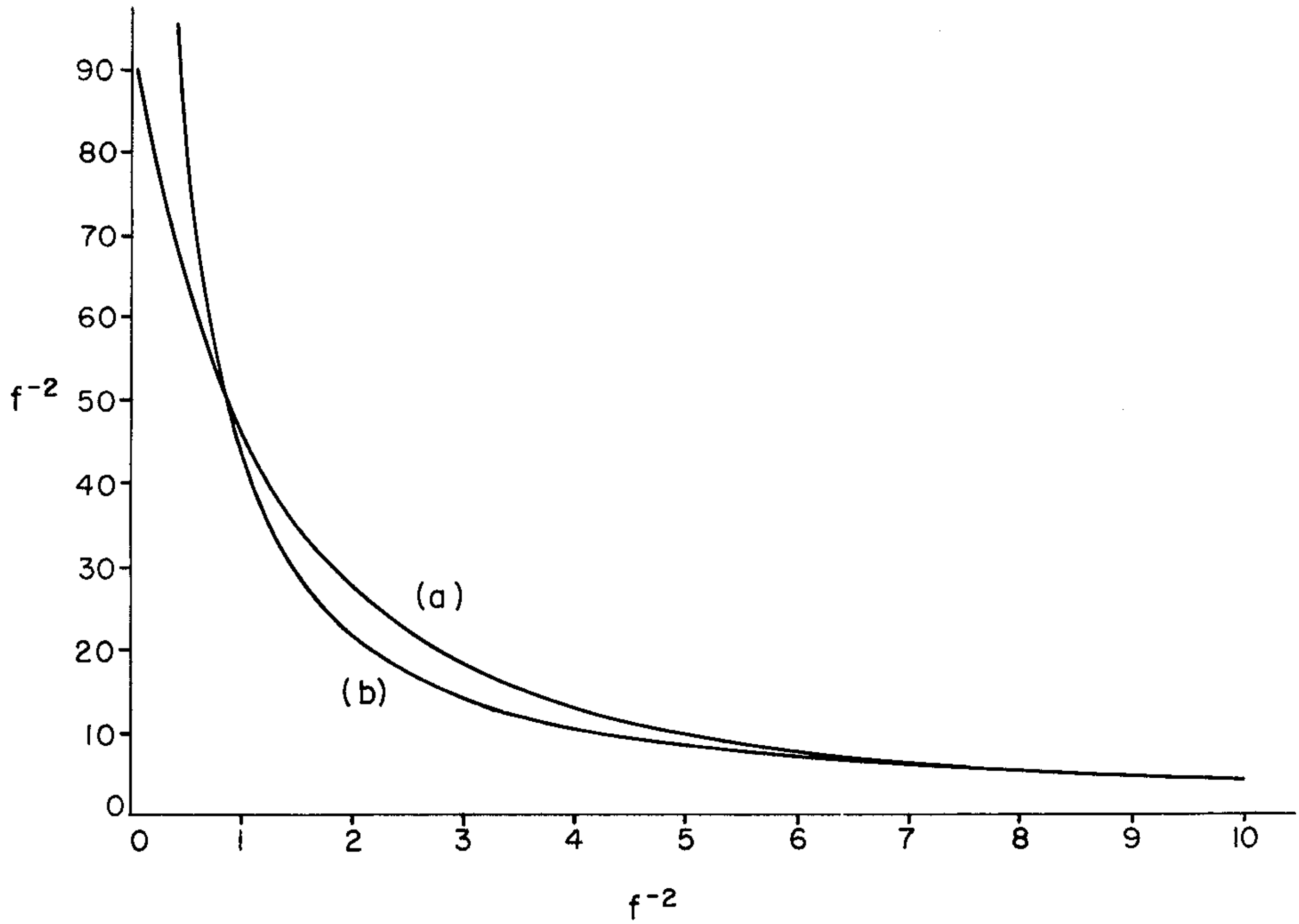
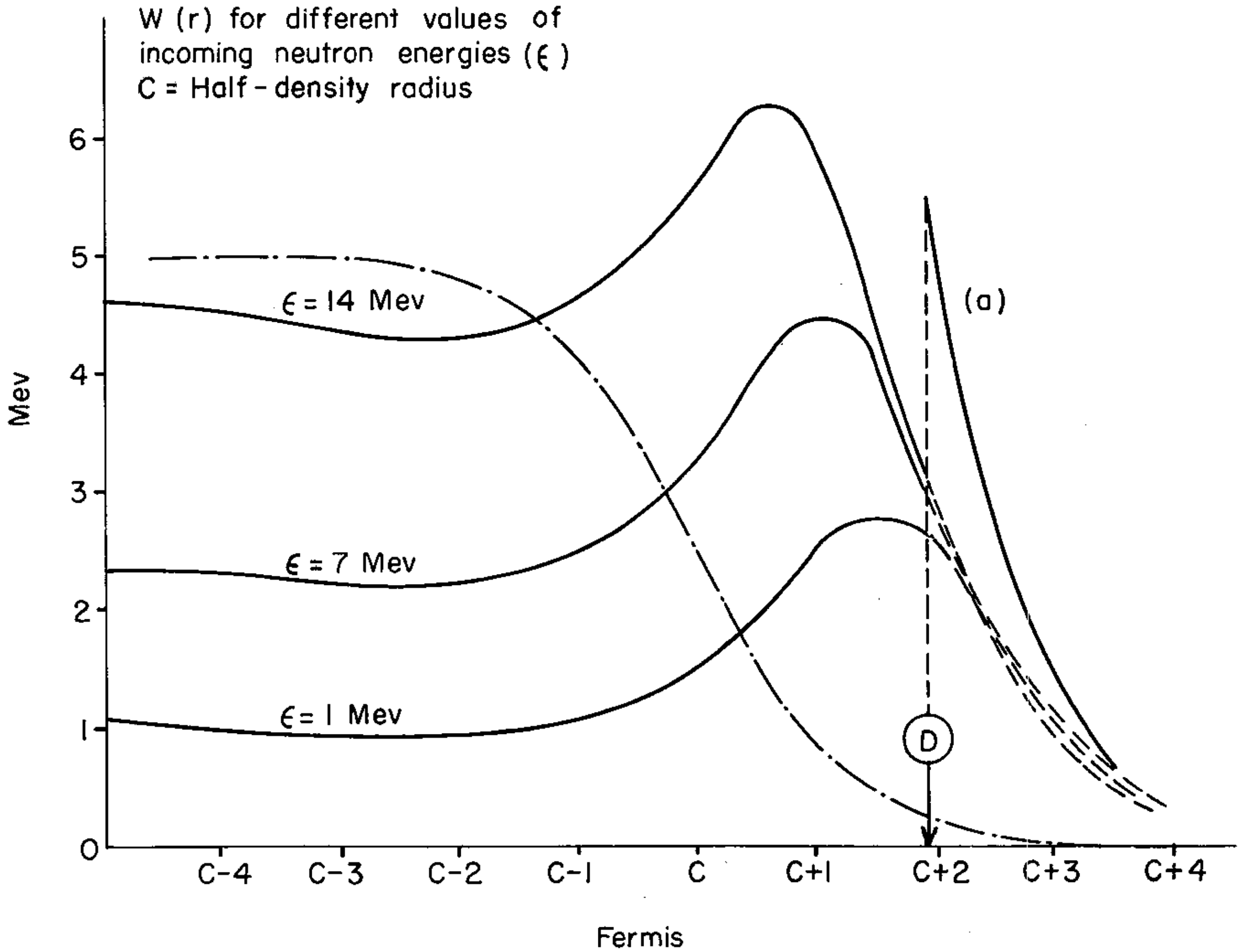


Figure 1 - Curve (a) is $(S_{\alpha\beta})_{np}$ as defined in eq. (5) and (b), $\sigma_{\alpha\beta}$, the scattering cross-section for an isolated pair. The vertical scale is in units of f^2 (1 f = 10^{-13}cm) and the horizontal scale is $|\vec{k} - \vec{k}|^2 = 4k^2$ in f^{-2}.



<u>Figure 2</u> - The imaginary part W of the optical potential at the nuclear surface, for different values of incoming neutron energies (ϵ). The vertical scale is in Mev and the horizontal scale in f; C is the half density radius. The dashed and dotted curve is the density function $\rho(r)$ in arbitrary vertical scale. D is the "classical" turning point. Curve (a) is W in the classically forbidden region calculated neglecting the exclusion principle.