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**Pascual Jordan's legacy and the ongoing research in
quantum field theory**

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Abstract

After recalling Pascual Jordan's pathbreaking work in shaping quantum mechanics I explain his role as the protagonist of quantum field theory (QFT). Particular emphasis is given to the 1929 Kharkov conference where Jordan not only presents a quite modern looking panorama about the state of art, but were some of his ideas already preempt an intrinsic point of view about a future QFT liberated from the classical parallelism and quantum field theory, a new approach for which the conceptional basis began to emerge only 30 years later.

Two quite profound subjects in which Jordan was far ahead of his contemporaries will be presented in separate sections: "Bosonization and Re-fermionization instead of Neutrino theory of Light" and "Nonlocal gauge invariants and an algebraic monopole quantization".

The last section contains scientific episodes mixed with biographical details. It includes remarks about his much criticized conduct during the NS regime. Without knowing about his entanglement with the Nazis it is not possible to understand that such a giant of particle physics dies without having received a Nobel prize.

1 Preface

The biographical part of this essay was written under the title: "Pascual Jordan, biographical notes, his contributions to quantum mechanics and his role as a protagonist of quantum field theory" for a conference to the memory of Pascual Jordan which took place 2005 in Mainz, Germany, .

Already at that time I thought it would be even more interesting to present some of his work written in the years 1930-38 in more details in order to make it better known¹ and also to reveal its startling modernity by relating it to ongoing research and by creating conceptual and philosophical connections where either as a result of world war II or through other sociological reasons the continuous flux of ideas was interrupted or took a different turn.

With the launch of the new Springer journal EPJ H, dedicated to the historical illumination of present research problems, it is now possible to recreate some of these lost ideas and demonstrate their relevance for the actual research.

2 Contributions to Quantum Mechanics and to the first phase of Quantum Field Theory

The following section contains a more detailed account of the content of some of Jordan's most seminal contributions to quantum mechanics (QM) and quantum field theory (QFT).

The situation which Jordan was confronting after he was called upon by Born to collaborate on the mathematical and conceptual underpinnings of Heisenberg's pivotal observation, was as follows. Heisenberg in his paradigmatic 1924 paper had gone far beyond the previous somewhat vague correspondence principle of the Bohr-Sommerfeld quantum setting as a result of his proposal to replace the canonical classical particle variables q, p by a novel set of observables consisting of a kind of complex amplitudes subjected to a non-commutative multiplication law. These quantities were supposed to satisfy a "quantum condition" which vaguely resembles the Thomas-Kuhn sum rule for harmonic oscillators, but which in terms of the new quantum mechanics was to be regarded as a basic law of universal validity.

It is tempting to compare the ensuing paradigmatic change with that from Lorentz's transformation formula in Einstein's special relativity. Whereas in the latter case the change is purely interpretational without any necessity to change the notation in passing from Lorentz's pre-Einstein transformation formula to its new relativistic interpretation, the paradigmatic change from the semi-classical Thomas-Kuhn sum rule to the new quantum mechanics affects the interpretation as well as the formal appearance. In fact in Heisenberg's paper the mathematical nature of the involved objects still remained somewhat opaque, precisely partially because a good notation which harmonizes with the paradigmatic conceptual change was not yet in place.

It was Max Born who proposed to identify the new quantities as (infinite dimensional) matrices. He conjectured that Heisenberg's new quantum law is the statement that the

¹These were the years in which according to a remark by Peter Bergmann a publication in *Zeitschrift fuer Physik* "was like a first class burial".

position q and the momentum p of a particle are to be represented by matrices whose product is non-commuting, such that $pq - qp$ is equal to $\frac{\hbar}{i}$ times the identity matrix, None of the Goettingen physicists at that time had sufficient knowledge about the mathematics of unbounded operators in Hilbert space in order to deal properly with this problem, although certainly some of these concepts were already familiar to the Goettingen mathematicians around Hilbert. In Born's matrix reading of Heisenberg's quantum mechanics he initially was only able to convince himself that the diagonal part of the p, q commutation relation had the conjectured form, the vanishing of the off-diagonal contribution was still an open problem.

His new collaborator Jordan (who also helped Courant when he was working on the famous Courant-Hilbert book about mathematical methods in physics) quickly succeeded to close this gap; he proved the vanishing of the off-diagonal elements as a consequence of the equations of motion and an ingenious algebraic argument [1]. From now on the (matrix-form of the) commutation relation was the new principle of quantum mechanics.

Shortly before this collaboration with Born Jordan defended his thesis "on quantum radiation" which referred heavily on Einstein's 1917 work, but also brought him into an interesting controversy with the great master (see last section).

The subsequent "Dreimännerarbeit" [2] extended the setting of quantum mechanics to systems with many degrees of freedom.

Shortly after this paper, Jordan together with Heisenberg [3] demonstrated the power of the new formalism by presenting the first fully quantum-mechanical treatment of the anomalous Zeeman effect based on the new electron spin hypothesis of Goudsmit and Uhlenbeck. In this article the authors left no doubt that they considered this as a temporary working hypothesis falling short of a relativistic description of the electrons intrinsic angular momentum; a problem which still had to wait three more years before Dirac finally gave it the final shape. In Heisenberg-Jordan paper all relevant matrixelements of the later Pauli matrices for spin 1/2 appear. Jordan also knew already at this time that the quaternions have a faithful representations by 2×2 matrices. So the question arises why did he not write the Pauli equation of motion before Pauli? I think that the answer is that such a differential equation at that early time would have been too much against the Goettingen spirit of matrix mechanics in the direction of Schroedinger's version. At that time the transformation theory showing the equivalence was not yet worked out. Pauli on the other hand, having been a student of Sommerfeld in Munich, did not have such an emotional commitment.

In this context it is interesting to add that E. C. Darwin already before Pauli described the electron in terms of a *two-component wave function*². His equation contained the Pauli term and the spin-orbit coupling. Pauli even mentions this paper but criticizes the "vectorlike wave function"!

As mentioned before, the first modern formulation of quantum mechanics in terms of operators in Hilbert space, in which the equivalence between Heisenberg's and Schroedinger's formulation was clarified, appears in the work of Fritz London [4]. substituted matrices by operators; apart from the name "rotation in Hilbert space" for unitary operator, the terminology is quite modern. Although Jordan praised this work of London, it was

²I owe this information to Gernot Muenster.

apparently overlooked by most of his contemporaries who attributed the Hilbert space formulation of quantum theory to the later papers of von Neumann. A sociological explanation may be that Fritz London was an assistant at the TH Stuttgart at a the time i.e. at place outside the "quantum dialogue" between the great centers at which quantum theory started. Taking Jordan's passion for matching fundamental physical insights with the right type of profound mathematics into consideration, it is suggestive to think that he was impressed to see for the first time the use of unitary operators in a Hilbert space in London's formulation of the "Transformation Theory".

His "skirmish" with Einstein on the content of his thesis had a radical impact; Jordan completely adopted Einstein's at that time not yet accepted viewpoint that photons and waves are two sides of the same coin and became a partisan of wave quantization for all quantum matter and not just photons. Heeding Einstein's suggestion Jordan showed that the quantization of electromagnetic waves delivers exactly Einstein's photons as covariant objects with a well defined energy-momentum (Einstein's "needle radiation"). All his publications after 1926 into the first half of the 30s deal with "quantization of wave fields" i.e. with aspects of quantum field theory.

There is however one important later contribution which addresses the foundations of quantum physics and led to an algebraic structure which bears his name, the so-called Jordan algebras. If one starts from von Neumann's axiomatic framework of quantum theory, which identifies observables with Hermitian operators acting in a Hilbert space, one would like to have at one's disposal a multiplication law which converts two observables into a new composed observable. This is achieved by taking the anti-commutator and Jordan posed the problem of unravelling the algebraic structure which one obtains if one disposes of the Hilbert space setting and axiomatizes this abstract algebraic composition. The result was a commutative but not distributive new structure, the so-called Jordan algebras [5]. This attracted the interest of von Neumann and Wigner and led to some profound mathematical results. In a joint paper [6] Jordan, von Neumann and Wigner proved that apart from a special exceptional finite dimensional Jordan algebra, all finite dimensional Jordan algebras are in fact ordinary matrix algebras with the anti-commutator composition law. This showed that the standard quantum theory setting of operator algebras in Hilbert space as axiomatized by von Neumann was more natural and stable against modifications than one had reasons to expect on the basis of Heisenberg's credo to limit quantum theoretical arguments to observables.

This work was later extended to the infinite dimensional realm (by making suitable topological assumptions about Jordan algebras) without encountering any additional exception [7]. Works on Jordan algebras, to the extent that they are physically motivated, should be viewed in line with other attempts to obtain a better understanding of the *origin of the superposition principle*³ in quantum theory. The latter expresses the fact that states in the sense of positive linear forms on algebras permit an (in general non-unique) interpretation as expectation values in vector states (or in terms of density matrices in case the states are impure) of a Hilbert space on which the of abstract algebra acts as a concrete operator algebra, so that the linear combination of two state vectors defines

³The total Hilbert space of a quantum physical system decomposes into coherent Hilbert subspaces which are subject to the superposition principle (i.e. the von Neumann axiom that with two physically realizable vectors any vector in their linear span is also physically realizable) is valid.

again a (pure) state vector. Since quantum states are conceptually very different from classical linear waves, the analogy to the superposition of classical linear waves is not of great help. The dual algebra-state relation permits to explore the problem of a more basic understanding of the quantum superposition principle either on the algebraic side as in the spirit of the Jordan algebras or on the side of the structure of the state space [8].

The farthest going results on the side of states was obtained decades later by Alain Connes [9] who succeeded to characterize operator (von Neumann) algebras and their Hilbert space superselection structure together with the operator algebra properties as arising from of the “facial substructure” of their convex state spaces (the pre-dual of an algebra). The work with von Neumann and Wigner was Jordan’s last international collaboration. His sympathies for the Nazis made a continuation of such collaborations very difficult, even if he did not share their anti-Semitism. It seems that already several years before the war Jordan had lost contact with those frontiers of research which he himself started.

In the remainder of this section I recall Jordan’s most important and enduring discovery namely that of quantum field theory. The conceptual difference of what Jordan did as compared with Dirac’s “second quantization” is somewhat subtle. The latter was an artful transcription (the Fock space formalism was not yet available) of the Schrödinger multi-particle setting into a form of what in modern terminology is called the occupation number representation; for the formal backup of this step Dirac used his version of the transformation theory. Jordan on the other hand had the bold idea to bypass quantum mechanics and obtain the discontinuous structure of quantum matter from a quantization process which viewed the one-particle Schrödinger wave function as a classical wave equation to be subjected to wave quantization rules in ordinary space i.e. a quantization imposed on the classical Lagrangian field formalism. He showed by a detailed calculation [10] that this procedure, projected onto the n -particle subspace, is equivalent to Dirac’s occupation number description. At that time Jordan met Oscar Klein in Copenhagen, both of them were guests of Niels Bohr. In a widely acclaimed paper they jointly extended the field quantization formalism to the case with interactions [11].

According to Darrigol [12], the idea of matter wave quantization in the presence of interactions was already on Jordan’s mind in 1925 at the time Schroedinger’s manuscript arrived in Goettingen; in fact it may have been a reaction to Einstein’s comments on Jordan’s 1924 thesis. His uncompromising field quantization point of view was initially rejected for massive matter by his contemporaries (quantizing something which according to Schroedinger is already “quantal”?) in favor of Dirac’s quantization approach which, apart from electromagnetic waves, started from classical particles. Dirac contributed pivotal ideas to relativistic quantum theory as the Dirac equation, but he only fully accepted Jordan’s unified field quantization for light and matter around 1950. For Jordan and Klein the equivalence of field quantization to the Dirac multi-particle approach in the nonrelativistic setting was settled in their 1927 joint paper.

Dirac’s reluctance to use wave quantization beyond light for massive matter had to do with the fact that the classical theory of massive matter was a theory of particles whose quantization led to a multiparticle theory in a high-dimensional configuration space. Dirac did not want to generalize the quantization procedure to concepts which have no classical reality. Jordan’s quantization of waves in actual space amounted to a quantization

of something which had no autonomous classical reality i.e. it is invented only for the quantization formalism to be applied to it. Dirac's criticism was a generalization of the already mentioned more specific argument "why quantize something which is already quantal" against Jordan's quantization of the Schroedinger wave function. His underlying philosophy, far from being an obstacle to the development of QFT, actually enriched its understanding. This is particularly evident in his positron theory, a particle theory in which the anti-particles of Fermions are pictured as holes in a "Dirac sea" of occupied negative energy states. However its importance was mainly transitory in that it led to intuitive geometrical arguments which in turn suggested rather straightforward calculations leading to experimentally verified correct perturbative results. The first textbooks on QFT were based on Dirac's particle-hole formalism [13]. But despite its historic importance Dirac's positron theory is not consistent as a particle theory; it runs into problems with vacuum polarization and it is not possible to base renormalized QED on it. In order to have a theory which is well-defined at least in each renormalized perturbative order one must use Jordan's charge-symmetric wave quantization which is automatically obtained by considering the Dirac field as a classical wave field subjected to the wave field quantization procedure. In this description the negative energy electrons are replaced by positive energy excitations with the opposite charge. More on the fascinating and fruitful controversy between Jordan and his adversary Dirac can be found in [12].

Jordan considered it as a significant advantage that his field quantization viewpoint allowed to incorporate "Pauli statistics" alternatively to Bose-Einstein statistics into one and the same formalism; in fact some of the wave field quantization ideas were already contained in the concluding sections of [14] which dealt with anti-commutation relations. He returned to this subject in a joint work with Wigner (submitted in January 1928) which contains significant extensions and clarifications [15]. This joint work was for a long time considered as incomprehensibly mathematical.

Nowadays the Jordan-Wigner paper is not only quoted as an alternative approach to Dirac's presentation of anti-commutation relations, but their method of construction also received particular attention in connection with nonlocal transformations which are capable of changing commutation relations. This is because these authors discovered an abstract (highly ambiguous) construction to write Fermions in terms of "Paulions" i. e. Pauli matrices (which however as a result of Jordan's lifelong love for quaternions appear in a quaternionic camouflage). The ordering prescription which they need in order to write concrete Fermion formulas in the Hilbert space of a discrete array of Pauli spin matrices loses its arbitrariness and becomes physically unique in case of the presence of a natural spatial ordering as in the transfer matrix formalism of the 2-dim. Lenz-Ising model [16]. This kind of nonlocal formula involving a line integral (a sum in the case of a lattice model) became the prototype of statistics changing transformations (Bosonization/Fermionization) in $d \neq +1$ models of quantum field theory [17][18] and condensed matter physics [19] which will be presented in more detail in the next section.

It is interesting to return to Dirac's reaction to the Jordan-Klein paper; different from the general enthusiastic acceptance of their approach he was somewhat disappointed that Jordan's version of second quantization did not yield more results beyond the ones he himself had worked out using his multiparticle configuration space description. He also was not much impressed by the formal incorporation of the anti-commutation structure

into Jordan's wave field quantization setting. His first complaint was of course rendered unjustified as soon as the new field quantization approach was applied to the relativistic setting [20] where the interaction-caused vacuum polarization and real particle creation left no alternative than to finally abandon a quantum mechanical particle quantization picture. But without vacuum polarization (e.g. for particles described by the nonrelativistic Schroedinger equation) Dirac's particle quantization is indeed equivalent to Jordan's field quantization. It seems that his second point was more of an esthetical and philosophical nature since it appeared to him somewhat unnatural to invent a fictitious classical reality (example: Grassmann Fermions in functional integral representations).

A very important conceptual addition, even if initially not recognized as such, was Heisenberg's 1932 introduction of the nuclear isospin which is the hour of birth of "internal symmetry". This is a concept for which there was no requisition in classical physics, so it was natural to inquire about its origin. After all spacetime symmetries were since Einstein perfectly accepted, but the internal symmetries of particle physics had an aura of mystery. At that time nobody, not even Jordan, had a chance to unravel the whole story how inner symmetries arise from the most fundamental locality principle of QFT. Nevertheless he did come halfway by concluding that the representation theory of the symmetry group [21] played an important part. It is interesting to compare this with the modern theory of superselection sectors whose final step is the explanation of the origin of internal symmetries.

The complete unravelling of the origin of inner symmetry is only possible in the modern autonomous formulation of QFT, often referred to as Local Quantum Physics (LQP) [22] or Algebraic Quantum Field theory (AQFT). Its characteristic feature is that it abandons the quantization parallelism to classical fields in favor of a characterization in terms of a few quantum principles which are viewed as physically motivated requirements on a *local net* of operator algebras i.e. on a family of spacetime indexed operator algebras. At the start the operator algebras are consisting of neutral observables which commute for spacelike distances (quantum Einstein causality). One then classifies all unitarily inequivalent (but locally equivalent) representations. Within each representation sector one encounters the unrestricted superposition principle, whereas between such sectors the superselection principle prevents coherent superpositions. The first step in combining the different sectors leads to the para-fermi or para-boson representation of permutation group. The ultimate step consists in rearranging the algebraic structure in such a way that the para-statistics aspect is traded against group theory. The resulting group is uniquely determined by the structure of the starting observable algebra, and the latter re-emerges as the fixed point algebra of the field algebra under the action of the group [23]. Clearly the important step in understanding the origin of inner symmetry was to start from a dichotomy between local observables and localized charge-carrying fields and view the latter as manifestations of the former.

Thanks to the DHR theory, the mystery of inner symmetry at the time of Heisenberg and Jordan has been lifted. But although nowadays there is no mystery on this issue, there is still the aura of surprise that the theory of compact groups⁴ appears at a place which at least on the surface has nothing to do with group theory but rather with localization

⁴Each compact group has an observable algebra which produces in in the indicated way.

structure of observables. If group theory would not have been invented by mathematicians almost two centuries ago, Physicists pursuing local quantum physics would have found it in their conceptual frame. But this was certainly not the way in which Heisenberg and Jordan approached the problem of inner symmetries in QT.

In view of the magnitude of the task, Jordan's attempt [21] to bring the representation theory of the symmetry group to bear on the inner symmetry problem appears quite valiant.

The continuation of the field quantization saga is well known. After the 1928 Jordan-Pauli paper on the spacetime treatment (overcoming the artificially caused separation into space and time of the canonical formalism) in which the free field Jordan-Pauli commutator functions appeared for the first time; afterwards the field quantization torch was taken over by Heisenberg and Pauli. The subsequent study of properties of the vacuum problems in QFT and the apparent divergencies brought the QFT train to a grinding halt.

After the second world war the new locomotive of renormalization led to a remarkable recovery and a new faith in the underlying principles of QFT but the team running the train had changed. Instead of the meanwhile grown up members of the "Knabenphysik" of the Goettingen days, the physicists in the driving seat consisted to a large degree of young Americans; the physics language changed from German to English.

It became gradually clear that the ubiquitous presence of so-called vacuum polarization clouds created in the presence of interactions, which convert interacting QFT into an infinite particle problem, has no counterpart in quantum mechanics. Since quantum fields are, even in the absence of interactions, inevitably rather singular objects, the perturbative approach was initially ridden with conceptual and computational problems which only more than two decades after the discovery of field quantization were finally understood.

It is interesting to dedicate a separate section to the 1929 Kharkov conference since this was the high point in the first phase of QFT and the culmination of Jordan's career in QFT. It is also the swansong of Germany's leading role in physics; in fact the last conference with the lingua franca in physics being German.

3 The 1929 Kharkov conference

In 1929 at a conference in Kharkov⁵, Jordan gave a remarkable plenary talk [24]. In a way it marks the culmination of the first pioneering phase of QFT; but it already raised some of the questions which were only taken up and partially answered almost 20 years later in the second phase of QFT (i.e. renormalized perturbation theory and its application to gauge theory). In his talk Jordan reviews in a very profound and at the same time simple fashion the revolutionary steps from the days of matrix mechanics to the subsequent formulation of basis-independent abstract operators (the transformation theory which he shares with Dirac and London) and steers then right into the presentation of the most im-

⁵Landau, after his return from a visit to Copenhagen, went to the university of Kharkov which for a short time became the "Mecca" of particle physics in the USSR.

The conference language at that time was still German

portant and characteristic of all properties which set QFT apart from QM: Commutation Relations in agreement with Locality and Causality as well as the inexorably related Vacuum Polarization. Already one year before in his Habilitationsschrift [25] he identified the two aspects of relativistic causality namely the statistical independence for spacelike separations (Einstein causality, commutation of observables) and the complete determination of events in timelike directions (the causal shadow property) as playing a crucial role in the new quantum field theory. He ends his presentation by emphasizing that even with all the progress already achieved and that expected to clarify some remaining unsatisfactory features of gauge invariance (*Die noch bestehenden Unvollkommenheiten, betreffs Eichinvarianz, Integrationstechnik usw., duerften bald erledigt sein*), one still has to confront the following problem: *Man wird wohl in Zukunft den Aufbau in zwei getrennten Schritten ganz vermeiden muessen, und in einem Zuge, ohne klassisch-korrespondenzmaessige Kruecken, eine reine Quantentheorie der Elektrizitaet zu formulieren versuchen. Aber das ist Zukunftsmusik.* (In the future one perhaps will have to avoid the construction in two separated steps and rather have to approach the problem of formulating a pure quantum theory of electricity (a pure quantum field theory) in one swoop, without the crutches of classical correspondences. But this is part of a future tune.)

He returns on this point several times, using slightly different formulations (*....muss aus sich selbst heraus neue Wege finden*) for a plea towards a future autonomous formulation of QFT which does not have to take recourse to quantization which requires starting with an (at least imagined) classical analog.

These statements are even more remarkable if one realizes that they come from the protagonist of field quantization only four years after this pivotal discovery. A similar surprise was caused when in the 70's some proceedings of a 1939 conference in Warsaw were rediscovered which contain a talk by Oskar Klein, Jordan's collaborator from the Copenhagen time, on aspects of nonabelian gauge theories and their possible applications to particle physics. It seems that the knowledge about QFT in pre-war Europe was more advanced than hitherto thought.

Jordan's critical attitude towards his own brain child of wave-field quantization has the same philosophical origin as his antagonism to Dirac's particle approach. Being a positivist, he had no problems with the physical non-existence of a classical structure to be quantized, as long as their quantization gave the experimentally verifiable behavior of quantum matter. To him quantization from a uniform setting of classical matter waves was preferable to Dirac's dual wave quantization of light and particle quantization for massive matter. What bothered Jordan about field quantization was the apparent necessity of a parallelism to classical physics which is inherent in the very procedure of quantization, be it that of particles or that of fields. Clearly a fundamental physical theory should stand on its own feet and explain the classical behavior in certain limits and not the other way around, as the various quantization approaches. Neither Dirac's nor Jordan's own approach was able to meet this plea for an autonomous approach to QFT.

In retrospect it is clear that an autonomous approach to QFT had no chance before the formalism of interacting field quantization was fully developed to the level of conceptual sophistication of renormalized perturbation theory, as in the work of Tomonaga, Feynman, Schwinger and Dyson in the end of the 40's. Only after this work the mathematical status of the inherent singular nature of pointlike fields became gradually known,

and arguments which permit to avoid infinities in intermediate computational steps were proposed. An autonomous formulation of QFT which is capable to classify and to construct models does not yet exist. What is referred to as LQP [22] or AQFT is rather a set of physically motivated and mathematically well-formulated requirements which allow to derive interesting general consequences. Only in $d=1+1$ one knows additional structures which permit a partial classification and construction of models. But the progress obtained during recent years nourishes the hope that a complete knowledge of autonomous QFT including a classification of models and a proof of their mathematical existence is possible. None of these recent attempts in LQP can be traced back to Jordan's 1929 plea for a QFT; the time lapse is too great and the worldwar II has destroyed any potential continuity. Nevertheless it is encouraging to know that the first pleas for an autonomous understanding of QFT are even older than renormalization theory.

Jordan's expectation about a rapid understanding of the "imperfections of gauge theories" at the time of his Kharkov talk may have been a bit optimistic since the Gupta-Bleuler formalism (much later followed by the more general BRST approach) only appeared 20 years later. But it is interesting to note that for Jordan gauge theory was an important issue already in 1929.

Finally one should add one more remark which shows that Jordan and Dirac had a very similar taste for what both considered the important problems. In 1935 Dirac presented a beautiful geometric argument which establishes that it is possible to introduce a magnetic monopole into quantum electrodynamics as long as the strength of the monopole times the value of the electric charge fulfill a quantization law. Shortly afterwards Jordan came up with an interesting very different algebraic argument for the magnetic monopole quantization which he based on the algebraic structure of bilinear gauge invariants [26]. He published a short note in *Zeitschrift fuer Physik* at a time when its international reputation had suffered from the deteriorating political situation⁶. Hence it is not very surprising that this paper was hardly noticed. The independent re-discovery of its main content (including Jordan's tetrahedron argument) by R. Jackiw [27] certainly shows that the problem remained non-trivial and interesting almost 70 years later. The problem of how these kind of arguments have to be amended in order to take account of renormalization has according to my best knowledge not been satisfactorily answered. Jordan's monopole paper is one in series of publications in which gauge invariance based on exponential functions of line-integrals over vectorpotentials play a central role (see next section). It is clear that what Jordan really wanted to achieve was a gauge invariant formulation of the basic dynamical equations of quantum electrodynamics. This he did not achieve, inasmuch as other attempts 30 years later in particular by Stanley Mandelstam also failed. Although gauge theory aims at gauge invariant observables and as such fulfills Heisenberg's dictum that the use of non-observables must be avoided since it does use unobservable negative metric ghosts in intermediate steps. Their use is reminiscent of a catalyzer in a chemical reaction which disappears after having done its job. Whereas their use in chemistry is well-understood, there is no fundamental understanding why one needs such ghost catalyzers for the description of interactions involving higher spin equations.

⁶According to Peter Bergmann (who left Germany and build up a very active and successful relativity group in Syracuse during the 50's and during that time also visited Jordan in Hamburg) "a publication in *Zeitschrift fuer Physik* after 1934 was a first class burial".

Their use is obviously helpful for the construction of gauge invariant local observables but the necessarily nonlocal physical charge-carrying fields remain outside this formalism.

Since Jordan left the area of QFT in the middle of the 30s and became disconnected from the discoveries thereafter; we do not know how he would have reacted to the amazing progress after the war brought about by renormalization theory. Probably contrary to expectations of most of the leading theorist during the 30's and in particular to Jordan's, this impressive progress after the world war II was obtained by a careful conceptual distinction between formal and physical (observed) Lagrangian parameters, as well as by a systematic replacement of the old quantum mechanic inspired formalism by a more intrinsic field theoretic relativistically covariant setting; in the words of Weinberg the success was achieved in a rather conservative manner [29].

Renormalization theory may have been too conservative in order to fit Jordan's radical expectations; in strange contrast to his political stance, in physics Jordan was a visionary revolutionary.

4 "Bosonization" instead of "neutrino theory of light"

Starting around 1935 Jordan began to publish a series of papers under the title "On the neutrino theory of light" [37]. The idea that photons may be bound states of $\nu\bar{\nu}$ was not entirely new since de Broglie had vaguely formulated a similar thought without presenting an argument. The statistics of particles in terms of commutation relations of fields was a still an unaccustomed subject and therefore problems which nowadays are considered as part of kinematics and to be done away with in a few lines, at that time filled the main part of a paper. There was the general belief that it is sufficient to illustrate an idea in a low-dimensional QFT. This explains to some extent why Jordan's contemporaries had no problem with the fact that he took a two-dimensional model instead of arguing in the realistic setting of 4-dimensional spacetime. Jordan started from a two-dimensional massless Weyl fermion, his neutrino model, from which he formed a bosonic current and its potential which was his two-dimensional analog of the photon field.

His model amounts to what since the 70s is called *bosonization* and *fermionization* is its inverse; both procedures only work in two dimensions and shed no light on a higher dimensional neutrino theory of light. But although Jordan was wrong in his central claim, he discovered the mathematics of an interesting new structure for which however there was no demand in QFT before the 70s. Before I present Jordan's model in more detail, some additional remarks on the state of QFT at the time of Jordan's participation in it are in order.

Using a modern notation and terminology his main points in [30] become more accessible. Starting from a $u = t + x$ chiral free fermion $\psi(u)$ one may define the u-component of a chiral current

$$\begin{aligned} \psi(u) &= \frac{1}{(2\pi)^{1/2}} \int_0^\infty dp (e^{ipu} a(p) + e^{-ipu} b^*(p)) \\ j(u) &=: \psi^*(u) \psi(u) : = \partial_u V(u) \end{aligned} \quad (1)$$

The u-lightray component (one can play this game equally with $v=t-x$) of the spinor has a v counterpart which is not needed here, but would be required for the massive

spinor which depends on both u and v (for which however the commutation properties are less simple). The double dot denotes as usual the ordering in which all annihilation parts appear in the right of the creation operators (with a sign factor for each fermion commutation). The only commutation relation one has to know in order to compute all the others is that between (Wigner) momentum space creation and annihilation operators are the standard ones $\{a(p)a^*(p')\} = \delta(p - p')$ and similarly for the b ...

The surprise is the result of the $j(u)$ commutator. One would naively think that the commutator of an expression which itself is bilinear in fermions would contain in addition to a c-number term (complete contraction) also a bilinear terms but against naive expectation this operator term, which is present in higher dimensions, is absent for 2-dimensional massless fermions [31]. In fact this is the only case where this simplification occurs in all other cases the bilinear current of a Fermi field fulfills never canonical commutation relations. This tricky business brought Jordan into a conflict with Fock [32]. In the abstract of the above cited paper Jordan vigorously (and correctly) refutes Fock's critique.

It is worthwhile to write the form of the current commutation relation

$$[j(u), j(u')] = c\delta'(u - u') \quad (2)$$

Jordan always performed his calculation in the filled Dirac sea i.e. in the charge symmetric prescription; the use of the *hole theory* would have caused a serious confusion in particular in such calculations. This derivative term in a commutation relation was equivalent to the presence of the so-called Schwinger terms of 1959 [33] i.e. derivative of delta functions in the mixed space-time components of currents in any dimension.

The cited paper of Jordan also treats the inversion of bosonization, namely the re-fermionization starting from the potentials $V(u)$

$$\begin{aligned} \Psi(u, \alpha) &\equiv e^{i\alpha V(u)} = e^{i\alpha \int_{-\infty}^u j(u) du} \\ [j(u), \Psi(u', \alpha)] &= \alpha \delta(u - u') \Psi(u', \alpha) \end{aligned} \quad (3)$$

where Jordan only considers the case $\alpha = 1$ which leads back to the canonical fermion with dimension 1/2. From a modern point of view these fields are charge-carrying fields of charge α associated with the current operator $j(u)$. Each charge defines a superselection sector i.e. there are continuously many. The Ψ are fields which turn out to be conformal covariant and therefore possess a scale dimension which is proportional to α^2 (the proportionality constant depends on the normalization of V). Together with the anomalous scale dimension the charge α determines also the anomalous (conformal) spin and through it the statistics which turns out to be "anyonic" i.e. that associated with an abelian braid group representation. There is one value of α for which the conformal spin is 1/2 and the anyonic commutation relation becomes fermionic; this is the value which Jordan used for re-fermionization and for which he showed that the Ψ coalesces with the original fermion ψ .

What one does not find in the neutrino papers of Jordan are calculations of states or correlation functions. In this respect he was lucky because that would have opened Pandora's box. It is doubtful that he would have had the conceptual resources one needs

to extract the charge superselection rules

$$\langle \Psi(u_1, \alpha_1) \Psi(u_2, \alpha_2) \dots \Psi(u_n, \alpha_n) \rangle \neq 0 \text{ only for } \sum_{i=1}^n \alpha_i = 0 \quad (4)$$

from the infrared divergencies of the $V(u, \alpha)$. This certainly cannot be understood in terms of simple minded quantum mechanical computational rules. A more demanding method based on the DHR (Doplicher-Haag-Roberts) superselection method of algebraic QFT can be found in [34].

As already mentioned the presentation of a new structure in low spacetime dimension coupled with the claim of its validity in higher dimensions as self-evident was common practice at that time. Almost all quantum mechanical calculation to illustrate matters of principle were for reasons of simplicity done in one dimensional models; most fundamental papers at the beginning of QM were written in this way. Nobody seem to have seriously doubted Jordan about his tacit generalization to 4 dimensions; in fact it is much more probable that most of his contemporaries did not understand his two-dimensional "magic" of starting with fermions and ending up with bosons. Those few who believed to have understood part of his calculations thought that there was a computational mistake, as in the before mentioned reaction of Fock. Nowadays we know better that, whereas the change of dimensions in QM does not lead to structural changes (at least if only a finite number of particles are involved), this is very different in QFT where already the classification of positive energy particle states according to Wigner depend crucially on spacetime dimensions. Nowadays one uses low dimensional QFT as a *theoretical laboratory*, but being more careful about the extracted messages for particle physics in higher dimensions.

Nevertheless there was the feeling that with the neutrino theory of light Jordan had let his imagination go overboard. Hence a little rubdown was in store. It came as a carnivalesque "Spottlied" with the following text (the melody is that of Mack the Knife) [35]:

"Und Herr Jordan	"Mr. Jordan
Nimmt Neutrinos	takes neutrinos
Und daraus baut	and from those he
Er das Licht	builds the light.
Und sie fahren	And in pairs they
Stets in Paaren	always travel.
Ein Neutrino sieht man nicht."	One neutrino's out of sight".

One would suppose that this rather good humored song was presented at the end of a conference or during a conference dinner, but Pais does not comment on this. Insofar as the mock song refers to the misleading title it is well-aimed, however as a presentation of the Bosonization/Fermionization idea the paper is not only correct but even far ahead of its time!

Our modern viewpoint on this issue is that although the photon cannot be viewed as a ν - $\bar{\nu}$ bound state an interacting QFT is condemned (essentially by its locality principle) to follow the local quantum physical adaptation of "Murphy's law" [36]: *what is not forbidden (by superselection rules) to couple does couple!*

Applied to the problem at hand this means among other things

$$\langle 0 | F_{\mu\nu}(0) | p, \vec{p}' \rangle^{in} \neq 0 \quad (5)$$

In words: the formfactor of the photon field between a $\nu\text{-}\bar{\nu}$ state with momenta p and p' and the vacuum is nonzero (but as a result of the presence of weak and electromagnetic interaction it is extremely tiny). The label in/out on particle states of formfactors is important because the connection between particles and fields in the presence of interactions was not at all understood at the time of Jordan but starting from the 60s we know that interacting QFT has no particle at finite times, they only have an asymptotic reality for infinite times in the sense of scattering theory. This is sufficient to secure their existence as states in the physical Hilbert space, but not for generating particles in compact spacetime regions. One consequence of what has been metaphorically referred to as Murphy's law is *nuclear democracy* namely that the quantum mechanical hierarchy between elementary particles and bound states disappears in QFT; the only remaining hierarchy is that between fundamental and fused "charges".

There is an interesting connection of the discrete "Paulion" formalism in the Jordan-Wigner paper [15] and the fermionization (3) which for generic charge α is really an "anyonization". As mentioned in the first section the Jordan-Wigner transformation formalism becomes more concrete in $d=1+1$ when the abstract ordering passes to a concrete linear ordering. In the continuous limit one obtains exponentials of line integrals which make the nonlocal character of the "anyonic" (braid group) commutation relation explicit. Jordan has various footnotes [37] to the Jordan-Wigner paper but he does not elaborate the connection between the two formalisms.

Jordan's model has an interesting relation with the later Schwinger model [38]. The Schwinger model is a 2-dimensional massless quantum electrodynamics which Schwinger proposed in order to illustrate that a gauge theory is not necessarily describing photons and free electric charges, rather its observable content under certain circumstances may consist of massive vectormesons and screened electric charges. The gauge invariant content of the Schwinger model is described by field which is the exponential of a massive free field i.e. it is of the form (3) except the field Φ_{schw} is now a massive⁷ free field in $d=1+1$ which depends on space and time (and not just on the light-ray combination $u=t+x$). The model loses its screening aspect for short distances when

$$e^{i\alpha\Phi_{schw}(x,t)} \xrightarrow{s.d.} e^{i\alpha(\Phi(u)+\Phi(v))}, \quad u = t + x, \quad v = t - x \quad (6)$$

The short distance limit is carried out on the n -point correlation functions by scaling the fields in such a way that no correlation diverges. With this role there are many n -point function which go to zero namely all those which violate the charge superselection rule (4). It turns out that the short distance limit of the massive Schwinger model is precisely the massless Jordan model.

The appearance of the charge superselection in the Jordan limit i.e. the emergence of the Jordan model in the short distance limit of the Schwinger models has been interpreted as a two dimensional analog of the still unsolved problem of quark and gluon

⁷The mass is actually proportional to the square of the coupling strength.

confinement in 4-dimensional quantum chromodynamics (QCD). The liberation of charges in the asymptotic short distance region of the Schwinger model corresponds to the perturbatively established asymptotic freedom in the nonabelian gauge description of QCD has been since and case of the Schwinger model the observable particle content consists of interaction-free scalar massive particles.

Such a simple illustration of screening/confinement versus short distance charge liberation is only possible in $d=1+1$; free massless fields in higher dimensions do not carry a charge structure. Hence although the Jordan model, different from the intentions of its protagonist, has no bearing on a neutrino theory of light (for whose validity there is not the slightest hint within the Standard Model, which is our presently best particle theory), it is believed to serve as a useful analogy for the great unsolved problem of quantum chromodynamics which is the problem of quark confinement. A recent description of the Jordan model and its appearance in the massless limit Schwinger model can be found in [39].

The re-fermionization formula (3) of the Jordan model describes a chiral fermion only for one value of α which corresponds the operator dimension $\dim\Psi = 1/2^8$. For this value the Ψ can be written as a Fourier transform in terms of Wigner particle creation and annihilation operators $a^*(p), a(p)$ and their charge-conjugate antiparticles. For other values of α one encounters a particle like entity which received the name "infraparticle" in the beginning 60s [40]. The reason is that for those values the pointlike field description was lost in favor of an infinite stringlike behavior as in (3). The model on which this was first noticed was a two-dimensional massive Dirac field coupled to the derivative of a massless scalar field. The long range interaction leading to a solution of the form

$$\psi(x) = \psi(x)_0 e^{ia\varphi(x)} = \psi(x)_0 e^{ia \int_{-\infty}^x \partial_\mu \varphi(x) dx^\mu} \quad (7)$$

Here ψ_0 is massive free Dirac field and since the infrared divergent zero mass scalar field is better interpreted as a string localized free field we prefer the second representation. The momentum space manifestation of the stringlike localization shows an interesting modification at the mass shell where in all theories describing particles there would be a $\delta(\kappa^2 - m^2)$ function. In terms of the Kallen-Lehmann spectral function the infraparticle spectral density for a massive $d=1+1$ infraparticle is instead

$$\begin{aligned} \langle \psi(x) \overline{\psi(y)} \rangle &= \frac{1}{2\pi} \int d\kappa^2 \rho(\kappa^2) \int \frac{dp}{2\sqrt{p^2 + \kappa^2}} p^\mu \gamma_\mu e^{ip(x-y)} \\ \rho(\kappa^2) &= c(\alpha) \theta(\kappa^2 - m^2) (\kappa^2 - m^2)^{-d(a)} \end{aligned} \quad (8)$$

The only important aspect of this formula is that a) for vanishing coupling strength α the infraparticle power law of the spectral density passes into the free field result $\rho(\kappa^2) = \delta(\kappa^2 - m^2)$ and b) it is not possible to write the spectral function as $\rho(\kappa^2) = \delta(\kappa^2 - m^2) + \text{rest}$ without violating the unitarity of the rest i.e. In an infraparticle Hilbert space there is no possibility to sneak in a particle through the back door.

The infrared behavior of the perturbative treatment of this model are reminiscent of those encountered in QED which led to the strong suspicion that the minimal interaction

⁸The normalizations used in Jordan's work is different from that used in more recent times. We found it convenient to state the results without fixing normalizations.

of electron/positrons with photons is so strong in the infrared (for long distances) that the structure of the particle itself is affected i.e. the changes are more radical (see next section) than in the Coulomb interaction in QM where the modification of time dependent scattering theory leaves the structure of particle unaffected.

The study of chiral conformal QFT started in the beginning of the 70s with the Jordan model but without knowing about Jordan's work since the title under which he published it did not sound trustworthy. All the global conformal block decomposition results involving the universal conformal covering representation were checked for the Jordan model. Only with the appearance of the family of minimal models by Belavin, Polyakov and Zamolodchikov it became clear that the prior structural work was not in vain, the expected richness of chiral conformal QFT really materialized.

By now the wealth of very nontrivial results is staggering, the world of chiral models is meanwhile under impressive mathematical control. This and the infinite family of massive factorizing models are the only families where existence proofs of models have been achieved. It seems that Jordan had the right conceptual-mathematical instinct in emphasizing these models in many of his publication.

The multi-component extension of the Jordan model

$$\begin{aligned} j_k(u) &=: \psi_k^*(u)\psi_k(u) := \partial\Phi_k(u), \quad k = 1, \dots, n \\ \Psi(u, \vec{\alpha}) &= e^{i \sum_{k=1}^n \alpha_k \Phi_k(u)} \end{aligned} \tag{9}$$

is the starting point of a rather simple family of models whose maximal local extensions are characterized by even lattices. Their representations are classified by the finite number of dual lattices and there is a finite number of selfdual lattices which are connected with finite exceptional groups, the largest being the so-called *moonshine group*.

Another application in which one quantizes Φ_k with a n-dimensional zero mode attached to a n-dimensional quantum mechanics p_k, q_k $k = 1, \dots, n$ was used for the operator formulation of the dual resonance model which led to string theory.

The history of the Jordan model in conjunction with the Schwinger model reveals that conceptually rich models develop a life of their own and are even able to survive flawed reasons which served their original introduction. One can be sure that at the time of the "neutrino theory of light" mock song neither Jordan nor his satyric colleagues had any firm idea about what message this 2-dim. model was supposed to reveal. In the 30's the extraordinary subtle conceptual and mathematical problems posed by interacting QFT was not yet appreciated and the idea of studying soluble models as a kind of theoretical laboratory in order to learn something about the classification and construction of interacting particles was still in the distant future. After Jordan's series of papers on the neutrino theory of light there were several other authors who published papers under the heading of neutrino theory of light without even mentioning how this can be achieved in the realistic $d=1+3$ case of physical photons and neutrinos. There were several authors who continued to publish articles on this 2-dim. model under the title "neutrino theory of light" without bothering how to get to light and neutrinos in $d=1+3$ QFT world. It remains somewhat incomprehensible why in none of these papers commented on the discrepancy between title and content.

5 Nonlocal gauge invariants and an algebraic monopole quantization

Jordan was one of the first who realized that the passing from the classical electrodynamic to its quantum counterpart brought about a loss of locality. More specifically besides the local observables which in the gauge theoretical setting are by definition the quantum counterparts of the classical (second kind) gauge invariants any physical charge-carrying object cannot be better localized than a spacelike semiinfinite string. That the algebra of all physical fields is larger than that generated by local observables is not exceptional, but that electric charge-carrying operators cannot be compactly localized is unusual in a theory which has the name "local" is unusual. It raises the question which structure is responsible that beyond charge neutral local observables the charge sectors of a theory are nonlocal in that strong sense. It can be shown that any renormalizable theory which a charged current is related to zero mass $s=1$ field strength through a Maxwell equation as in QED falls into this class.

The best localization for a charged generating field is that of a semiinfinite Dirac-Jordan-Mandelstam string (DJM) characterized *formally* by the well-known expression

$$\Psi(x; e) = \text{"}\psi(x)e^{\int_0^\infty ie_{el}A^\mu(x+\lambda e)e_\mu d\lambda}\text{"} \quad (10)$$

$$\Phi(x, y; e) = \text{"}\psi(x)e^{\int_0^1 ie_{el}A^\mu(x+\lambda(x-y))(x-y)_\mu d\lambda}\bar{\psi}(y)\text{"} \quad (11)$$

Such objects involving line integrals over vectorpotentials or their bilocal counterparts with a connecting "gauge bridge" (11) appear already in Jordan's work on attempts to quantize in a gauge invariant manner [42]. As an ardent positivist and a fierce defender of Heisenberg's maxim to use observables throughout, he even tried to formulate the dynamics of quantum electrodynamics solely in terms of gauge invariant operators. As a similar attempt more than 20 years later by Stanley Mandelstam, the effort fell short of what the authors had expected.

There is however one very pretty fall-out of these attempts which is worth mentioning. Being impressed by Dirac's magnetic monopole quantization, but not by his too classically looking geometric method of presentation, Jordan published a very different algebraic operator derivation in the same year [26]. Three years later he returned to this topic, this time presenting his arguments with more details and some helpful drawings⁹. The argument in both papers is based on the use of the above "bridged" bilocals $\Phi(x, x')$. Starting from the commutation relations

$$[\Phi(x, x'), \Phi(y, y')] = \delta(y - x')\Phi(x, y')e^{i\omega(x, y', x')} - \delta(x - y')\Phi(y, x')e^{i\omega(y, x', x)} \quad (12)$$

$$\omega(x, y, z) = \text{magnetic flux through triangle}$$

$$\text{Jacobi identity for } \Phi's \curvearrowright e^{i\sum_{tetrahedron} \omega} = 1$$

The last line denotes the application of the Jacobi identity to the bridged bilocals whose validity turns out to be equivalent to the magnetic flux through a tetrahedron being integer valued (in certain unities). A similar method for monopole quantization where

⁹In 2005 Roman Jackiw rediscovered this algebraic argument [27].

the result also emerges from a cohomological argument involving a tetrahedron has been proposed by Roman Jackiw [27].

The quotation marks in the above formulas (10) highlights their formal aspects. Since these objects are not belonging to the local gauge invariant operators which appear in the formalism of n^{th} order renormalized perturbation theory, they have to be defined (and renormalized) by hand, a gruesome task which was carried out by Steinmann [28] who succeeded to attribute mathematical meaning to these expressions. This technical work is important because electrically charged fields cannot be better localized than along a semiinfinite spacelike string and this has radical implication for the associated charged particles.

It is fascinating and very informative to follow the idea of gauge invariant nonlocal semiinfinite string-localized fields and that of bridged bilocals a bit more through the history of particle physics. In an historically important paper, written about the same time as the appearance of the above string-localized fields in Jordan's work, Bloch and Nordsieck, using a simple model, argued that the scattering of photons off charged particles will lead to infrared divergencies unless one treats the problem in the way they proposed in their model. After the renormalization theory of QED was understood, Yennie, Frautschi and Suura (YFS) showed that although the scattering amplitudes are infrared divergent the inclusive cross section for a specified photon resolution Δ not; the artificially introduced cutoff from the "virtual" photons in the Feynman amplitude compensates with that coming from the integration over the infrared tail of the real photons to be summed over up to Δ .

From a pragmatic view the formalism of YFS (i.e. compensating two infrared divergencies against each other) would have been the end; a finite answer which agrees with the experimental data would signal for many physicists: mission accomplished. But Jordan and some of his contemporaries had a strong philosophical motivation and one can almost be sure that this kind of pragmatic reasoning would not have been the end of the infrared issue in QED. Rather it would have been very much in the spirit of Jordan to search for a deep connection between the formula for the string-localized generating fields describing charge-carrying fields (10) and charged "particles". The parenthesis is to indicate that electrically charged particles in QED are not particles in the usual (Wigner) sense of irreducible (m,s) representations of the Poincaré group¹⁰. In this way it becomes clear that the breakdown of the scattering theory is not just because the interaction is long ranged as the quantum mechanical Coulomb scattering where the breakdown of the standard scattering theory through the appearance of a logarithmic phase factor does not have any consequences for the structure of single particle states. Rather in QED the very existence of particles is affected, electrically charged particles are "infraparticles" and even in case of a one particle state the best one can do is prepare such a state that no photon with energy larger than Δ can emerge from such a state where Δ can be arbitrarily small.

In the previous section the issue of infraparticles came up in connection with a hidden infinite stringlike localization aspect of Jordan's two-dimensional model. In that case the two-point spectral function $\rho(\kappa^2)$ remains covariant. However in 4 dimensions the spacelike string direction defined in terms of a spacelike unit vector e certainly enters the

¹⁰The relativistic particle concept was laid down in Wigner's famous representation theoretical classification of 1939 [43].

covariance low for ρ . The characteristic dissolution of the mass shell delta function into a singular cut, which starts at $p^2 = m^2$ is a general feature of infraparticles. Unitarity puts a lid on the strength of the singularity, it must be milder than a delta function. This leads to a vanishing in/out LSZ scattering limit; the perturbative infrared divergence of the scattering amplitude is a perturbative phenomenon; by summing up the leading terms and letting $\Delta \rightarrow 0$, the infinity is converted into zero.

Hence the pointlike gauge dependent Dirac spinor $\psi(x)$ which enters the Lagrangian is an unphysical chimera which has to be tolerated as an intermediate computational tool as long as one does not know how to formulate the dynamics and the computations directly in terms of physical operators. There is simply no compact localized operator which applied to the vacuum generates a state with an electron or positron charge, only zero charge operators are compactly localizable. This is an illustration of the fact that the main and only physical principle is causal localization and its realization in Hilbert space (unitarity). Even the Lagrangian formalism and perturbation theory has to cede if it produces unphysical operators. The only known way up to know is to repair the perturbative situation by hand i.e. to construct the DJM object from the unphysical ψ and A_μ which is a gruesome enterprise. The fact that QFT is governed by one principle namely causal localization does not make life simpler.

There is no doubt that the knowledge of the local observables fix the remaining physical operators, including bridged bilocals, but the question which remained unanswered since the time of Jordan is how, by what formula? If the theory would not be a gauge theory but one which only involves $s=0$ and $s=1/2$ fields it would be less problematic to construct bilocals from locals. The bilocal $: A(x)A(y) :$ can be obtained from the product of two locals $: A^2(x) :$ by a lightlike limiting procedure [44] and recently the problem whether bridged bilocals can be obtained in this way came up [45]. Having such zero charge bilocals, the DJM would be expected to result from the "shifting the (unwanted) charge behind the moon" argument.

There is however another more radical (but also more promising) method. The origin of all the problems goes back to Wigner's 1939 classification of positive energy representations of the Poincaré group. The transition from Wigner's form to the covariant form of the representation is not unique. In the massive case the (m,s) $s=\text{halfinteger}$ Wigner representation can be described by infinitely many dotted/undotted spinorial fields for given spin s

$$\begin{aligned} \psi^{(A,\dot{B})}(x), \quad \left| A - \dot{B} \right| \leq s \leq A + \dot{B} \\ m = 0, \quad s = \left| A - \dot{B} \right| \end{aligned} \tag{13}$$

The second line contains the formula for the significantly reduced number of zero mass spinorial wave functions; the popular potentials A_μ for $s=1$ and $g_{\mu\nu}$ for $s=2$ are not backed up by representation theory which would allow field strength ($F_{\mu\nu}$ for $s=1$, $R_{\mu\nu\kappa\lambda}$ for $s=2$). In classical physics there is no such requirement because unitarity and Hilbert space requirement is no issue. If one uses pointlike covariant potentials for the formulation of interactions (the standard method in QED) then one has to rely on a quantum gauge formalism (Gupta-Bleuler or BRST) which allows a cohomological return to a physical subspace which does not contain charged particles.

But there is another way which is more physical. Its starting point is the realization that the full spinorial possibilities (13) can be recovered with semiinfinite stringlocalized covariant potentials $A_\mu(x, e), g_{\mu\nu}(x, e)$ e = spacelike string direction. In this way the origin of the string-localized charged fields and their infraparticle behavior near the old mass shell are not surprising since the semiinfinite localization aspect enters through the vectorpotentials from the beginning. Although all the fields are now living in a physical Hilbert space, the Epstein-Glaser iteration step is more complicated since the causal position of semiinfinite strings, which should preserve the string localization for counterterms, is now more involved. These new ideas may lead to the long overdue reformulation of gauge theory.

Closely connected with the string localization of the potentials is the Aharonov-Bohm effect. In fact there is a theorem which shows that behind this effect is the consequence of some basic structural difference between operator algebras generated by field strength associated to massless spin $s \geq 1$ as compared to their massive counterpart. Whereas for any compact simply connected spacetime region \mathcal{O} there holds Haag duality

$$\mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}') \quad (14)$$

i.e. the commutant of the algebra of \mathcal{O} -localized operators equals the algebra of operators localized in the causal complement of \mathcal{O} , if it comes to non simply connected regions (example toroidal spacetime regions \mathcal{T}) the $m = 0, s \geq 0$, the operator algebras show a violation [46]

$$\mathcal{A}(\mathcal{T})' \supset \mathcal{A}(\mathcal{T}') \quad (15)$$

of Haag duality which has no counterpart in the massive case. The explanation is that operator algebra $\mathcal{A}(\mathcal{T})$ generated by the field strength does not contain all physical operators, there are Bohm-Aharonov like nonlocal operators in \mathcal{T} , but thanks to the breakdown of Haag duality the algebra of the field strengths "knows about its own imperfection". For higher spin the Bohm-Aharonov phenomenon has a generalization affecting also algebras localized in multifold connected regions. The use of stringlocalized potentials $A_\mu(x, e), g_{\mu\nu}(x, e), \dots$ makes this nonlocal aspect manifest [48] which remains hidden in the standard formalism and only suddenly pops out e.g. when one tries to construct electrically charged operators.

It is quite surprising that a theory as QED, which already in the middle of the 30s showed a rich conceptual structure, has still not reached its conceptual closure. This is even more so for its nonabelian extension the Yang Mills theory and quantum chromodynamics (QCD). We have gotten accustomed to nice words as "gluon" and "quark" confinement about which we think we know their content, but our understanding goes hardly¹¹ beyond the small subset of gauge invariant local operators. When it comes to the description of physical charge-carrying operators our formalism foresakes us not to mention the string localized counterparts of the DJM operators and the question what happened to the gluon and quark degrees of freedom.

¹¹Whereas the vacuum correlations of gauge-invariant local observables are finite in the abelian case (and only on-shell formfactors and scattering amplitudes are infrared divergent) in the nonabelian case also the gauge invariant correlation functions are divergent.

Whereas the long stagnation on questions like this and on the standard model could be shrugged off by pointing to the complexity of the task, it is somewhat saddening to notice the loss of hard gained conceptional understanding. A typical illustration for the conceptual impoverishment is the story of infraparticles which begun with the infrared divergent scattering amplitudes and the recipe for calculating inclusive cross sections with a given inclusive resolution Δ . As mentioned, the conceptual conquest of this problem started with the realization that the root of the problem is not merely a long range modification of scattering, but a radical change of the particle concept. From Jordan's semiinfinite stringlike charged fields (10) to infraparticles is a long way. The first observation of a deviation from the standard particle structure was observed in a two-dimensional model [40] similar to Jordan's in the previous section. It was observed that the expansion of the cut which starts at the would be particle mass with respect to the coupling strength leads to similar terms as in the YFS work. The infraparticle aspect of electrically charged particles in QED was proven in the 80s, the most conceptual line of arguments in [41] established the infinite extension of electrically charged infraparticles as a consequence of the quantum adaptation of Gauss's law. At that point it became clear that the string-like extension is inexorably linked to the "dissolution" of the mass shell.

The loss of conceptional understanding in contemporary attempts to go beyond the standard (Wigner) particle concept becomes obvious in the flood of papers on "unparticles" (more than 400 by now) starting with [47]. The authors owe an answer how their ill-defined objects can be placed into the conceptual quite dense meshwork of string-localized fields and its momentum space properties in terms of dissolved mass shells¹² Even in those few cases where they cite the infraparticle work, it is clear that they do not understand its conceptual basis. They seem to think that the YFS kind of infrared problem is similar to the infrared aspects of Coulomb scattering where the one-particle remain those of standard particles. There seems to be nobody of sufficient knowledge of QFT whom they would listen to. The new generation of referees have the same background and are unable to lift the state of arts to where it has been in the past. This makes the question of the causes behind this derailment relevant, but this article about backtracks to Pascual Jordan is not the right setting to enter a critical analysis of the Zeitgeist.

6 Biographical Notes

Among the physicists whose biography contains glorious scientific achievements next to disturbing human weaknesses, Pascual Jordan certainly plays a role which is hard to be overlooked.

Born on October 18, 1902 in Hannover of mixed German-Spanish ancestry, he became (starting in the age of 22) a main architect of the conceptual and mathematical foundations of quantum theory and the protagonist of quantum field theory. Pascual Jordan owes his Spanish name to his great grandfather Pascual Jorda (unrelated to the biblical river), who came from the Alcoy branch (southern Spain) of the Jorda nobility

¹²Apparently the unparticle followers believe that the infrared divergencies in the scattering theory of electrically charged particles represent (like the quantum mechanical Coulomb scattering) just a small modification of scattering theory and not a radical change of the particle concept.

with a genealogy which can be traced back to the 9th century. His grandfather fought on the side of the British-Prussian coalition against Napoleon. After the British victory of Wellington over Napoleon, the family patriarch Pascual Jorda settled in Hannover where he continued his service to the British crown as a member of the “Koeniglich-Grossbritannisch-Hannoverschen Garde-Husaren Regiments” until 1833. Every first-born son of the Jordan (the n was added later) clan was called Pascual.

There is no doubt that *Erich Pascual Jordan* took the lead in the formulation of the conceptual and mathematical underpinnings of “Matrix Mechanics” in his important paper together with Max Born [1] entitled “Zur Quantenmechanik” and submitted for publication on 27. September 1925 (3 months after the submission of Heisenberg’s pivotal paper!). Jordan’s mathematical preparation, particularly in the area of algebra, was superb. He had taken courses at the Göttingen mathematics department from Richard Courant and became his assistant (helping in particular on the famous Courant-Hilbert book); through Courant he got to know Hilbert even before he met the 20 year older Max Born, the director of the Theoretical Physics Department of the Göttingen university. By that time Jordan already had gained his physics credentials as a coauthor of a book which he wrote together with James Franck [49].

After Max Born obtained Heisenberg’s manuscript, he tried to make sense of the new quantum objects. While he had the right intuition about their relation to matrices¹³, he felt that it would be a good idea to look for a younger collaborator with a strong mathematics background. After Pauli rejected his proposal and even expressed some reservations that Born’s more mathematically inclined program could stifle Heisenberg’s powerful physical intuition, the more corporative Jordan volunteered to collaborate in this problem [35][50]. Within a matter of days he confirmed that Born’s conjecture about the off-diagonal part in the p-q commutation relation was indeed consistent with the conjecture of an identity operator so that its full content was secured. Together with Born’s previous argument about the diagonal part this result completed the central structural relation of the new mechanics and made the content of Heisenberg’s paper more concrete. Probably as a consequence of the acoustic similarity of p-q with Pascual, the younger members of the physics department (the protagonists of the “Knabenphysik”) in their discussions often called it Pascual’s relation¹⁴. Max Born became Jordan’s mentor in physics. Jordan always maintained the greatest respect which withstood many later political and ideological differences although that relation was severely strained when Jordan a decade later joined the anti-Semitic Nazi party.

The year 1925 was a bright start for the 23-year-old Jordan. After the submission of the joint work with Max Born on Matrix Mechanics (the *Zweimaennerarbeit*), in which the p-q commutation relation appeared for the first time, there came the famous “*Dreimaennerarbeit*” [2] with Born and Heisenberg in November of the same year, only to conclude the year’s harvest with a paper by Jordan alone on the “Pauli Statistics”. Jordan’s manuscript apparently contained what is nowadays known as the Fermi-Dirac statistics; however it encountered an extremely unfortunate fate after its submission because it landed on the bottom of one of Max Born’s suitcases (in his role as the editor

¹³“Matrix-mechanics” preceded the more appropriate later formulation based on operator theory in Hilbert space due to Fritz London and John von Neumann.

¹⁴I am indebted to Anita Ehlers for permitting me to use her biographical notes on Pascual Jordan.

of the *Zeitschrift fuer Physik*) on the eve of an extended lecture tour to the US, where it remained for about half a year. When Born discovered this mishap after his return to Goettingen, the papers of Dirac and Fermi were already in the process of being published. In the words of Max Born [51][52] a quarter of a century later: "I hate Jordan's politics, but I can never undo what I did to him.....When I returned to Germany half a year later I found the paper on the bottom of my suitcase. It contained what one calls nowadays the Fermi-Dirac statistics. In the meantime it was independently discovered by Enrico Fermi and Paul Dirac. But Jordan was the first".

In Jordan's subsequent papers, including those with other authors such as Eugene Wigner and Oscar Klein, it was always referred to as "Pauli statistics" because for Jordan it resulted from a straightforward algebraization of Pauli's exclusion principle. As Stanley Deser remarked recently in a private communication, without Born's negligence the name "Jordanons" would possibly have added a Middle East flavour to the beginnings of modern particle physics.

From later writings of Born and Heisenberg we also know that Jordan contributed the sections on the Statistical Mechanics (or rather Kinetic Gas Theory) aspects to their joint papers on matrix mechanics; this is not surprising since the main point in his 1924 PhD thesis¹⁵ was the treatment of photons according to Planck's distribution whereas thermal aspects of matter were still described according to the classics Boltzmann statistics. He continued this line of research by introducing the "Stosszahlansatz" for photons and substituting for electrons and atoms the new Bose statistics [53]¹⁶; this work brought him praise by Einstein and led to an (unfortunately lost) correspondence between the two. In the following we will continue to briefly mention his scientific contributions in the biographical context and reserve a more detailed account about their scientific content to the next section.

The years 1926/27 were perhaps the most important years in Jordan's career in which he succeeded to impress his peers with works of astonishing originality. The key words are Transformation Theory [55][56] and Canonical Anti-Commutation Relations [14]. With these discoveries he established himself as the friendly competitor of Dirac on the continental side of the channel; in its printed form one finds an acknowledgment of Dirac's manuscript¹⁷. As an interesting sideline, one also notes that in a footnote at the beginning of the paper on transformation theory (which addresses the equivalence of the Heisenberg and Schroedinger quantum mechanics), Jordan mentions a "very clear and transparent treatment" of the same problem in a manuscript by Fritz London, a paper which he received after completing his own work and which was published in [4]. So it seems that the transformation theory was discovered almost simultaneously by the three authors.

¹⁵His thesis led to a small article in which Einstein criticised Jordan's point that the photon recoil and its emission with a sharp momentum (Einstein's "Nadelstrahlung") was not necessary for obtaining a thermal equilibrium. Einstein pointed out that there are other arguments why there are momentum carrying photons and after that incidence Jordan not only derived particles from waves, but became the most ardent defender of wave- instead of particle- quantization.

¹⁶This paper was submitted simultaneously with another paper in which Jordan coined the term "Pauli-Principle" [54]; but the relation to statistics was only seen later.

¹⁷In those days papers were presented in a factual and very courteous style; however verbal discussions and correspondences were sometimes more direct and less amiable (e.g. see some published letters of Pauli [35][59]).

As the result of a very influential textbook whose first edition appeared in 1930, most physicists are more familiar with Dirac's version. Although London's treatment remained less known, Jordan's praise is more than the old style politeness or a condescent attitude towards younger people outside the ongoing dialog. London's article introduced for the first time the modern Hilbert space operator setting as we know it and this several years before von Neumann published his famous work on the mathematical and conceptual structure of quantum theory.

Jordan's most seminal contribution is his discovery of "Quantization of Wave Fields" which marks the birth of QFT. The reader finds a description about the chronology of this most important of Jordan's discoveries, its relation to Dirac's radiation theory and its influence on the subsequent development of particle physics in the next section.

We are used to the fact that names for new concepts and formulas in modern publications should be taken with a grain of salt. But we trust that what the textbooks say about the beginnings of quantum mechanics can be taken literally. When we look at Jordan's paper ([55] page 811) we note that the relation between Born and the probability interpretation in the Schroedinger formulation of quantum mechanics is more indirect than the impression obtained from reading most textbooks. Max Born in his 1926 papers was calculating what is nowadays called the Born approximation of scattering theory and his proposal to associate a probability with scattering in modern terminology amounted to identifying the cross section as the main observable of quantum mechanical collision theory. According to Jordan the use of the probability interpretation for the absolute square of the x-space Schrödinger wave function is appears first in Pauli's work ¹⁸ who, although strongly influenced by Born's probability interpretation of the scattering amplitude, does not refer to Born's paper. It happened frequently that in such a surrounding of intense communication as the physics department of Goettingen during the 1920's new ideas were used freely in scientific discussions and that the later attachment of a person's name even if it represents the origin of a concept correctly cannot be taken *prime facie* when it comes to details. Although the x-space probability does not explicitly appear in Born's early publications, he certainly was the first who brought the all-important probability aspect into quantum physics.

Jordan's increasing detachment from the ongoing conceptual development of QFT after 1933, and his concentration on more mathematical and conceptual problems of quantum theory whose investigation can be pursuit without being instantly connected to the stream of new information happens at the time of his unfortunate political activities, as he lets himself be increasingly sucked into the mud of the rising Nazi-regime. The attempt to understand some of his increasing nationalistic and militaristic behavior in terms of his family background does not yield convincing clues. A more plausible reason is that Jordan's radicalization was a consequence of the general great post-war turmoil within the German society which led to extreme political polarization and finally to the demise of the democratic Weimar republic.

Pascual Jordan was brought up in a traditional Lutheran religious surrounding. At the age of 12 he apparently went through a soul-searching fundamentalist period (not uncommon for a bright youngster who tries to come to terms with rigid traditions) in

¹⁸This may partially explain why Pauli in his 1933 Handbuch article on wave mechanics introduced the spatial localization probability density without reference to Born.

which he wanted to uphold a literal interpretation of the bible against the materialistic Darwinism (which he experienced as a "quälendes Aergernis", a painful calamity), but his more progressive teacher of religion convinced him that there is basically no contradiction between religion and the sciences. This then became a theme which accompanied him throughout his life; he wrote many articles and presented innumerable talks on the subject of religion and science.

When Jordan moved from his hometown Hannover to the University of Goettingen at the beginnings of the 1920's, the opinion that the treaty of Versailles with its exorbitant reparation requirements was unjust and threatened the new German democratic republic was quite widespread in German society and even in academic circles. Such a political view certainly would not have isolated Jordan from his Goettingen colleagues. But Jordan's political inclination went far beyond and became increasingly nationalistic and extreme right-wing. These were of course not very good prerequisites for resisting the propaganda of the NS movement, in particular since the conservative wing of the protestant church (to which he adhered¹⁹) started to support Hitler in the 30's; in fact the behavior of both of the traditional churches during the NS regime belongs to their darkest chapters. Hitler presented his war of aggression as a divine mission and considered himself as an instrument of God's predestination (göttliche Vorsehung), while almost all Christian churches were silent or even supportive.

Already in the late 20s Jordan published articles under a pseudonym of an aggressive and bellicose stance in journals dedicated to the spirit of German Heritage. It is unclear to what degree his more cosmopolitan academic peers in Göttingen knew about these activities, but the fact that he published under a pseudonym suggests that he wanted to keep his political activity hidden from them.

Jordan considered the October revolution and the founding of the Soviet Union as extremely worrisome developments. The strong anti-communist stance of the Nazis which they later mixed with their anti-Semitism was certainly attractive to persons with a nationalistic and bellicose ideology. But on the other hand the fact that most of Jordan's collaborators were of Jewish background makes it very difficult to understand his sympathies for a party which was so overtly anti-Semitic. The only clue (which he himself offered after the war) is his very naive idea that he could use his fame to influence the new regime; his most bizarre project in this direction was to convince some influential people in the NS establishment that modern physics, as represented by Einstein and especially the new Copenhagen brand of quantum theory, was the best antidote against the "materialism of the Bolsheviks". This explains perhaps why he joined NS organisations at an early date when there was yet no pressure to do so [57]. He of course failed in such attempts; despite verbal support²⁰ he gave to their nationalistic and bellicose propaganda and even despite their very strong anti-communist and anti-Soviet stance with which he fully agreed, the anti-Semitism of the Nazis did not permit such a viewpoint since they considered the acceptance of Einstein's relativity and the modern quantum theory with its Copenhagen interpretations as incompatible with their anti-Semitic propaganda; one

¹⁹The oldest son of the family patriarch Pascual Jorda was brought up in the Lutheran faith of his foster mother, whereas all the other children born within that marriage were raised in the Catholic faith.

²⁰In contrast to Heisenberg he did not directly work on any armament project but rather did most of his military service as a meteorologist.

can also safely assume that the intense collaboration with his Jewish colleagues which extended into the first half of the 30's made him appear less than trustworthy in the eyes of the regime. Apparently Pauli thought that Jordan's sharp intellect failed outside of the sciences and that in order to judge his action one must be willing to make some allowances. My own impression which I formed as a undergraduate student participating in his General Relativity seminar in 1957 (in which Juergen Ehlers, Wolfgang Kundt, Rainer Sachs and Engelbert Schuecking were the dominating active participants) was consistent with that of a very friendly but extremely clumsy person who, outside his area of competence, did not have a reliable guiding instinct. I do remember a scene where, after the seminar he wanted to wash off the chalk-dust from his hands but opened the water from the tap too much so that he could not adjust it without becoming completely wet. His pet version of general relativistic gravity at that time had a variable gravitational constant and he was collecting geological data in favor of a decreasing Newton gravitational strength; in fact I remember that he had at least one collaborator who was writing his PhD on this subject. This work never gained any significance comparable to his pivotal contributions to QM and QFT. We (the younger participants in that seminar) used to joke about the death of the dinosaurs being the most convincing support for Jordan's theory (the decrease of Kappa lead to a popping up of their massive bodies). One of the first scientific contacts he had was Peter Bergmann's visit. Bergmann was a relativist at the University in Freiburg when the anti-Semitic Nazi laws forced him to continue in the US where he created an internationally renown relativity group (Rainer Sachs was one of his younger collaborators). Those who knew Jordan were (like Pauli) inclined to ignore his verbal support of the new Nazi order as well as his membership in the party, there was less leniency towards Heisenberg who was neither a party-member nor the author of supporting statements and articles but who headed the German Uranium club.

Jordan's career during the NS times ended practically in scientific isolation at the small university of Rostock; his promotion to fill von Laue's position in Berlin in 1944 came too late for a new scientific start. Despite his Nazi sympathies he never received benefits for his pro-NS convictions and the sympathy remained one-sided. Unlike the mathematician Teichmueller, whose rabid anti-Semitism led to the emptying of the Göttingen mathematics department, Jordan inflicted the damage mainly on himself. The Nazis welcomed his verbal support, but he always remained a somewhat suspicious character to them since despite his support for their aggressive policies and their preemptive war, he never rescinded having collaborated with Jewish colleagues in the past nor lend public support to the anti-Semitism of the Nazis²¹. As a result, unlike the less political Heisenberg and others, he was not called upon to participate in war-related projects and spent most of those years in scientific isolation. This is somewhat surprising in view of the fact that Jordan, like nobody else in the scientific community, tried to convince influential people in the NS regime that fundamental research should receive more support because of its potential weapons-related applications; in these attempts he came closer to a "star wars" propagandist of the Nazis than Heisenberg who headed the German uranium program but kept a low political profile and never joined the Nazi party.

²¹However when he felt threatened by fellow Nazi sympathizers who cast doubts on his loyalties, he reacted by exposing their prior Jewish connections and accused them of opportunism (e.g. the Dingler affair).

Jordan's early party membership and his radical verbal support of the NS regime in several articles written during the war got him into a lot of trouble after 1945. For two years he was without any work and even after his re-installment as a university professor he had to wait up to 1953 for the reinstatement of his full rights (e.g. to advise PhD candidates). When his pre-war friend and colleague Wolfgang Pauli asked him after the war: "Jordan, how could you write such things?" Jordan retorted: "Pauli, how could you read such a thing?" Without Heisenberg's and Pauli's help he would not have been able to pass through the process of de-nazification (in the jargon of those days Jordan got a "Persilschein", i.e. a whitewash paper) and afterwards to be re-installed as a university professor. In Pauli's acerbic way of dealing with such problems he tried to explain Jordan's political aberrations as follows: "Jordan is in the possession of a pocket spectrometer by which he is able to distinguish intense brown from a deep red"²². "Jordan served every regime trustfully" is another of Pauli's comments. Nevertheless Pauli recommended Jordan for a reinstatement of his professorship at the University of Hamburg and he also suggested to him to stay away from politics and rather "worry about his pension".

Jordan did not heed Pauli's advice for long; during the time of Konrad Adenauer and the great debates about the re-armament of West Germany including the stationing of nuclear weapons he became a member of parliament for the CDU party. His speech problem (as mentioned before he sometimes fell into a stuttering mode which was quite painful for people who were not accustomed to him) which already caused a serious impediment during his scientific years in Göttingen, had more serious consequences in politics and prevented him from becoming a scientific speaker for the CDU party. At that time of the re-armament issue there was a manifesto by the "Göttingen 18" which was signed by many of the famous names of the early days of the university of Göttingen quantum theory, including Werner Heisenberg and Max Born. Jordan, whose signature was conspicuously absent, immediately wrote a counter article with the endorsement of the CDU party in which he severely criticized the 18 and claimed that by their action they endangered world peace and undermine stability in Europe. Max Born felt irritated by Jordan's article, but he did not react in public. What really annoyed him were Jordan's attempts to disclaim full responsibility for his article by arguing that some of the misunderstandings resulted from the fact that it was written in a hurry. But Born's wife Hedwig exposed her anger in a long letter to Jordan in which she blamed him for "deep misunderstanding of fundamental issues". She quoted excerpts from previous books by Jordan and wrote: "Reines Entsetzen packt mich, wenn ich in Ihren Büchern lese, wie da menschliches Leid abgetan wird" (pure horror overcomes me when I read in your books how human suffering is taken lightly). Immediately after this episode she collected all of Jordan's political writings and published them under the title: "Pascual Jordan, Propagandist im Sold der CDU" (Pascual Jordan, propagandist in the pay of the CDU) in the *Deutsche Volkszeitung*.

In the middle of the twenties the authors of the "Dreimaennerarbeit" were proposed twice for the Nobel prize by Einstein, but whereas Heisenberg obtained it in 1932, Dirac together with Schroedinger in 1933 and Born in 1954, the support for Jordan dwindled after the war. Even though he was not only a leading figure in the discovery of quantum

²²The members of the Nazi party appeared in black boots and brown uniforms in their meetings.

mechanics but also the only protagonist of an uncompromising radical version of quantum field theory which together with the post-war renormalization theory culminated in the modern form of quantum electrodynamics, his active sympathy for the NS regime seriously tainted him and affected his scientific reputation. Nevertheless, in 1979 it was his former colleague and meanwhile Nobel prize laureate Eugen Wigner who nominated Jordan as a candidate. But at that time the Nobel committee was already considering candidates associated with the second phase of QFT which started after the war with perturbative renormalization theory of quantum electrodynamics. A Nobel prize for the discovery of QFT at such a late date would also have upset the chronological order. Jordan did however receive (besides several other honors) the Max-Planck-medal of the German Physical Society in 1942. Nevertheless the situation begs the question why the protagonist of the quantization of wave fields remained its “unsung hero” [59] while his contemporaries and co-worker became Nobel-laureats. Part of the reason is certainly his speech problem which prevented him to eloquently present his ideas in talks and discussions. But the fact why his former collaborators did not support Jordan for a Nobel prize or (in case of Wigner) supported him very reluctantly cannot be explained entirely in terms of his speech handicap only. Although Jordan was less useful to the Nazis than Heisenberg, the fact that he joined an open anti-Semitic party at a time when there was no reason to secure his career for such a step is deeply disturbing and must have been a great disappointment to them. He never succeeded to renew the old ties and the high scientific esteem he enjoyed during his years in Goettingen and Copenhagen. It seems that starting from his years in Goettingen he was driven by a weird ideological idea which consisted in seeing a link between the post world war I “New Order” of fascist regimes and the unusual counter-intuitive (Copenhagen) interpretation of quantum theory and its revolutionary new conquest of reality [57][58]. Among all reasons for supporting the NS regime Jordan’s idea of a parallelism between the revolution in quantum physics and the new political order is certainly the most bizarre. The fact that Jordan did not fit the picture of an opportunistic Nazi follower but rather came closer to that of a deeply misguided intellectual who despite his overtures to the NS leadership created suspicion and was unable to advance his career²³. Probably the only one who tried to understand this bizarre aspect of Jordan’s personality was Pauli.

Although Jordan took, along with a majority of German physicists, a strong position against those supporting the racist “German Physics”²⁴ and in this way contributed to its demise, he defended on the other hand bellicose and nationalistic positions and he certainly supported Hitler’s war of aggression against the “Bolshevik peril”. The fact that he was a traditional religious person and that several of the leading bishops in the protestant church were pro Hitler had evidently a stronger effect on him than his friendship with his Jewish colleagues from the times in Goettingen, Hamburg and Copenhagen, who by that time had mostly joined the exodus of writers, scientist and artists to the Americas.

²³Obviously outside of the third Reich somebody with the scientific achievement of Jordan would not have to spend his most important years at a small university as Rostok.

²⁴It was Jordan’s opinion that nationalistic and racist views had no place in science; in his own bellicose style of ridicule (in this case especially directed against nationalistic and racist stance of the mathematician Biberbach): “The differences among German and French mathematics are not any more essential than the differences between German and French machine guns”.

In contrast to Pauli who contributed to the second post war phase of QFT and always followed the flow of ideas in QFT up to his premature death, Jordan's active participation in QFT stopped around the middle of the 30s and it seems that he did not follow the development in those areas which he helped to create which culminated shortly after the war in the first comprehensive quantum field theory of matter interacting with light i.e. the renormalized treatment of quantum electrodynamics. Beginning in the mid 30's and through the war he turned his attention to more mathematical and conceptual interdisciplinary problems touching upon biology [60] and psychology. His enduring interest in psychology was presumably related to the psychological origins of his stuttering handicap²⁵ which prevented him from using his elegant writing style in discussions with his colleagues and communications with a wider audience; this perhaps explains in part why even in the physics community his contributions are not as well known as they deserve to be. In fact this handicap even threatened his Habilitation (which was a necessary step for an academic career) in Göttingen. Jordan was informed by Franck (with whom he had coauthored a book) that Niels Bohr had arranged a small amount of money for Jordan which was to be used for getting some cure of his speech problem. Wilhelm Lenz, whose assistant Jordan was for a short time after Pauli left, suggested to visit the famous psychologist Adler. Jordan went to but we only know that he attended a lecture of Schrödinger and criticized his wave mechanics from the Göttingen point of view, there is no record of meetings with Adler.

His increasing withdrawal from the mainstream of quantum field theory and particle physics in the 30s may have partially been the result of his frustration that his influence on the NS regime was not what he had expected. After the defeat of Germany in 1945 his attempts to account for his membership in the Nazi party as well as the difficult task to make a living with the weight of his past NS sympathies (which cost him his position as a university professor for the first two years after the war) seriously impeded his scientific activities.

Unlike the majority of the German population, for which the early Allied re-education effort (which was abandoned after a few years) to rid society of aggressive militaristic and racist ideas was a huge success so that the subsequent change of US policy in favor of re-armament of West Germany ran into serious opposition during the Adenauer period, Jordan did not completely abandon his militaristic and rightwing outlook. In the 50s he joined the CDU party where he had to undergo the least amount of change²⁶, thus forgetting Pauli's admonitions in favor of political abstinence.

All the protagonists of those pioneering days of quantum physics have been commemorated in centennials except Pascual Jordan who, as the result of the history we have described, apparently remained a somewhat "sticky" problem despite Pauli's intercession by stating "it would be incorrect for West Germany to ignore a person like P. Jordan". His postwar scientific activities consisted mainly in creating and arranging material support (by grants from Academies and Industry as well as from the US Airforce) for a very successful group of highly motivated and talented young researchers in the area of General Relativity who became internationally known and attracted famous visitors es-

²⁵One also should keep in mind that the interest in psychology became a "fashion" among the Copenhagen physicists (notably Bohr and Pauli).

²⁶The leadership of the CDU recently supported Bush's war in Iraq (against the majority of its voters).

pecially from Peter Bergmann's group (Rainer Sachs,...). In this indirect way there is a connection between Jordan's post war activities in general relativity and the new Albert Einstein Institute in Golm (Potsdam). This somewhat meandering path leads from Jordan's Seminar in Hamburg through universities in Texas (where most of its members got positions), and then via the astrophysics in Garching (where Ehlers took up a position in 1971) to the AEI for Gravitational Physics of the MPI where Ehlers became the founding director in 1995.

Jordan died in 1980 while working on his post-war pet theory of gravitation with a time-dependent gravitational coupling; his post war work never reached the level of the papers from those glorious years 1925-1930 or his subsequent rather deep pre-war mathematical physics contributions. In the words of Silvan Schweber in his history of quantum electrodynamics, Jordan became the "unsung hero" of a glorious epoch of physics which led to the demise of one of its main architects.

It is however fair to note that with the exception of Max Born Jordan's other collaborators, especially von Neumann and Wigner, had a certain sympathy for an anti-communism position; Wigner later became an ardent defender of the Vietnam war. Since both of them came from a cosmopolitan Jewish family background, their anti-communist fervor probably had its roots in their experience with the radical post World War I Bela Kuhn regime in the Hungarian part of the decaying Habsburg empire. In fact Wigner was so disgusted by the communist takeover led by Bela Kuhn that he and his family converted to the Lutheran religion because Kuhn was Jewish.

The cultural and scientific achievements in a destroyed and humiliated Germany of the post World War I Weimar republic within a short period of 15 years belong to the more impressive parts of mankind's evolution and Jordan, despite his nationalistic political viewpoints is nevertheless part of that heritage.

There remains one unanswered question how could such a giant of 20th particle physics miss the Nobel prize? The answer lies clearly in his curriculum vitae. Einstein, Born and Wigner, truly no sympathisers of Jordans virulent political ideas saw the problem that this omission would cause on the reputation of the prize as such, but their effort came to late.

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