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A MODEL FIELD THEORY IN FIVE DIMENSIONS

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ABSTRACT

A theory for interacting spin 2, 1 and 0 fields is constructed by 4 + 1 decomposition of a symmetric tensor in five-dimensional flat space. The field equations are a natural consequence of the existing symmetry principles.

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Several years ago, Kaluza and Klein [1] proposed an unified theory of gravitation and electromagnetism. This theory represents up to the present time one of the most complete unitary field theories. According to these authors, the invariance group of the complete system was represented in a five dimensional curved manifold. Presently we intend to determine up to what extension it is possible to formulate a version of this theory in the flat space-time of special relativity.

The general aspects which may be re-interpreted in this way, as well as, those aspects which can not be assimilated in this flat space-time method, will be discussed in the course of this presentation.

We start by introducing the symmetric tensor [2] $\phi_{\mu\nu}$, and by extension of the usual relativistic teminology, we consider this field as the spin -2- field in five-dimensions. The five-dimensional space is taken as a flat manifold, with a metric that may have any one of the four possible signatures ± 1, ± 3. All important results which will be obtained are independent of any particular choice of the diagonal elements of the metric.

The fifteen component tensor $\phi_{\mu\nu}$ may be divided into the three objects ϕ_{ij}, ϕ_{i5}, and ϕ_{55} with respectively ten, four and one components. We impose that the invariance group in five-dimensions may be decomposed in sub-groups in such way that the above objects represent a tensor, a vector, and a scalar fields from the point of view of four-dimensions.

Under a coordinate transformation in five-dimensional space, we have

$$\phi'_{\mu\nu}(x'^{\alpha}) = \frac{\partial x^{\rho}}{\partial x'^{\mu}} \frac{\partial x^{\lambda}}{\partial x'^{\nu}} \phi_{\rho\lambda}(x) \ . \qquad (1)$$

The general group $x'^{\mu} = x'^{\mu}(x)$ is decomposed into two sub-groups

$$G_{(1)} = \begin{cases} x^{i} = f^{i}(x'^{j}) \\ x^{5} = x'^{5} \end{cases}$$

$$G_{(2)} = \begin{cases} x^{i} = x'^{i} \\ x^{5} = x'^{5} - f(x^{i}) \end{cases}$$

It may be shown that under $G_{(1)}$ the geometrical objects

$$\psi_{ij} = \phi_{ij} - \frac{\phi_{i5}\,\phi_{j5}}{\phi_{55}} \qquad (2)$$

$$\psi_{i} = \frac{\phi_{i5}}{\phi_{55}} \qquad (3)$$

$$\phi_{55} \qquad (4)$$

transforms as a tensor, a vector, and a scalar respectively. Under $G_{(2)}$, ψ_{ij} and ϕ_{55}, transform as a set of eleven scalars, and ψ_{i} transforms as [2]

$$\psi'_{i}(x^{j}, x'^{5}) = \psi_{i}(x^{j}, x^{5}) + f_{,i}(x^{j}) \ . \qquad (5)$$

In order to interpret (5) as a gauge transformation on the potentials [2], $\psi_{i,5} = 0$ is further necessary to impose, which implies that ψ_i is independent of the fifth coordinate in the transformed frame. By symmetry, $\psi_{ij,5} = 0$, and $\phi_{55,5} = 0$ are also required. Thus, relation (5) is a gauge transformation on the ψ, under which the tensor field $\psi_{i,j} - \psi_{j,i}$ is invariant. The group $G_{(2)}$ is then interpreted as the gauge group for the vector field. The group $G_{(1)}$ further simplified by imposing $f^i(x'^j) = L^i_{\ j} x'^j$ where $L^i_{\ j}$ is a Lorentz matrix, will represent the homogeneous Lorentz group.

In this way, we may introduce symmetry principles which allow us to construct the fields ψ_{ij}, ψ_i and ϕ_{55} according to (2), (3) and (4) in analogy to the method of Klein but now in flat space.

The structure of the Lagrangian density will then be constructed out of these fields.

However, we still have the gauge group for the tensor field which implies in the transformation law

$$\psi'_{ij}(x^l) = \psi_{ij}(x^l) + \varphi_{i,j}(x^l) + \varphi_{j,i}(x^l) \; . \tag{7}$$

Breaking with the previous symmetric way of introducing the invariance groups, the spin -2 gauge group have to be postulated. In other words, we have to look for a Lagrangian density which is a function of $\psi_{ij,k} - \psi_{ik,j}$. This represents one point in which differs from Klein's general relativistic theory. This is due to the fact that in Klein's theory the gauge group for spin 2 is written together with the general relativistic transformation law for $G_{(1)}$, that is, gauge transformations and coordinate transformations are treated as the same geometrical transformations.

Now, with the fields $\psi_{i,j} - \psi_{j,i}$, $\psi_{ij,k} - \psi_{ik,j}$ and ϕ_{55} we construct the most general Lagrangian density. This Lagrangian will invariant under all types of transformations seen up to here.

Therefore, a theory for spins 2, 1 and 0 interacting fields may be constructed, and solely by symmetry requirements. Similarly we can construct theories for other spins, for instance, considering a vector $\phi_\mu(x^\alpha)$ instead of a symmetric tensor we may introduce a theory for spins 1 and 0 at four-dimensions. In this form, we may go beyond Klein's original proposition, which was limited to symmetric tensors due to its intention of describing the gravitational field as one of the present dynamical systems.

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REFERENCES

1. Th. Kaluza, Sitzungsber, d. Preuss. Akad. Wiss, p. 966, 1921; O. Klein, Z. Physic 46, 188 (1927); Arkiv Mat. Astr. Fysik 25A, nº 15 (1936).
2. Greek Letters take values 1, 2, 3, 4, 5 and latin letters 1, 2, 3, 4. By convenience we use a comma to denote a ordinary derivative.