φ_4^4-Theory for Antisymmetric Tensor Matter Fields in Minkowski Space-Time

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Abstract

The nonabelian generalization of a recently proposed abelian axial gauge model for tensor matter fields is obtained. In both cases the model can be derived from a φ^4-type theory for antisymmetric fields obeying a complex self-dual condition.

Key-words: Field theory; Renormalization.

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1 Introduction

In a recent paper L. V. Avdeev and M. V. Chizhov [1] have analysed the properties of an abelian axial gauge model containing antisymmetric second rank tensor fields as matter fields. The model is formulated in Minkowski flat space-time and exihibits several interesting features which allow for many applications both from phenomenological [2] and theoretical [3] point of view.

Let us underline, in particular, the asymptotically free ultraviolet behaviour of the abelian axial gauge interaction. As shown by the authors [1] with an explicit one-loop computation, this is due to the fact that the contribution of the tensor fields to the gauge $\beta-$function is negative. This particularly attractive feature motivates further efforts in understanding tensor matter fields.

The aim of this work, which is the continuation of a previous paper [4] where the renormalizability of the model has been discussed to all orders of perturbation theory, is to study the geometrical properties of the matter tensor action, i.e. to investigate the guide principle which underlies its gauge formulation. In particular, it turns out that the model can be obtained in a very simple and suggestive way from a φ^4-theory for tensor fields satisfying a complex self-dual condition, namely

$$\varphi_{\mu\nu} = i\widetilde{\varphi}_{\mu\nu} \ , \qquad \widetilde{\varphi}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\varphi^{\rho\sigma} \ , \tag{1.1}$$

$\varphi_{\mu\nu}$ being an antisymmetric complex tensor field and $\varepsilon_{\mu\nu\rho\sigma}$ the Levi-Civita symbol.

As we shall see in detail, the complex self-dual condition (1.1) uniquely fixes the Lorentz contractions of the tensor φ^4-Lagrangian, reproducing thus the action of Avdeev and Chizhov. Moreover, the formulation of the model as a kind of φ^4-theory will give us a straightforward way of obtaining its nonabelian generalization which, so far, has not yet been established.

Let us also remark that complex self-dual conditions of the type of eq.(1.1) have been known and used since several years. As an example we mention the complex self-dual connection[1] used by A. Ashtekar in its formulation of gravity [5].

The paper is organized as follows. In Sect.2, after introducing the complex self-dual condition (1.1), we show how it can be used to recover the abelian model of Avdeev and Chizhov. Sect.3 is then devoted to a detailed discussion of the nonabelian generalization. Concerning the latter case, we shall limit here only to classical aspects. The renormalization properties of the nonabelian model, i.e. its stability under radiative corrections and the absence of anomalies, will be reported in a future work.

[1] In this case the complex self-dual condition refers to the internal indices of $SO(3,1)$ rather than space-time indices.

2 Tensor matter fields: the abelian case

Let us begin this section by introducing the notations and some properties of the Levi-Civita tensor $\varepsilon_{\mu\nu\rho\sigma}$. We shall work in a flat Minkowski space-time with metric $g_{\mu\nu} = diag(+,-,-,-)$.

The totally antisymmetric tensor $\varepsilon_{\mu\nu\rho\sigma}$ is normalized as [6]

$$\varepsilon_{1234} = 1 \ , \qquad \varepsilon^{1234} = -1 \ . \tag{2.1}$$

It obeys the property

$$\varepsilon_{\mu_1\mu_2\mu_3\mu_4}\varepsilon^{\nu_1\nu_2\nu_3\nu_4} = -\delta^{[\nu_1}_{\mu_1}...\delta^{\nu_4]}_{\mu_4} \ , \tag{2.2}$$

from which it follows that

$$\varepsilon^{\mu\nu\rho\sigma}\varepsilon_{\rho\sigma\tau\omega} = -2(\delta^{\mu}_{\tau}\delta^{\nu}_{\omega} - \delta^{\mu}_{\omega}\delta^{\nu}_{\tau}) \ . \tag{2.3}$$

2.1 Complex self-dual fields in Minkowski space-time

A well known feature of the Minkowski space-time is that it does not allow for self-dual (or antiself-dual) fields

$$\eta_{\mu\nu} = \widetilde{\eta}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}\eta^{\rho\sigma} \ . \tag{2.4}$$

Indeed, due to eq.(2.3), one has

$$\widetilde{\widetilde{\eta}}_{\mu\nu} = -\eta_{\mu\nu} \ , \tag{2.5}$$

which is incompatible with the self-duality condition (2.4).

Instead, equation (2.4) is replaced by a complex self-dual condition involving a complex tensor field $\varphi_{\mu\nu}$:

$$\varphi_{\mu\nu} = i\widetilde{\varphi}_{\mu\nu} \ . \tag{2.6}$$

The factor i in eq.(2.6) is needed in order to compensate the minus sign coming from eq.(2.3). In fact, making the dual of eq.(2.6) one has

$$\widetilde{\varphi}_{\mu\nu} = i\widetilde{\widetilde{\varphi}}_{\mu\nu} = -i\varphi_{\mu\nu} \ , \tag{2.7}$$

i.e. one consistently goes back to eq.(2.6).

The complex self-dual condition (2.6) is easily solved. Writing $\varphi_{\mu\nu}$ as

$$\varphi_{\mu\nu} = T_{\mu\nu} + iR_{\mu\nu} \ , \tag{2.8}$$

with T and R real antisymmetric fields, one gets $R = \widetilde{T}$, i.e.

$$\varphi_{\mu\nu} = T_{\mu\nu} + i\widetilde{T}_{\mu\nu} \ . \tag{2.9}$$

2.2 Coupling to abelian gauge fields

Let us now try to couple the complex self-dual field $\varphi_{\mu\nu}$, considered as a matter field, to a gauge potential A_μ. In order to do this we require that under an abelian gauge transformation

$$\delta A_\mu = \partial_\mu \alpha \ , \tag{2.10}$$

the antisymmetric field $\varphi_{\mu\nu}$ transforms as an ordinary matter field according to

$$\delta\varphi_{\mu\nu} = i\alpha\varphi_{\mu\nu} \ , \qquad \delta{\varphi^\dagger}_{\mu\nu} = -i\alpha{\varphi^\dagger}_{\mu\nu} \ . \tag{2.11}$$

As usual, for the covariant derivative we get

$$\nabla_\sigma\varphi_{\mu\nu} = \partial_\sigma\varphi_{\mu\nu} - iA_\sigma\varphi_{\mu\nu} \ , \tag{2.12}$$

and

$$\delta(\nabla_\sigma\varphi_{\mu\nu}) = i\alpha(\nabla_\sigma\varphi_{\mu\nu}) \ , \qquad \delta(\nabla_\sigma\varphi_{\mu\nu})^\dagger = -i\alpha(\nabla_\sigma\varphi_{\mu\nu})^\dagger \ . \tag{2.13}$$

The next natural step is then to discuss the action. In order to find such an invariant action let us forget, for the time being, the Lorentz structure of the antisymmetric field φ. It is apparent thus that an action of the φ^4-type theory yields an invariant action:

$$S = \int d^4x \left((\nabla\varphi)^\dagger(\nabla\varphi) - \frac{q}{8}(\varphi^\dagger\varphi)^2 \right) \ . \tag{2.14}$$

Let us now try to take into account the Lorentz indices of the field $\varphi_{\mu\nu}$ and of the covariant derivative ∇_σ. Owing to the tensorial nature of $\varphi_{\mu\nu}$ one could expect many possible Lorentz contractions, both in the kinetic and in the quartic self-interaction term. Of course, this would spoil the interest and the meaning of the action (2.14).

However, it is a remarkable fact that the complex self-dual condition (2.6) completely fixes the Lorentz structure of (2.14), giving rise to a unique term both in the kinetic and in the self-interaction sector. In other words, condition (2.6) singles out a unique invariant action. This nice feature is due to the following property

$$\varphi^{\mu\nu}\mathcal{M}\varphi^\dagger_{\mu\nu} = 0 \ , \tag{2.15}$$

$\mathcal{M}$ denoting an arbitrary operator depending on the gauge potential A_μ and on the space-time derivatives ∂_μ. As one can easily understand, eq.(2.15) is a direct consequence of the complex self-dual condition (2.6) and of eq.(2.3). Indeed

$$\begin{aligned} \varphi^{\mu\nu}\mathcal{M}\varphi^\dagger_{\mu\nu} &= \widetilde{\varphi}^{\mu\nu}\mathcal{M}\widetilde{\varphi}^\dagger_{\mu\nu} = \frac{1}{4}\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\mu\nu\lambda\delta}\varphi_{\alpha\beta}\mathcal{M}\varphi^{\dagger\lambda\delta} \\ &= -\varphi^{\lambda\delta}\mathcal{M}{\varphi^\dagger}_{\lambda\delta} \ . \end{aligned} \tag{2.16}$$

It is worth to notice that condition (2.15) forbids the existence of a mass term $\varphi^{\mu\nu}{\varphi^\dagger}_{\mu\nu}$, i.e. $(\mathcal{M} = 1)$.

Concerning now the kinetic part of the action (2.14) it is easily verified that the unique nonvanishing Lorentz contraction is given, modulo integrations by parts, by

$$\int d^4x \; (\nabla_\mu \varphi^{\mu\nu})(\nabla_\sigma {\varphi^\sigma}_\nu)^\dagger \; . \tag{2.17}$$

Let us now turn to the quartic self-interaction part. In this case eq.(2.15) selects three possible terms, given respectively by

$$i)\; \varphi^{\dagger\mu\nu} {\varphi^\dagger}_{\mu\nu} \varphi^{\alpha\beta} \varphi_{\alpha\beta} \; , \quad ii)\; \varphi^{\dagger\mu\nu} \varphi_{\nu\alpha} \varphi^{\dagger\alpha\beta} \varphi_{\beta\mu} \; , \quad iii)\; \varphi^{\dagger\mu\nu} {\varphi^\dagger}_{\nu\alpha} \varphi^{\alpha\beta} \varphi_{\beta\mu} \; . \tag{2.18}$$

However, it turns out that these three terms are, in fact, equivalent. This can be proven by making use of the following identity, valid in four dimension

$$\varepsilon_{\alpha\beta\mu\nu} \Xi_{\sigma\ldots} + \varepsilon_{\sigma\alpha\beta\mu} \Xi_{\nu\ldots} + \varepsilon_{\nu\sigma\alpha\beta} \Xi_{\mu\ldots} + \varepsilon_{\mu\nu\sigma\alpha} \Xi_{\beta\ldots} + \varepsilon_{\beta\mu\nu\sigma} \Xi_{\alpha\ldots} = 0 \; , \tag{2.19}$$

where $\Xi_{\mu\ldots}$ denotes an arbitrary tensor. Equation (2.19) stems from the fact that in four dimension the antisymmetrization with respect to five Lorentz indices automatically vanishes.

As an example, let us prove the equivalence between the terms *i)* and *iii)* of (2.18). We have

$$\varphi^{\dagger\mu\nu} {\varphi^\dagger}_{\mu\nu} \varphi^{\alpha\beta} \varphi_{\alpha\beta} = -i \varphi^{\dagger\mu\nu} \widetilde{\varphi}^\dagger_{\mu\nu} \varphi^{\alpha\beta} \varphi_{\alpha\beta} = -\frac{i}{2} \varepsilon_{\mu\nu\lambda\sigma} \varphi^{\dagger\mu\nu} \varphi^{\dagger\lambda\sigma} \varphi^{\alpha\beta} \varphi_{\alpha\beta} \; . \tag{2.20}$$

Moreover, antisymmetrization with respect to the indices $(\mu, \nu, \lambda, \sigma)$ and α yields

$$\begin{aligned} \varphi^{\dagger\mu\nu} {\varphi^\dagger}_{\mu\nu} \varphi^{\alpha\beta} \varphi_{\alpha\beta} = & \; \frac{i}{2} \varphi^{\dagger\mu\nu} \varphi^{\dagger\lambda\sigma} \varphi^{\alpha\beta} \Big(\varepsilon_{\alpha\mu\nu\lambda} \varphi_{\sigma\beta} + \varepsilon_{\sigma\alpha\mu\nu} \varphi_{\lambda\beta} \Big) \\ & + \frac{i}{2} \varphi^{\dagger\mu\nu} \varphi^{\dagger\lambda\sigma} \varphi^{\alpha\beta} \Big(\varepsilon_{\lambda\sigma\alpha\mu} \varphi_{\nu\beta} + \varepsilon_{\nu\lambda\sigma\alpha} \varphi_{\mu\beta} \Big) \\ = & \; 4 \, \varphi^{\dagger\mu\nu} {\varphi^\dagger}_{\nu\alpha} \varphi^{\alpha\beta} \varphi_{\beta\mu} \; , \end{aligned} \tag{2.21}$$

showing then the equivalence.

Summarizing, the complex self-dual condition (2.6) uniquely fixes the Lorentz structure of the invariant action (2.14). The latter, including also the Maxwell term, is given by

$$\begin{aligned} S_{inv} = & \; -\frac{1}{4g^2} \int d^4x \, F_{\mu\nu} F^{\mu\nu} \\ & - \int d^4x \left((\nabla_\mu \varphi^{\mu\nu})(\nabla_\sigma {\varphi^\sigma}_\nu)^\dagger + \frac{q}{8} (\varphi^{\dagger\mu\nu} \varphi_{\nu\alpha} \varphi^{\dagger\alpha\beta} \varphi_{\beta\mu}) \right) \; . \end{aligned} \tag{2.22}$$

2.3 The Avdeev-Chizhov abelian action

In order to recover the action of Avdeev and Chizhov [1], let us rewrite the expression (2.22) in components, i.e. let us make use of eq.(2.9) ($\varphi = T + i\widetilde{T}$). Concerning the gauge transformations (2.10), (2.11), they split as

$$\delta A_\mu = \partial_\mu \alpha \ , \qquad \delta T_{\mu\nu} = -\alpha \widetilde{T}_{\mu\nu} \ , \qquad \delta \widetilde{T}_{\mu\nu} = \alpha T_{\mu\nu} \ . \tag{2.23}$$

Analogously, for the covariant derivative $\nabla_\sigma \varphi_{\mu\nu}$ we get

$$\nabla_\sigma \varphi_{\mu\nu} = \nabla_\sigma T_{\mu\nu} + i \nabla_\sigma \widetilde{T}_{\mu\nu} \ , \tag{2.24}$$

with

$$\nabla_\sigma T_{\mu\nu} = \partial_\sigma T_{\mu\nu} + A_\sigma \widetilde{T}_{\mu\nu} \ , \qquad \nabla_\sigma \widetilde{T}_{\mu\nu} = \partial_\sigma \widetilde{T}_{\mu\nu} - A_\sigma T_{\mu\nu} \ , \tag{2.25}$$

and

$$\delta(\nabla_\sigma T_{\mu\nu}) = -\alpha(\nabla_\sigma \widetilde{T}_{\mu\nu}) \ , \qquad \delta(\nabla_\sigma \widetilde{T}_{\mu\nu}) = \alpha(\nabla_\sigma T_{\mu\nu}) \ . \tag{2.26}$$

We recover then the two covariant derivatives $(\nabla T, \nabla \widetilde{T})$ already introduced in [4].

Finally, after a straightforward calculation, for the invariant action we get

$$\begin{aligned} S_{inv} = & -\frac{1}{4g^2}\int d^4x \, F_{\mu\nu}F^{\mu\nu} \\ & -\int d^4x \left((\nabla_\mu T^{\mu\nu})(\nabla_\sigma T^\sigma_{\ \nu}) + (\nabla_\mu \widetilde{T}^{\mu\nu})(\nabla_\sigma \widetilde{T}^\sigma_{\ \nu}) \right. \\ & \qquad \left. + \frac{q}{4}\left(2T_{\mu\nu}T^{\nu\rho}T_{\rho\lambda}T^{\lambda\mu} - \frac{1}{2}(T_{\mu\nu}T^{\mu\nu})^2 \right) \right) \\ = & -\frac{1}{4g^2}\int d^4x \, F_{\mu\nu}F^{\mu\nu} \\ & +\int d^4x \left(\frac{1}{2}(\partial_\lambda T_{\mu\nu})^2 - 2(\partial_\mu T^{\mu\nu})^2 + 2A_\mu\left(T^{\mu\nu}\partial_\lambda \widetilde{T}^\lambda_{\ \nu} - \widetilde{T}^{\mu\nu}\partial_\lambda T^\lambda_{\ \nu} \right) \right. \\ & \qquad + \left(\frac{1}{2}(A_\lambda T_{\mu\nu})^2 - 2(A^\mu T_{\mu\nu})^2 \right) \\ & \qquad \left. - \frac{q}{4}\left(2T_{\mu\nu}T^{\nu\rho}T_{\rho\lambda}T^{\lambda\mu} - \frac{1}{2}(T_{\mu\nu}T^{\mu\nu})^2 \right) \right) \ , \end{aligned} \tag{2.27}$$

with

$$\delta S_{inv} = 0 \ . \tag{2.28}$$

Expression (2.27) is nothing but the original abelian action proposed by Avdeev and Chizhov [1]. We see thus that, as announced, the model can be derived in a very simple and suggestive way from a φ^4-theory for tensor fields obeying a complex self-dual condition.

3 The nonabelian case

In order to obtain the nonabelian generalization of the invariant action (2.27), we proceed as before and we treat the antisymmetric tensor field φ as an ordinary bosonic matter field belonging to some finite representation $(\lambda^a)^{ij}$ of a Lie group G, assumed to be semisimple (the index a labels the generators of G, while the indices (ij) specify the representation). For reasons which will be clear later on, the representation identified by the hermitian matrices $(\lambda^a)^{ij}$ will be required to be a complex representation, i.e.

$$\lambda^a = \lambda^a_R + i\lambda^a_I \ , \tag{3.1}$$

λ^a_R and λ^a_I denoting respectively the real and the imaginary part of λ^a. In particular, from the commutation relations

$$\left[\lambda^a, \lambda^b\right] = if^{abc}\lambda^c \ , \tag{3.2}$$

we get

$$\begin{aligned} \left[\lambda^a_R, \lambda^b_R\right] - \left[\lambda^a_I, \lambda^b_I\right] &= -f^{abc}\lambda^c_I \ , \\ \left[\lambda^a_R, \lambda^b_I\right] + \left[\lambda^a_I, \lambda^b_R\right] &= f^{abc}\lambda^c_R \ , \end{aligned} \tag{3.3}$$

and from the hermiticity condition $\lambda^a = \lambda^{a\dagger}$

$$(\lambda^a_R)^{ij} = (\lambda^a_R)^{ji} \ , \qquad (\lambda^a_I)^{ij} = -(\lambda^a_I)^{ji} \ . \tag{3.4}$$

3.1 Coupling to Yang-Mills fields

As said before, in order to couple the nonabelian complex self-dual field $\varphi^i_{\mu\nu}$

$$\varphi^i_{\mu\nu} = i\widetilde{\varphi}^i_{\mu\nu} \ , \qquad \varphi^i_{\mu\nu} = T^i_{\mu\nu} + i\widetilde{T}^i_{\mu\nu} \ , \tag{3.5}$$

to Yang-Mills fields A^a_μ, we treat it as a bosonic matter field which transforms according to the usual nonabelian gauge transformations, here written for convenience as BRS transformations [7, 8]

$$\begin{aligned} sA^a_\mu &= \partial_\mu c^a + f^{abc}A^b_\mu c^c \ , \\ s\varphi^i_{\mu\nu} &= ic^a(\lambda^a)^{ij}\varphi^j_{\mu\nu} \ , \\ s{\varphi^\dagger}^i_{\mu\nu} &= -ic^a{\varphi^\dagger}^j_{\mu\nu}(\lambda^a)^{ji} \ , \\ sc^a &= -\frac{1}{2}f^{abc}c^bc^c \ , \qquad s^2 = 0 \ . \end{aligned} \tag{3.6}$$

In complete analogy with the previous abelian case, for the nonabelian covariant derivative $(\nabla_\sigma\varphi_{\mu\nu})^i$ we get

$$(\nabla_\sigma\varphi_{\mu\nu})^i = \partial_\sigma\varphi^i_{\mu\nu} - iA^a_\sigma(\lambda^a)^{ij}\varphi^j_{\mu\nu} \ , \tag{3.7}$$

and

$$
\begin{aligned}
s(\nabla_\sigma\varphi_{\mu\nu})^i &= ic^a(\lambda^a)^{ij}(\nabla_\sigma\varphi_{\mu\nu})^j \ , \\
s(\nabla_\sigma\varphi_{\mu\nu})^{\dagger i} &= -ic^a(\nabla_\sigma\varphi_{\mu\nu})^{\dagger j}(\lambda^a)^{ji} \ .
\end{aligned}
\tag{3.8}
$$

Of course, property (2.15) remains unchanged, implying thus the following expression for the BRS invariant nonabelian action:

$$
\begin{aligned}
S_{inv} = &-\frac{1}{4g^2}\int d^4x\, F^a_{\mu\nu}F^{a\mu\nu} \\
&-\int d^4x \left((\nabla_\mu\varphi^{\mu\nu})^i(\nabla_\sigma\varphi^\sigma{}_\nu)^{\dagger i} + \frac{q}{8}(\varphi^{\dagger\mu\nu i}\varphi^i_{\nu\alpha}\varphi^{\dagger\alpha\beta j}\varphi^j_{\beta\mu}) \right) \ ,
\end{aligned}
\tag{3.9}
$$

where the Yang-Mills term has been included.

3.2 Components

For a better understanding of the above nonabelian invariant action let us rewrite, as done before, the expression (3.9) in terms of the component fields $(T^i_{\mu\nu}, \widetilde{T}^i_{\mu\nu})$ of eq.(3.5). Considering first the BRS transformations (3.6), we obtain

$$
\begin{aligned}
&sA^a_\mu = \partial_\mu c^a + f^{abc}A^b_\mu c^c \ , \qquad sc^a = -\frac{1}{2}f^{abc}c^bc^c \\
&sT^i_{\mu\nu} = -c^a\left((\lambda^a_R)^{ij}\widetilde{T}^j_{\mu\nu} + (\lambda^a_I)^{ij}T^j_{\mu\nu} \right) \ , \\
&s\widetilde{T}^i_{\mu\nu} = c^a\left((\lambda^a_R)^{ij}T^j_{\mu\nu} - (\lambda^a_I)^{ij}\widetilde{T}^j_{\mu\nu} \right) \ .
\end{aligned}
\tag{3.10}
$$

Their nilpotency easily follows from the algebraic relations (3.3).

One should notice that the choice of a complex representation, i.e. $\lambda^a_R \neq 0$, allows for a nontrivial mixing between the chiral components $(T, \widetilde{T})$ of the complex self-dual field φ, yielding then the nonabelian generalization of the Avdeev-Chizhov chiral transformations (2.23).

For the covariant derivative (3.7) we get

$$
(\nabla_\sigma\varphi_{\mu\nu})^i = (\nabla_\sigma T_{\mu\nu})^i + i(\nabla_\sigma\widetilde{T}_{\mu\nu})^i \ , \tag{3.11}
$$

with

$$
\begin{aligned}
(\nabla_\sigma T_{\mu\nu})^i &= \partial_\sigma T^i_{\mu\nu} + A^a_\sigma(\lambda^a_I)^{ij}T^j_{\mu\nu} + A^a_\sigma(\lambda^a_R)^{ij}\widetilde{T}^j_{\mu\nu} \ , \\
(\nabla_\sigma\widetilde{T}_{\mu\nu})^i &= \partial_\sigma\widetilde{T}^i_{\mu\nu} + A^a_\sigma(\lambda^a_I)^{ij}\widetilde{T}^j_{\mu\nu} - A^a_\sigma(\lambda^a_R)^{ij}T^j_{\mu\nu} \ .
\end{aligned}
\tag{3.12}
$$

Their BRS transformations read

$$
\begin{aligned}
s(\nabla_\sigma T_{\mu\nu})^i &= -c^a\left((\lambda^a_R)^{ij}(\nabla_\sigma\widetilde{T}_{\mu\nu})^j + (\lambda^a_I)^{ij}(\nabla_\sigma T_{\mu\nu})^j \right) \ , \\
s(\nabla_\sigma\widetilde{T}_{\mu\nu})^i &= c^a\left((\lambda^a_R)^{ij}(\nabla_\sigma T_{\mu\nu})^j - (\lambda^a_I)^{ij}(\nabla_\sigma\widetilde{T}_{\mu\nu})^j \right) \ .
\end{aligned}
\tag{3.13}
$$

Finally, for the invariant action (3.9) one has

$$
\begin{aligned}
S_{inv} = & -\frac{1}{4g^2}\int d^4x\, F^a_{\mu\nu}F^{a\mu\nu} \\
& -\int d^4x \left((\nabla_\mu T^{\mu\nu})^i(\nabla_\sigma T^\sigma{}_\nu)^i + (\nabla_\mu \widetilde{T}^{\mu\nu})^i(\nabla_\sigma \widetilde{T}^\sigma{}_\nu)^i \right. \\
& \qquad \left. +\frac{q}{4}\left(2(T^i_{\mu\nu}T^{i\nu\rho})^2 - \frac{1}{2}(T^i_{\mu\nu}T^{i\mu\nu})^2 \right) \right) \\
= & -\frac{1}{4g^2}\int d^4x\, F^a_{\mu\nu}F^{a\mu\nu} + \int d^4x \left(\frac{1}{2}(\partial_\lambda T_{\mu\nu})^2 - 2(\partial_\mu T^{\mu\nu})^2 \right) \\
& -2\int d^4x\, A^a_\mu \left((\partial_\sigma T^{\sigma\nu})\lambda^a_R \widetilde{T}^\mu{}_\nu - (\partial_\sigma \widetilde{T}^{\sigma\nu})\lambda^a_R T^\mu{}_\nu \right) \\
& -2\int d^4x\, A^a_\mu \left((\partial_\sigma T^{\sigma\nu})\lambda^a_I T^\mu{}_\nu + (\partial_\sigma T^{\mu\nu})\lambda^a_I T^\sigma{}_\nu \right) \\
& +\int d^4x\, A^a_\mu A^b_\sigma \left(T^{\mu\nu}\lambda^a_I\lambda^b_I T^\sigma{}_\nu + T^{\mu\nu}\lambda^a_I\lambda^b_R \widetilde{T}^\sigma{}_\nu \right) \\
& -\int d^4x\, A^a_\mu A^b_\sigma \left(\widetilde{T}^{\mu\nu}\lambda^a_R\lambda^b_I T^\sigma{}_\nu + \widetilde{T}^{\mu\nu}\lambda^a_R\lambda^b_R \widetilde{T}^\sigma{}_\nu \right) \\
& +\int d^4x\, A^a_\mu A^b_\sigma \left(\widetilde{T}^{\mu\nu}\lambda^a_I\lambda^b_I \widetilde{T}^\sigma{}_\nu - \widetilde{T}^{\mu\nu}\lambda^a_I\lambda^b_R T^\sigma{}_\nu \right) \\
& +\int d^4x\, A^a_\mu A^b_\sigma \left(T^{\mu\nu}\lambda^a_R\lambda^b_I \widetilde{T}^\sigma{}_\nu - T^{\mu\nu}\lambda^a_R\lambda^b_R T^\sigma{}_\nu \right) \\
& -\frac{q}{4}\int d^4x \left(2(T_{\mu\nu}T^{\nu\rho})^2 - \frac{1}{2}(T_{\mu\nu}T^{\mu\nu})^2 \right) ,
\end{aligned}
\tag{3.14}
$$

where the implicit notation $T\lambda^a_R\widetilde{T} = T^i(\lambda^a_R)^{ij}\widetilde{T}^j$, etc...., has been used.

Expression (3.14) represents thus the nonabelian generalization of the Avdeev-Chizhov model. In particular one remarks that, contrary to the abelian case (2.27), the Levi-Civita tensor $\varepsilon_{\mu\nu\rho\sigma}$ is now present also in the quartic ($AATT$) interaction term.

4 Conclusion

The abelian axial model proposed by Avdeev and Chizhov has been proven to be interpreted as a φ^4-type theory for tensor fields obeying a complex self-dual condition. This formulation allows us to obtain in a simple and elegant way the corresponding nonabelian generalization.

Many aspects of the tensor matter field theories remain still to be discussed and clarified. Exemples of them are, for instance, the unitarity properties of the degrees of freedom associated to the antisymmetric tensor fields (see also ref [3] for a study of the Fock space) and the renormalizability of the nonabelian model. Any progress on these aspects will be reported in a detailed work.

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