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Chern-Simons AdS_5 supergravity in a Randall-Sundrum background

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Abstract

Chern-Simons AdS supergravity theories are gauge theories for the super-AdS group. These theories possess a fermionic symmetry which differs from standard supersymmetry. In this paper, we study five-dimensional Chern-Simons AdS supergravity in a Randall-Sundrum scenario with two Minkowski 3-branes. After making modifications to the $D = 5$ Chern-Simons AdS supergravity action and fermionic symmetry transformations, we obtain a $\mathbb{Z}_2$-invariant total action $S = \tilde{S}_{bulk} + S_{brane}$ and fermionic transformations $\tilde{\delta}_\epsilon$. While $\tilde{\delta}_\epsilon \tilde{S}_{bulk} = 0$, the fermionic symmetry is broken by S_{brane}. Our total action reduces to the original Randall-Sundrum model when $\tilde{S}_{bulk}$ is restricted to its gravitational sector. We solve the Killing spinor equations for a bosonic configuration with vanishing $su(N)$ and $u(1)$ gauge fields.

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1 Introduction

Chern-Simons AdS supergravity [1, 2, 3] theories can be constructed only in odd spacetime dimensions. As the name implies, they are gauge theories for supersymmetric extensions of the AdS group.[1] They have a fiber bundle structure and hence are potentially renormalizable [2]. The dynamical fields form a single adS superalgebra-valued connection and hence the supersymmetry algebra closes automatically *off-shell* without requiring auxiliary fields [4]. The Lagrangian in dimension $D = 2n-1$ is a Chern-Simons $(2n-1)$-form for the super-adS connection and is a polynomial of order n in the corresponding curvature. Unlike standard supergravity theories, there can be a mismatch between the number of bosonic and fermionic degrees of freedom.[2] For this reason, the 'supersymmetry' of Chern-Simons AdS supergravity theories is perhaps better referred to as a fermionic symmetry.

$D = 11$, $N = 1$ *Chern-Simons AdS supergravity* may correspond to an off-shell supergravity limit of M-theory [2, 3]. It has expected features of M-theory which are not shared by $D=11$ *Cremmer-Julia-Scherk (CJS) supergravity* [5]. These features include an $osp(32|1)$ superalgebra [6] and higher powers of curvature [7]. *Hořava-Witten theory* [8] is obtained from CJS supergravity by compactifying on an $S^1/\mathbb{Z}_2$ orbifold and requiring gauge and gravitational anomalies to cancel. This theory gives the low energy, strongly coupled limit of the heterotic $E_8 \times E_8$ string theory. In light of the above discussion, it would be interesting to reformulate Hořava-Witten theory with $D=11$, $N=1$ Chern-Simons AdS supergravity.

Reformulating Hořava-Witten theory as described above may prove to be difficult. It is simpler to compactify the five-dimensional version of Chern-Simons AdS supergravity on an $S^1/\mathbb{Z}_2$ orbifold and ignore anomaly cancellation issues. Canonical sectors of $D = 5$ Chern-Simons AdS supergravity have been investigated in locally AdS_5 backgrounds possessing a spatial boundary with topology $S^1 \times S^1 \times S^1$ located at infinity [9]. In this paper, as a preamble to reformulating Hořava-Witten theory, we will study $D = 5$ Chern-Simons AdS supergravity in a Randall-Sundrum background with two Minkowski 3-branes [10]. We choose coordinates $x^\mu = (x^{\bar\mu}, x^5)$ to parameterize the five-dimensional spacetime manifold.[3] In terms of these coordinates, the background metric takes the form

$$g_{\mu\nu}dx^\mu dx^\nu = \mathfrak{a}^2(x^5)\eta^{(4)}_{\bar\mu\bar\nu}dx^{\bar\mu}dx^{\bar\nu} + (dx^5)^2, \tag{1.1}$$

where $\eta^{(4)}_{\bar\mu\bar\nu} = \mathrm{diag}(-1,1,1,1)_{\bar\mu\bar\nu}$, $\mathfrak{a}(x^5) \equiv \exp(-|x^5|/\ell)$ is the *warp factor*, and ℓ is the AdS_5 curvature radius. The coordinate x^5 parameterizes an $S^1/\mathbb{Z}_2$ orbifold, where the circle S^1 has radius ρ and $\mathbb{Z}_2$ acts as $x^5 \to -x^5$. We choose the range $-\pi\rho \le x^5 \le \pi\rho$ with the endpoints identified as $x^5 \simeq x^5 + 2\pi\rho$. The Minkowski 3-branes are located at

[1] The AdS group in dimension $D \ge 2$ is $SO(D-1,2)$. The corresponding super-AdS groups are given in [3]. For $D=5$ and $D=11$, the super-AdS groups are respectively $SU(2,2|N)$ and $OSp(32|N)$.

[2] For example, in $D=5$ *Chern-Simons AdS supergravity* [1], the number of bosonic degrees of freedom (N^2+15) is equal to the number of fermionic degrees of freedom $(8N)$ only for $N=3$ and $N=5$.

[3] We use indices $\mu,\nu,\ldots = 0,1,2,3,5$ for local spacetime and $a,b,\ldots = \dot 0,\dot 1,\dot 2,\dot 3,\dot 5$ for tangent spacetime. The corresponding metrics, $g_{\mu\nu}$ and $\eta_{ab} = \mathrm{diag}(-1,1,1,1,1)_{ab}$, are related by $g_{\mu\nu} = e_\mu{}^a e_\nu{}^b \eta_{ab}$, where $e_\mu{}^a$ is the fünfbein. Barred indices $\bar\mu,\bar\nu,\ldots = 0,1,2,3$, and $\bar a,\bar b,\ldots = \dot 0,\dot 1,\dot 2,\dot 3$ denote the four-dimensional counterparts of $\mu,\nu,\ldots$ and $a,b,\ldots$, respectively.

the $\mathbb{Z}_2$ fixed points $x^5 = 0$ and $x^5 = \pi\rho$. These 3-branes have corresponding tensions $\mathcal{T}^{(0)}$ and $\mathcal{T}^{(\pi\rho)}$ and may support $(3+1)$-dimensional field theories.

This paper is organized as follows: In Section 2, we construct a $\mathbb{Z}_2$-invariant bulk theory. This bulk theory is obtained by making modifications to the $D=5$ Chern-Simons AdS supergravity action and fermionic symmetry transformations which allow consistent orbifold conditions to be imposed. The variation of the resulting bulk action S_{bulk} under the resulting fermionic transformations δ_ϵ vanishes everywhere except at the $\mathbb{Z}_2$ fixed points. We calculate $\delta_\epsilon S_{bulk}$ in Section 3. In Section 4, we modify S_{bulk} and δ_ϵ to obtain a modified $\mathbb{Z}_2$-invariant bulk theory. The modified bulk action $\tilde{S}_{bulk}$ is invariant under the modified fermionic transformations $\tilde{\delta}_\epsilon$. In Section 5, we complete our model by adding the brane action S_{brane}. We show in Section 6 that our total action

$$S = \tilde{S}_{bulk} + S_{brane} \tag{1.2}$$

reduces to the original Randall-Sundrum model [10] when $\tilde{S}_{bulk}$ is restricted to its gravitational sector. In Section 7, we solve the Killing spinor equations for a purely bosonic configuration with vanishing $su(N)$ and $u(1)$ gauge fields. Our concluding remarks are given in Section 8. Finally, in the Appendix, we work out the fünfbein, spin connection, curvature 2-form components, Ricci tensor, and Ricci scalar for our metric (1.1).

2 $\mathbb{Z}_2$-invariant bulk theory

In this section, we construct a $\mathbb{Z}_2$-invariant bulk theory. The bulk theory is obtained by making modifications to the $D = 5$ Chern-Simons AdS supergravity [1] action and fermionic symmetry transformations which allow consistent orbifold conditions to be imposed.

The field content of $D=5$ Chern-Simons AdS supergravity is the fünfbein $e_\mu{}^a$, the spin connection $\omega_\mu{}^{ab}$, the $su(N)$ gauge connection $A_\mu = A^i_\mu \tau_i$, the $u(1)$ gauge connection B_μ, and N complex gravitini $\psi_{\mu r}$ which transform as Dirac spinors in a vector representation of $su(N)$.[4] These fields form a connection for the adS superalgebra $su(2,2|N)$. The action and fermionic symmetry transformations are given in [9] in terms of the AdS_5 curvature radius ℓ. The only free parameter in the action is a dimensionless constant k. To allow consistent $\mathbb{Z}_2$ orbifold conditions to be imposed, we make the following modifications:

1. Rescale the $su(N)$ and $u(1)$ gauge connections:

$$A \to g_A A, \quad B \to g_B B.$$

[4]We use indices $i, j, \ldots = 1, \ldots, N^2-1$ to label the $N \times N$-dimensional $su(N)$ generators τ_i. The indices $r, s, \ldots = 1, \ldots, N$ label vector representations of $su(N)$. We will use the notation $A^r_s \equiv A^i (\tau_i)^r_s$. Spinor indices $\alpha, \beta, \ldots$ will sometimes be suppressed.

2. Replace g_A, g_B, ℓ^{-1}, and k by the $\mathbb{Z}_2$-odd expressions[5]

$$G_A \equiv g_A \,\mathrm{sgn}(x^5), \quad G_B \equiv g_B \,\mathrm{sgn}(x^5), \quad L^{-1} \equiv \ell^{-1} \,\mathrm{sgn}(x^5),$$
$$K \equiv k \,\mathrm{sgn}(x^5).$$

In this manner, we obtain the bulk action

$$S_{bulk} = S_{grav} + S_{su(N)} + S_{u(1)} + S_{ferm}, \tag{2.1}$$

where

$$\begin{aligned}
S_{grav} &= \int \tfrac{1}{8} K \varepsilon_{abcde} \left(\tfrac{1}{L} R^{ab} R^{cd} e^e + \tfrac{2}{3L^3} R^{ab} e^c e^d e^e + \tfrac{1}{5L^5} e^a e^b e^c e^d e^e \right), \\
S_{su(N)} &= \int iK \;\mathrm{str}\left(G_A^3 A F^2 - \tfrac{1}{2} G_A^4 A^3 F + \tfrac{1}{10} G_A^5 A^5 \right), \\
S_{u(1)} &= \int K \Big[-\left(\tfrac{1}{4^2} - \tfrac{1}{N^2} \right) G_B^3 B (dB)^2 + \tfrac{3}{4L^2} \big(T^a T_a - \tfrac{L^2}{2} R^{ab} R_{ab} \\
&\qquad - R^{ab} e_a e_b \big) G_B B - \tfrac{3}{N} G_A^2 G_B F^r_s F^s_r B \Big], \\
S_{ferm} &= \int \tfrac{3}{2i} K \left(\bar{\psi}^r_\alpha \mathcal{R}^\alpha_\beta \nabla \psi^\beta_r + \bar{\psi}^s_\alpha \mathcal{F}^r_s \nabla \psi^\alpha_r \right) + c.c.,
\end{aligned} \tag{2.2}$$

and the transformations

$$\begin{aligned}
\delta_\epsilon e^a &= -\tfrac{1}{2} (\bar{\psi}^r \Gamma^a \epsilon_r - \bar{\epsilon}^r \Gamma^a \psi_r), & \delta_\epsilon \omega^{ab} &= \tfrac{1}{4} (\bar{\psi}^r \Gamma^{ab} \epsilon_r - \bar{\epsilon}^r \Gamma^{ab} \psi_r), \\
\delta_\epsilon \psi_r &= -\nabla \epsilon_r, & \delta_\epsilon \bar{\psi}^r &= -\nabla \bar{\epsilon}^r, \\
\delta_\epsilon A^r_s &= i \left(\bar{\psi}^r \epsilon_s - \bar{\epsilon}^r \psi_s \right), & \delta_\epsilon B &= i \left(\bar{\psi}^r \epsilon_r - \bar{\epsilon}^r \psi_r \right).
\end{aligned} \tag{2.3}$$

In these expressions, Γ^a are the Dirac matrices[6], $\Gamma^{ab} \equiv \frac{1}{2}\left(\Gamma^a \Gamma^b - \Gamma^b \Gamma^a \right)$, $R^{ab} = d\omega^{ab} + \omega^{ac} \omega_c{}^b$ is the curvature 2-form, $T^a = de^a + \omega^a{}_b e^b$ is the torsion 2-form, $F = dA + G_A A^2 = F^i \tau_i$ is the $su(N)$ curvature,

$$\begin{aligned}
\mathcal{R}^\alpha_\beta &\equiv \tfrac{1}{2L} T^a (\Gamma_a)^\alpha_\beta + \tfrac{1}{4} (R^{ab} + \tfrac{1}{L^2} e^a e^b) (\Gamma_{ab})^\alpha_\beta + \tfrac{i}{4} G_B \, dB \, \delta^\alpha_\beta - \tfrac{1}{2} \psi^\alpha_s \bar{\psi}^s_\beta, \\
\mathcal{F}^r_s &\equiv F^r_s + \tfrac{i}{N} G_B \, dB \, \delta^r_s - \tfrac{1}{2} \bar{\psi}^r_\beta \psi^\beta_s,
\end{aligned} \tag{2.4}$$

str is a symmetrized trace satisfying $\mathrm{str}(\tau_i \tau_j \tau_k) \equiv \frac{1}{2i} \mathrm{tr}\left(\{\tau_i, \tau_j\} \tau_k \right)$, ∇ is the $adS_5 \times su(N) \times u(1)$ covariant derivative, and

$$\begin{aligned}
\nabla \psi_r &\equiv \left(d + \tfrac{1}{4} \omega^{ab} \Gamma_{ab} + \tfrac{1}{2L} e^a \Gamma_a \right) \psi_r - G_A A^s_r \psi_s + i \left(\tfrac{1}{4} - \tfrac{1}{N} \right) G_B B \psi_r, \\
\nabla \epsilon_r &\equiv \left(d + \tfrac{1}{4} \omega^{ab} \Gamma_{ab} + \tfrac{1}{2L} e^a \Gamma_a \right) \epsilon_r - G_A A^s_r \epsilon_s + i \left(\tfrac{1}{4} - \tfrac{1}{N} \right) G_B B \epsilon_r.
\end{aligned} \tag{2.5}$$

Note that the results in the Appendix can be used to show that the torsion vanishes for our metric.

We impose the following orbifold conditions:

[5]The *signum function* $\mathrm{sgn}(x^5)$ is $+1$ for $0 < x^5 < \pi\rho$ and -1 for $-\pi\rho < x^5 < 0$. It obeys $\mathrm{sgn}^2(x^5) = 1$ and $\partial_5 \,\mathrm{sgn}(x^5) = 2[\delta(x^5) - \delta(x^5 - \pi\rho)]$.

[6]We choose a chiral basis for the 4×4 Dirac matrices

$$\Gamma^a = \left(\Gamma^{\bar{a}}, \Gamma^{\dot{5}} \right) = \left(\begin{bmatrix} \mathbf{0} & -i\sigma^{\bar{a}} \\ -i\bar{\sigma}^{\bar{a}} & \mathbf{0} \end{bmatrix}, \begin{bmatrix} \mathbf{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \right),$$

where $\sigma^{\bar{a}} = (\mathbf{1}, \vec{\sigma})$ and $\bar{\sigma}^{\bar{a}} = (\mathbf{1}, -\vec{\sigma})$. These matrices satisfy $\mathrm{tr}\left(\Gamma_a \Gamma_b \Gamma_c \Gamma_d \Gamma_e \right) = -4i \varepsilon_{abcde}$, where ε_{abcde} is the Levi-Civita tensor and $\varepsilon^{\dot{0}\dot{1}\dot{2}\dot{3}\dot{5}} = 1$.

1. *Periodicity on S^1.* The fields and the fermionic parameters ϵ_r, denoted generically by ϕ, are required to be periodic on the circle S^1. That is,

$$\phi(x^{\bar{\mu}}, x^5) = \phi(x^{\bar{\mu}}, x^5 + 2\pi\rho). \tag{2.6}$$

2. $\mathbb{Z}_2$ *parity assignments.* The bosonic field components

$$\Phi = e_\mu{}^{\bar{a}}, e_5{}^{\dot{5}}, A_5^i, B_5, \quad \Theta = e_{\bar{\mu}}{}^{\dot{5}}, e_5{}^{\bar{a}}, A_{\bar{\mu}}^i, B_{\bar{\mu}}$$

are chosen to satisfy

$$\Phi(x^\mu, x^5) = +\Phi(x^\mu, -x^5), \quad \Theta(x^\mu, x^5) = -\Theta(x^\mu, -x^5). \tag{2.7}$$

That is, the Φ components are $\mathbb{Z}_2$-even and the Θ components are $\mathbb{Z}_2$-odd. For the gravitini, we require

$$\begin{aligned} \Gamma^{\dot{5}}\, \psi_{\bar{\mu}r}(x^{\bar{\mu}}, x^5) &= +\, \psi_{\bar{\mu}r}(x^{\bar{\mu}}, -x^5), \\ \Gamma^{\dot{5}}\, \psi_{5r}(x^{\bar{\mu}}, x^5) &= -\, \psi_{5r}(x^{\bar{\mu}}, -x^5). \end{aligned} \tag{2.8}$$

Finally, the fermionic parameters ϵ_r are required to satisfy

$$\Gamma^{\dot{5}}\, \epsilon_r(x^{\bar{\mu}}, x^5) = +\, \epsilon_r(x^\mu, -x^5). \tag{2.9}$$

These conditions imply that the $\mathbb{Z}_2$-odd quantities vanish at the orbifold fixed points. It is straightforward to check that S_{bulk} is $\mathbb{Z}_2$-even and that the transformations (2.3) are consistent with the $\mathbb{Z}_2$ parity assignments.

3 Calculation of $\delta_\epsilon S_{bulk}$

The $D = 5$ Chern-Simons AdS supergravity action is invariant (up to a boundary term) under its fermionic symmetry transformations. In Section 2, we modified this action and its fermionic transformations to obtain a $\mathbb{Z}_2$-invariant bulk theory. Due to the signum functions introduced by the modifications, $\delta_\epsilon S_{bulk}$ contains terms which have no counterpart in the unmodified theory. More specifically, the extra terms arise from ∂_5 acting on the signum functions to yield delta functions. Such 'delta function' contributions to $\delta_\epsilon S_{bulk}$ can potentially spoil the fermionic symmetry only at the $\mathbb{Z}_2$ fixed points. Thus, S_{bulk} is invariant under its fermionic transformations everywhere except perhaps at the $\mathbb{Z}_2$ fixed points. In this section, we will calculate $\delta_\epsilon S_{bulk}$.

For our metric and $\mathbb{Z}_2$ parity assignments, the uncancelled variation $\delta_\epsilon S_{bulk}$ arises from the variation of the 4-Fermi terms. The 4-Fermi terms are

$$
\begin{aligned}
S_{\psi^4} &= \tfrac{3i}{4}\int K\left(\bar\psi^r_\alpha\psi^\alpha_s\bar\psi^s_\beta\nabla\psi^\beta_r + \bar\psi^s_\alpha\bar\psi^r_\beta\psi^\beta_s\nabla\psi^\alpha_r\right) + c.c.\\
&= \tfrac{3i}{2}\int K\bar\psi^r_\alpha\psi^\alpha_s\bar\psi^s_\beta\nabla\psi^\beta_r + c.c.\\
&= \tfrac{3i}{2}\int K\left(\bar\psi^r\psi_s\right)\left(\bar\psi^s\nabla\psi_r\right) + c.c.\\
&= \tfrac{3i}{2}\tfrac{1}{5!}\int d^5x\,\varepsilon^{\mu\nu\rho\sigma\lambda}\,K\left(\bar\psi^r_\mu\psi_{\nu s}\right)\left(\bar\psi^s_\rho\nabla_\sigma\psi_{\lambda r}\right) + c.c.\\
&= \tfrac{3i}{2}\tfrac{1}{4!}\int d^5x\,K\Big[\varepsilon^{5\bar\nu\bar\rho\bar\sigma\bar\lambda}\left(\bar\psi^r_5\psi_{\bar\nu s}\right)\left(\bar\psi^s_{\bar\rho}\nabla_{\bar\sigma}\psi_{\bar\lambda r}\right) + \varepsilon^{\bar\mu 5\bar\rho\bar\sigma\bar\lambda}\left(\bar\psi^r_{\bar\mu}\psi_{5s}\right)\left(\bar\psi^s_{\bar\rho}\nabla_{\bar\sigma}\psi_{\bar\lambda r}\right)\\
&\quad+\varepsilon^{\bar\mu\bar\nu 5\bar\sigma\bar\lambda}\left(\bar\psi^r_{\bar\mu}\psi_{\bar\nu s}\right)\left(\bar\psi^s_5\nabla_{\bar\sigma}\psi_{\bar\lambda r}\right) + \varepsilon^{\bar\mu\bar\nu\bar\rho 5\bar\lambda}\left(\bar\psi^r_{\bar\mu}\psi_{\bar\nu s}\right)\left(\bar\psi^s_{\bar\rho}\nabla_5\psi_{\bar\lambda r}\right)\\
&\quad+\varepsilon^{\bar\mu\bar\nu\bar\rho\bar\sigma 5}\left(\bar\psi^r_{\bar\mu}\psi_{\bar\nu s}\right)\left(\bar\psi^s_{\bar\rho}\nabla_{\bar\sigma}\psi_{5r}\right)\Big] + c.c.
\end{aligned}
\tag{3.1}
$$

Let us now compute $\delta_\epsilon S_{bulk}$ by applying δ_ϵ to (3.1) and dropping all terms which contribute no delta functions. For our metric and $\mathbb{Z}_2$ parity assignments, we can drop all but

1. The ∂_μ part of ∇_μ.
2. The $-\partial_\mu\epsilon_r$ part of $\delta_\epsilon = -\nabla_\mu\epsilon_r$.

The only contributing terms are thus contained in the expression

$$
\begin{aligned}
Q \equiv -\tfrac{3i}{2}\tfrac{1}{4!}\int d^5x\,K\Big\{&\varepsilon^{5\bar\nu\bar\rho\bar\sigma\bar\lambda}\left(\partial_5\bar\epsilon^r\psi_{\bar\nu s}\right)\left(\bar\psi^s_{\bar\rho}\partial_{\bar\sigma}\psi_{\bar\lambda r}\right)\\
&+\varepsilon^{\bar\mu 5\bar\rho\bar\sigma\bar\lambda}\left(\bar\psi^r_{\bar\mu}\partial_5\epsilon_s\right)\left(\bar\psi^s_{\bar\rho}\partial_{\bar\sigma}\psi_{\bar\lambda r}\right) + \varepsilon^{\bar\mu\bar\nu 5\bar\sigma\bar\lambda}\left(\bar\psi^r_{\bar\mu}\psi_{\bar\nu s}\right)\left(\partial_5\bar\epsilon^s\partial_{\bar\sigma}\psi_{\bar\lambda r}\right)\\
&+\varepsilon^{\bar\mu\bar\nu\bar\rho 5\bar\lambda}\Big[\left(\partial_{\bar\mu}\bar\epsilon^r\psi_{\bar\nu s}\right)\left(\bar\psi^s_{\bar\rho}\partial_5\psi_{\bar\lambda r}\right) + \left(\bar\psi^r_{\bar\mu}\partial_{\bar\nu}\epsilon_s\right)\left(\bar\psi^s_{\bar\rho}\partial_5\psi_{\bar\lambda r}\right)\\
&\qquad+\left(\bar\psi^r_{\bar\mu}\psi_{\bar\nu s}\right)\left(\partial_{\bar\rho}\bar\epsilon^s\partial_5\psi_{\bar\lambda r}\right) + \left(\bar\psi^r_{\bar\mu}\psi_{\bar\nu s}\right)\left(\bar\psi^s_{\bar\rho}\partial_5\partial_{\bar\lambda}\epsilon_r\right)\Big]\\
&+\varepsilon^{\bar\mu\bar\nu\bar\rho\bar\sigma 5}\left(\bar\psi^r_{\bar\mu}\psi_{\bar\nu s}\right)\left(\bar\psi^s_{\bar\rho}\partial_{\bar\sigma}\partial_5\epsilon_r\right)\Big\} + c.c.
\end{aligned}
\tag{3.2}
$$

More specifically, the delta function terms contained in Q are obtained by integrating by parts and keeping only the terms in which ∂_5 acts on K. Thus,

$$
\begin{aligned}
\delta_\epsilon S_{bulk} = \tfrac{3i}{2}\tfrac{1}{4!}\int d^5x\,\partial_5 K\Big\{&\varepsilon^{5\bar\nu\bar\rho\bar\sigma\bar\lambda}\left(\bar\epsilon^r\psi_{\bar\nu s}\right)\left(\bar\psi^s_{\bar\rho}\partial_{\bar\sigma}\psi_{\bar\lambda r}\right)\\
&+\varepsilon^{\bar\mu 5\bar\rho\bar\sigma\bar\lambda}\left(\bar\psi^r_{\bar\mu}\epsilon_s\right)\left(\bar\psi^s_{\bar\rho}\partial_{\bar\sigma}\psi_{\bar\lambda r}\right) + \varepsilon^{\bar\mu\bar\nu 5\bar\sigma\bar\lambda}\left(\bar\psi^r_{\bar\mu}\psi_{\bar\nu s}\right)\left(\bar\epsilon^s\partial_{\bar\sigma}\psi_{\bar\lambda r}\right)\\
&+\varepsilon^{\bar\mu\bar\nu\bar\rho 5\bar\lambda}\Big[\left(\partial_{\bar\mu}\bar\epsilon^r\psi_{\bar\nu s}\right)\left(\bar\psi^s_{\bar\rho}\psi_{\bar\lambda r}\right) + \left(\bar\psi^r_{\bar\mu}\partial_{\bar\nu}\epsilon_s\right)\left(\bar\psi^s_{\bar\rho}\psi_{\bar\lambda r}\right)\\
&\qquad+\left(\bar\psi^r_{\bar\mu}\psi_{\bar\nu s}\right)\left(\partial_{\bar\rho}\bar\epsilon^s\psi_{\bar\lambda r}\right) + \left(\bar\psi^r_{\bar\mu}\psi_{\bar\nu s}\right)\left(\bar\psi^s_{\bar\rho}\partial_{\bar\lambda}\epsilon_r\right)\Big]\\
&+\varepsilon^{\bar\mu\bar\nu\bar\rho\bar\sigma 5}\left(\bar\psi^r_{\bar\mu}\psi_{\bar\nu s}\right)\left(\bar\psi^s_{\bar\rho}\partial_{\bar\sigma}\epsilon_r\right)\Big\} + c.c.,
\end{aligned}
\tag{3.3}
$$

where

$$
\partial_5 K = 2k\left[\delta(x^5) - \delta(x^5-\pi\rho)\right]. \tag{3.4}
$$

4 Modified $\mathbb{Z}_2$-invariant bulk theory

The result (3.3) for $\delta_\epsilon S_{bulk}$ demonstrates that S_{bulk} is not invariant under the fermionic transformations δ_ϵ. In this section, we will modify S_{bulk} and δ_ϵ by replacing the $adS_5 \times su(N) \times u(1)$ covariant derivative ∇ with $\widetilde{\nabla}$, where

$$\begin{aligned} \widetilde{\nabla}_\sigma \psi_{\lambda r} &\equiv \nabla_\sigma \psi_{\lambda r} + 2\delta^5_\sigma \delta^{\bar{\lambda}}_\lambda \left[\delta(x^5) - \delta(x^5 - \pi\rho)\right] \mathrm{sgn}(x^5) \Gamma_{\dot{5}} \psi_{\bar{\lambda} r}, \\ \widetilde{\nabla}_\sigma \epsilon_r &\equiv \nabla_\sigma \epsilon_r + 2\delta^5_\sigma \left[\delta(x^5) - \delta(x^5 - \pi\rho)\right] \mathrm{sgn}(x^5) \Gamma_{\dot{5}} \epsilon_r. \end{aligned} \tag{4.1}$$

We will show that the modified bulk action

$$\tilde{S}_{bulk} \equiv S_{bulk}\big(\nabla \to \widetilde{\nabla}\big) \equiv S_{bulk} + \Delta S_{bulk} \tag{4.2}$$

is invariant under the modified transformations

$$\tilde{\delta}_\epsilon \equiv \delta_\epsilon\big(\nabla \to \widetilde{\nabla}\big) \equiv \delta_\epsilon + \Delta\delta_\epsilon. \tag{4.3}$$

That is, we will show that

$$\tilde{\delta}_\epsilon \tilde{S}_{bulk} = \delta_\epsilon S_{bulk} + (\Delta\delta_\epsilon)\, S_{bulk} + \tilde{\delta}_\epsilon \left(\Delta S_{bulk}\right) \tag{4.4}$$

vanishes. It is straightforward to check that $\tilde{S}_{bulk}$ is $\mathbb{Z}_2$-invariant and the transformations $\tilde{\delta}_\epsilon$ are consistent with our $\mathbb{Z}_2$ parity assignments.

We begin by computing $(\Delta\delta_\epsilon)\, S_{bulk}$. For our metric and $\mathbb{Z}_2$ parity assignments, the only part of S_{bulk} which is not invariant under $\Delta\delta_\epsilon$ is S_{ψ^4} (given by (3.1)). Note that

$$(\Delta\delta_\epsilon)\, \psi_{\lambda r} = -2\delta^5_\lambda \left[\delta(x^5) - \delta(x^5 - \pi\rho)\right] \mathrm{sgn}(x^5) \Gamma_{\dot{5}} \epsilon_r. \tag{4.5}$$

Thus, after using $K\,\mathrm{sgn}(x^5) = k$, (2.9), and (3.4), we obtain

$$\begin{aligned} (\Delta\delta_\epsilon)\, S_{bulk} = -\tfrac{3i}{2}\tfrac{1}{4!} \int d^5x\, \partial_5 K \Big[&\varepsilon^{5\bar{\nu}\bar{\rho}\bar{\sigma}\bar{\lambda}} \left(\bar{\epsilon}^r \psi_{\bar{\nu}s}\right) \left(\bar{\psi}^s_{\bar{\rho}} \partial_{\bar{\sigma}} \psi_{\bar{\lambda}r}\right) \\ &+ \varepsilon^{\bar{\mu}5\bar{\rho}\bar{\sigma}\bar{\lambda}} \left(\bar{\psi}^r_{\bar{\mu}} \epsilon_s\right) \left(\bar{\psi}^s_{\bar{\rho}} \partial_{\bar{\sigma}} \psi_{\bar{\lambda}r}\right) + \varepsilon^{\bar{\mu}\bar{\nu}5\bar{\sigma}\bar{\lambda}} \left(\bar{\psi}^r_{\bar{\mu}} \psi_{\bar{\nu}s}\right) \left(\bar{\epsilon}^s \partial_{\bar{\sigma}} \psi_{\bar{\lambda}r}\right) \\ &+ \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}\bar{\sigma}5} \left(\bar{\psi}^r_{\bar{\mu}} \psi_{\bar{\nu}s}\right) \left(\bar{\psi}^s_{\bar{\rho}} \partial_{\bar{\sigma}} \epsilon_r\right) \Big] + c.c. \end{aligned} \tag{4.6}$$

Now, let us compute $\tilde{\delta}_\epsilon \left(\Delta S_{bulk}\right)$. For our metric and $\mathbb{Z}_2$ parity assignments, the only part of S_{bulk} which is changed by the replacement $\nabla \to \widetilde{\nabla}$ is S_{ψ^4}. After using $K\,\mathrm{sgn}(x^5) = k$, (2.8), and (3.4), we obtain

$$\Delta S_{bulk} = \tfrac{3i}{2}\tfrac{1}{4!} \int d^5x\, \partial_5 K\, \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}5\bar{\lambda}} \left(\bar{\psi}^r_{\bar{\mu}} \psi_{\bar{\nu}s}\right) \left(\bar{\psi}^s_{\bar{\rho}} \psi_{\bar{\lambda}r}\right) + c.c. \tag{4.7}$$

Applying $\tilde{\delta}_\epsilon$ to (4.7) yields

$$\begin{aligned} \tilde{\delta}_\epsilon \left(\Delta S_{bulk}\right) = -\tfrac{3i}{2}\tfrac{1}{4!} \int d^5x\, \partial_5 K\, \varepsilon^{\bar{\mu}\bar{\nu}\bar{\rho}5\bar{\lambda}} \Big[&\left(\partial_{\bar{\mu}} \bar{\epsilon}^r \psi_{\bar{\nu}s}\right) \left(\bar{\psi}^s_{\bar{\rho}} \psi_{\bar{\lambda}r}\right) + \left(\bar{\psi}^r_{\bar{\mu}} \partial_{\bar{\nu}} \epsilon_s\right) \left(\bar{\psi}^s_{\bar{\rho}} \psi_{\bar{\lambda}r}\right) \\ &+ \left(\bar{\psi}^r_{\bar{\mu}} \psi_{\bar{\nu}s}\right) \left(\partial_{\bar{\rho}} \bar{\epsilon}^s \psi_{\bar{\lambda}r}\right) + \left(\bar{\psi}^r_{\bar{\mu}} \psi_{\bar{\nu}s}\right) \left(\bar{\psi}^s_{\bar{\rho}} \partial_{\bar{\lambda}} \epsilon_r\right) \Big] + c.c. \end{aligned} \tag{4.8}$$

Using the results (3.3), (4.6), and (4.8) in (4.4) yields

$$\tilde{\delta}_\epsilon \tilde{S}_{bulk} = 0. \tag{4.9}$$

5 Brane action

To complete our model, we add the brane action S_{brane}. In the absence of particle excitations, the brane action consists of brane tensions. That is,

$$S_{brane} = -\int d^5x\, e^{(4)} \left[\mathcal{T}^{(0)}\delta(x^5) + \mathcal{T}^{(\pi\rho)}\delta(x^5 - \pi\rho)\right] + \text{excitations}, \tag{5.1}$$

where $e^{(4)} \equiv \det(e_{\bar{\mu}}{}^{\bar{a}})$. As discussed in Section 2, $\mathbb{Z}_2$-odd quantities vanish at the $\mathbb{Z}_2$ fixed points. Thus, it is clear that S_{brane} is $\mathbb{Z}_2$-even. Further discussion of 3-brane actions can be found in [11].

6 Connection with original RS model

In this section, we will show that our total action $S = \tilde{S}_{bulk} + S_{brane}$ reduces to the original Randall-Sundrum model [10] when $\tilde{S}_{bulk}$ is restricted to its gravitational sector.

The gravitational sector of $\tilde{S}_{bulk}$ is S_{grav}, given by the first equation of (2.2). S_{grav} consists of three terms:

1. The 'Gauss-Bonnet' term $\int \frac{1}{8} K \varepsilon_{abcde} R^{ab} R^{cd} e^e / L$.
2. The 'Einstein-Hilbert' term $\int \frac{1}{8} \cdot \frac{2}{3} K \varepsilon_{abcde} R^{ab} e^c e^d e^e / L^3$.
3. The 'cosmological constant' term $\int \frac{1}{8} \cdot \frac{1}{5} K \varepsilon_{abcde} e^a e^b e^c e^d e^e / L^5$.

For our metric, the first term can be expressed as a linear combination of the other two. Summing the three terms yields an effective Einstein-Hilbert term and an effective cosmological constant term. To demonstrate this explicitly, let us evaluate S_{grav} for our metric. Using the results in the Appendix, we obtain

$$\begin{aligned} \varepsilon_{abcde} R^{ab} R^{cd} e^e &= d^5x\, e \left(-\frac{120}{\ell^4} + \frac{192}{\ell^3} \left[\delta(x^5) - \delta(x^5 - \pi\rho)\right] \right), \\ \varepsilon_{abcde} R^{ab} e^c e^d e^e &= d^5x\, e\, (-6R)\,, \\ \varepsilon_{abcde} e^a e^b e^c e^d e^e &= d^5x\, e\, (-5!)\,, \end{aligned} \tag{6.1}$$

where $e \equiv \det(e_\mu{}^a)$. Thus,

$$
\begin{aligned}
S_{grav} &= \int d^5x\, e\, \frac{1}{8}\left\{\frac{k}{\ell}\left(-\frac{120}{\ell^4}+\frac{192}{\ell^3}\left[\delta(x^5)-\delta(x^5-\pi\rho)\right]\right)\right. \\
&\qquad \left. +\frac{2k}{3\ell^3}(-6R)+\frac{k}{5\ell^5}(-5!)\right\} \\
&= \int d^5x\, e\, \frac{k}{\ell^3}\left\{-\frac{15}{\ell^2}+\frac{24}{\ell}\left[\delta(x^5)-\delta(x^5-\pi\rho)\right]-\frac{1}{2}R-\frac{3}{\ell^2}\right\} \\
&= \int d^5x\, e\, \frac{k}{\ell^3}\left\{\frac{3}{2}\left(-\frac{20}{\ell^2}+\frac{16}{\ell}\left[\delta(x^5)-\delta(x^5-\pi\rho)\right]\right)-\frac{1}{2}R+\frac{12}{\ell^2}\right\} \\
&= \int d^5x\, e\, \frac{k}{\ell^3}\left(R+\frac{12}{\ell^2}\right) \\
&= \int d^5x\, e\left(2M^3R-\Lambda\right), \qquad (6.2)
\end{aligned}
$$

where M is the five-dimensional gravitational mass scale[7], Λ is the bulk cosmological constant, and

$$M^3=\frac{k}{2\ell^3}, \quad \Lambda=-\frac{24M^3}{\ell^2}. \qquad (6.3)$$

Combining the result (6.2) with (5.1) , we obtain the action of the original Randall-Sundrum model. It is shown in [10] that the five-dimensional vacuum Einstein's equations for this system,

$$R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-\frac{1}{4M^3}\left\{g_{\mu\nu}\Lambda+\frac{e^{(4)}}{e}\delta^{\bar\mu}_\mu\delta^{\bar\nu}_\nu g_{\bar\mu\bar\nu}\left[\mathcal{T}^{(0)}\delta(x^5)+\mathcal{T}^{(\pi\rho)}\delta(x^5-\pi\rho)\right]\right\}, \qquad (6.4)$$

are solved by our metric provided that the relations

$$\mathcal{T}^{(0)}=-\mathcal{T}^{(\pi\rho)}=24M^3/\ell, \quad \Lambda=-24M^3/\ell^2 \qquad (6.5)$$

are satisfied.

7 Killing spinors

In this section, we will solve the Killing spinor equations for a purely bosonic configuration with vanishing $su(N)$ and $u(1)$ gauge fields. In this case, the Killing spinor equations reduce to

$$
\begin{aligned}
0 &= \delta_\epsilon\psi_{\bar\mu r}=-\partial_{\bar\mu}\epsilon_r-\frac{1}{2}\frac{\mathfrak{a}'}{\mathfrak{a}}\Gamma_{\bar\mu}\left(\Gamma_{\dot 5}-1\right)\epsilon_r, \\
0 &= \delta_\epsilon\psi_{5r}=-\partial_5\epsilon_r+\frac{1}{2}\frac{\mathfrak{a}'}{\mathfrak{a}}\Gamma_{\dot 5}\epsilon_r-2\left[\delta(x^5)-\delta(x^5-\pi\rho)\right]\mathrm{sgn}(x^5)\Gamma_{\dot 5}\epsilon_r. \qquad (7.1)
\end{aligned}
$$

[7] M is related to the four-dimensional gravitational mass scale $M_{(4)}=2.43\times10^{18}$ GeV by $M^2_{(4)}=M^3\int_{-\pi\rho}^{+\pi\rho}dx^5\,\mathfrak{a}^2(x^5)=M^3\ell\left[1-\exp(-2\pi\rho/\ell)\right]$. The effective mass scales on the 3-branes at $x^5=0$ and $x^5=\pi\rho$ are respectively $M_{(4)}$ and $M_{(4)}e^{-\pi\rho/\ell}$. If the Standard Model fields live on the 3-brane at $x^5=\pi\rho$, then $M_{(4)}e^{-\pi\rho/\ell}$ can be associated with the electroweak scale.

To solve these equations, split ϵ_r into $\mathbb{Z}_2$-even (ϵ_r^+) and $\mathbb{Z}_2$-odd (ϵ_r^-) pieces:

$$\epsilon_r = \epsilon_r^+ + \epsilon_r^-, \tag{7.2}$$

where

$$\epsilon^\pm \equiv \tfrac{1}{2}\left(\epsilon_r \pm \Gamma_{\dot{5}}\epsilon_r\right) = \pm\Gamma_{\dot{5}}\epsilon_r^\pm. \tag{7.3}$$

We obtain the following system of equations:

$$\begin{aligned}
\partial_{\bar{\mu}}\epsilon_r^+ &= -\left(\mathfrak{a}'/\mathfrak{a}\right)\Gamma_{\bar{\mu}}\Gamma_{\dot{5}}\epsilon_r^-,\\
\partial_{\bar{\mu}}\epsilon_r^- &= 0,\\
\partial_5\epsilon_r^+ &= +\tfrac{1}{2}\left(\mathfrak{a}'/\mathfrak{a}\right)\epsilon_r^+ - 2\left[\delta(x^5) - \delta(x^5 - \pi\rho)\right]\mathrm{sgn}(x^5)\epsilon_r^+,\\
\partial_5\epsilon_r^- &= -\tfrac{1}{2}\left(\mathfrak{a}'/\mathfrak{a}\right)\epsilon_r^- + 2\left[\delta(x^5) - \delta(x^5 - \pi\rho)\right]\mathrm{sgn}(x^5)\epsilon_r^-.
\end{aligned} \tag{7.4}$$

These equations are solved by

$$\begin{aligned}
\epsilon_r^+ &= \mathfrak{a}^{1/2}\left[-\left(\mathfrak{a}'/\mathfrak{a}^2\right)x^{\bar{\mu}}\Gamma_{\bar{\mu}}\Gamma_{\dot{5}}\,\mathrm{sgn}(x^5)\,\chi_r^{-(0)} + \chi_r^{+(0)}\right]\\
&= \mathfrak{a}^{1/2}\left[(1/\ell)\,x^{\bar{\mu}}\delta_{\bar{\mu}}{}^{\bar{a}}\Gamma_{\bar{a}}\Gamma_{\dot{5}}\,\chi_r^{-(0)} + \chi_r^{+(0)}\right],\\
\epsilon_r^- &= \mathfrak{a}^{-1/2}\,\mathrm{sgn}(x^5)\,\chi_r^{-(0)},
\end{aligned} \tag{7.5}$$

where $\chi_r^{+(0)}$ and $\chi_r^{-(0)}$ are constant (projected) spinors.[8] Thus, our solution for the Killing spinors is

$$\epsilon_r = \mathfrak{a}^{1/2}\chi_r^{+(0)} + \mathfrak{a}^{-1/2}\,\mathrm{sgn}(x^5)\left(1 - \frac{\mathfrak{a}'}{\mathfrak{a}}x^{\bar{\mu}}\Gamma_{\bar{\mu}}\Gamma_{\dot{5}}\right)\chi_r^{-(0)}. \tag{7.6}$$

8 Conclusion

We have constructed a Randall-Sundrum scenario from $D=5$ Chern-Simons AdS supergravity. Our total action $S = \tilde{S}_{bulk} + S_{brane}$ is $\mathbb{Z}_2$-invariant. $\tilde{S}_{bulk}$ is invariant under the fermionic transformations $\tilde{\delta}_\epsilon$. However,

$$\tilde{\delta}_\epsilon S_{brane} = -\int d^5x\,\tilde{\delta}_\epsilon e^{(4)}\left[\mathcal{T}^{(0)}\delta(x^5) + \mathcal{T}^{(\pi\rho)}\delta(x^5 - \pi\rho)\right] + \cdots, \tag{8.1}$$

where

$$\tilde{\delta}_\epsilon e^{(4)} = e^{(4)}\left[-\tfrac{1}{2}\left(\bar{\psi}^r_{\bar{\mu}}\Gamma^{\bar{\mu}}\epsilon_r - \bar{\epsilon}^r\Gamma^{\bar{\mu}}\psi_{\bar{\mu}r}\right)\right]. \tag{8.2}$$

Thus, the fermionic symmetry is broken by S_{brane}. Nevertheless, the Killing spinors of Section 7 are globally defined.

Our model reduces to the original Randall-Sundrum model [10] when $\tilde{S}_{bulk}$ is restricted to its gravitational sector. The original Randall-Sundrum model involves the fine-tuning relations

$$\mathcal{T}^{(0)} = -\mathcal{T}^{(\pi\rho)} = 24M^3/\ell, \quad \Lambda = -24M^3/\ell^2.$$

---

[8] It is straightforward to check that (7.5) satisfies the first, second, and fourth equations of (7.4). There is, however, a subtlety in checking that (7.5) satisfies the third equation of (7.4). Unlike ϵ_r^-, ϵ_r^+ is a smooth function of x^5. Thus, the second term on the right side of the third equation of (7.4) contributes nothing.

Randall-Sundrum scenarios constructed from standard $D=5$ supergravity theories yield these relations (up to an overall normalization factor) as a consequence of local supersymmetry (some examples are given in [12]). In our case, the relation $\Lambda = -24M^3/\ell^2$ follows from our metric choice. We do not obtain the relations $\mathcal{T}^{(0)} = -\mathcal{T}^{(\pi\rho)} = 24M^3/\ell$ as a consequence of local fermionic symmetry. These are fine-tuning relations in our model.

A Appendix

In this Appendix, we work out the fünfbein, spin connection, curvature 2-form components, Ricci tensor, and Ricci scalar for our metric (1.1). For the fünfbein, we obtain

$$\begin{aligned} &e_{\bar\mu}{}^{\bar a} = \mathfrak{a}\delta_{\bar\mu}{}^{\bar a}, \quad e_{\bar\mu\bar a} = e_{\bar\mu}{}^{\bar b}\eta_{\bar b\bar a}, \quad e^{\bar\mu\bar a} = g^{\bar\mu\bar\nu}e_{\bar\nu}{}^{\bar a}, \\ &e_{\bar a}{}^{\bar\mu} = \mathfrak{a}^{-1}\delta_{\bar a}{}^{\bar\mu}, \quad e_{\bar a\bar\mu} = e_{\bar a}{}^{\bar\nu}g_{\bar\nu\bar\mu}, \quad e^{\bar a\bar\mu} = \eta^{\bar a\bar b}e_{\bar b}{}^{\bar\mu}, \\ &e_5{}^{\dot 5} = e_{5\dot 5} = e^{5\dot 5} = 1, \quad e_{\dot 5}{}^5 = e_{\dot 5 5} = e^{\dot 5 5} = 1. \end{aligned} \tag{A.1}$$

Our conventions for the spin connection, curvature 2-form components, Ricci tensor, and Ricci scalar are respectively

$$\begin{aligned} \omega_\mu{}^{ab} &= \tfrac{1}{2}e^{\nu a}(\partial_\mu e_\nu{}^b - \partial_\nu e_\mu{}^b) - \tfrac{1}{2}e^{\nu b}(\partial_\mu e_\nu{}^a - \partial_\nu e_\mu{}^a) \\ &\quad - \tfrac{1}{2}e^{\rho a}e^{\sigma b}(\partial_\rho e_{\sigma c} - \partial_\sigma e_{\rho c})e^c{}_\mu, \\ R_{\mu\nu}{}^{ab} &= \partial_\mu\omega_\nu{}^{ab} - \partial_\nu\omega_\mu{}^{ab} + \omega_\mu{}^{ac}\omega_{\nu c}{}^b - \omega_\nu{}^{ac}\omega_{\mu c}{}^b, \\ R_{\nu\sigma} &= R_{\mu\nu}{}^{ab}e_a{}^\mu e_{b\sigma}, \quad R = e_a{}^\mu e_b{}^\nu R_{\mu\nu}{}^{ab}. \end{aligned}$$

For the metric (1.1), the nonzero quantities here are

$$\omega_{\bar\mu}{}^{\bar a\dot 5} = -\omega_{\bar\mu}{}^{\dot 5\bar a} = \mathfrak{a}'\delta_{\bar\mu}{}^{\bar a} = -e_{\bar\mu}{}^{\bar a}/L, \tag{A.2}$$

$$\begin{aligned} R_{\bar\mu\bar\nu}{}^{\bar a\bar b} &= -\mathfrak{a}'^2(\delta_{\bar\mu}{}^{\bar a}\delta_{\bar\nu}{}^{\bar b} - \delta_{\bar\mu}{}^{\bar b}\delta_{\bar\nu}{}^{\bar a}) = -(e_{\bar\mu}{}^{\bar a}e_{\bar\nu}{}^{\bar b} - e_{\bar\mu}{}^{\bar b}e_{\bar\nu}{}^{\bar a})/\ell^2, \\ R_{5\bar\mu}{}^{\bar a\dot 5} &= \mathfrak{a}''\delta_{\bar\mu}{}^{\bar a} = e_{\bar\mu}{}^{\bar a}\left\{1/\ell^2 - 2[\delta(x^5) - \delta(x^5-\pi\rho)]/\ell\right\}, \end{aligned} \tag{A.3}$$

$$\begin{aligned} R_{\bar\mu\bar\nu} &= -(\mathfrak{a}\mathfrak{a}'' + 3\mathfrak{a}'^2)\eta_{\bar\mu\bar\nu} = -\left\{4/\ell^2 - 2[\delta(x^5) - \delta(x^5-\pi\rho)]/\ell\right\}g_{\bar\mu\bar\nu}, \\ R_{55} &= -4\mathfrak{a}^{-1}\mathfrak{a}'' = -\left\{4/\ell^2 - 8[\delta(x^5) - \delta(x^5-\pi\rho)]/\ell\right\}, \end{aligned} \tag{A.4}$$

$$R = -8\mathfrak{a}^{-1}\mathfrak{a}'' - 12\mathfrak{a}^{-2}\mathfrak{a}'^2 = -20/\ell^2 + 16[\delta(x^5) - \delta(x^5-\pi\rho)]/\ell, \tag{A.5}$$

and those related to (A.3) by $R_{\mu\nu}{}^{ab} = -R_{\nu\mu}{}^{ab} = -R_{\mu\nu}{}^{ba}$. The prime symbol $\prime$ denotes partial differentiation with respect to x^5.

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