CBPF - CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Rio de Janeiro

Notas de Física

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Bert Schroer

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Bert Schroer

CBPF, Rua Dr. Xavier Sigaud 150

22290-180 Rio de Janeiro, Brazil

and Institut fuer Theoretische Physik der FU Berlin, Germany

December 2009

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Abstract

The crossing property, which originated more than 5 decades ago in the aftermath of dispersion relations, was the central new concept which opened an S-matrix based line of research in particle theory. Many constructive ideas in particle theory outside perturbative QFT, among them the S-matrix bootstrap program, the dual resonance model and the various stages of string theory have their historical roots in this property.

The crossing property is perhaps the most subtle aspect of the particle-field relation. Although it is not difficult to state its content in terms of certain analytic properties relating different matrixelements of the S-matrix or formfactors, its relation to the localization- and positive energy spectral principles requires a level of insight into the inner workings of QFT which goes beyond anything which can be found in typical textbooks on QFT. This paper presents a recent account based on new ideas derived from "modular localization" including a mathematic appendix on this subject.

The main content is an in-depth criticism of the dual model and its string theoretic extension. The conceptual flaws of these models are closely related to misunderstandings of the true meaning of crossing. The correct interpretation of string theory is that of a dynamic infinite component wave function or pointlike field i.e. a theory which under irreducible Poincare decomposition into an infinite mass/spin tower but which also contains operators which do not commute with the generators of the Poincaré group but rather intertwine between different mass/spin levels.

1   The increasing gap between foundational work and particle theory

There has always existed a tendency to romanticize the past when criticizing the present. But the importance of interpretational and philosophical ideas for the development of quantum theory (QT) in the first three decades of particle theory, starting in quantum mechanics (QM) and escorting the beginnings of quantum field theory (QFT), as compared to their superficial role or absence in the ongoing particle theory is hard to be overlooked. Most of the foundational concepts in relativistic QT can be traced back to developments before 1980. One can hardly think of any other branch of physics in which the correct interpretation of observational results was dependent on the outcome of a delicate balance between speculative innovations being followed by critical foundational work in which questions of conceptual aspects and philosophal consistency were the main driving force.

The strength of this connection between descriptive and conceptual aspects in the beginning of quantum physics was a result of the protagonist's (Bohr, Heisenberg, Schroedinger,..) intense interests in conceptual and philosophical questions of the protagonists of quantum theory. Almost the entire arsenal of foundational concepts, including those iconized Gedankenexperiments as *Schroedinger's cat* and Fermi's two-atom experiment in QED (arguing that c as the maximal velocity survives the quantization of electrodynamics), were introduced in order to highlight the philosophical consequences of their discoveries and to facilitate a critical approach for others.

But this does not mean that all this impressive grand design was an inevitable outcome of the innovative potential of the protagonists. Even the greatest intellectual brilliance is no insurance for finding the "diretissima" for scientific progress; already a slight change in the chronological ordering of important discoveries could have led to a time-consuming detour.

Just imagine that Feynman's path integral would have entered before matrix mechanics and transformation theory; as a result of the conceptual proximity of an integral over classical orbits with the Bohr-Sommerfeld framework of the largely quasi-classical old quantum theory, there is hardly anything more natural than to contemplate such a direct connection. The resulting formalism would have unified all the quasi-classical results of the old quantum theory and lifted it to a new level. It would have streamlined most previous calculations and presented an elegant way how to do computations around quantum oscillators, but it would have missed the important dichotomy between observables and states. Even worse, the elaboration of the Hilbert space formalism and operators acting in it, as well as all the understanding of those important integrable systems as the hydrogen atom (which even with all the present hindsight about path integrals remained a nontrivial endeavour) without whose operator presentation a course on QM is unthinkable, all these important contributions would have appeared much later and in a very different and probably more involved form.

Fortunately this was not the way things unfolded; by the time Feynman proposed his path representation, the conceptual level of operator QT was mature enough to resist the temptation of a fallacious short-sighted interpretation of this elegant but often conceptually and computational unsafe formalism. In this way many years of confusion in quantum physics were avoided and the path integral could be explored for those purposes for which it is powerful, namely quasiclassical approximations, keeping track of combinatorial aspects of renormalized perturbation theory and for presenting a flexible metaphoric top soil on which innovative ideas can sprout and specific computational problems be formulated. Many operator results on the other hand are either out of reach of the path integral, or can only be obtained by imposing artificial tricks which do not follow from its measure theoretic foundation and are less trustworthy than direct operator methods. The conceptual-mathematical control is limited to QM and certain (superrenormalizable) models in low dimensional QFTs, but this does not diminish its value as an intuitive guide and a social cohesion-creating construct in discussions among particle physicists with different backgrounds.

Taking into account that progress in particle physics it is not only the result of the intellectual capacity and the originality of the involved actors, but also requires an element of good fortune about taking the right turns at the right time on important cross roads, there is ample reason for considering the first three decades of particle physics in retrospect as the "good old days". The aim of this essay is to shed light on later developments, when innovation, critical analysis and luck began to drift apart. The best way to do this is to revisit the chain of events which started from the S-matrix bootstrap approach and culminated in string theory.

It is not difficult to localize the point of no return from where the present less fortunate direction in particle physics research took its beginning by following the events in the aftermath of the enormous successful perturbative renormalized quantum electrodynamics

(QED). The emerging difficulties to treat the nuclear interactions with the same methods led to a revival of S-matrix based ideas. This time the connection between relativistic local fields and asymptotic in/out particles were better understood than in Heisenberg's ill-fated first attempt [1] a decade before the S-matrix bootstrap.

Instead of investigating a concrete hadronic model, for which there existed at that time no computational framework, the most reasonable approach was to look for some experimentally accessible consequences of general principles. This led to the derivation of a form of the Kramers-Kronig dispersion known from optics but now adapted to particle physics. The derivation of these relations from first principles and their subsequent experimental verification in high energy collisions was the main aim in which many of the best brains of the 50s participated.

According to the best of my knowledge this was the only topic in post QED particle theory which can be characterized by the words "mission accomplished"; several years of dedicated work led to the solution of the problem, so that one could move on to other problems in an upbeat spirit without being obliged to revisit the problems in order to patch up conceptual holes left behind.

It was in the wake of these dispersion theory that the notion of the crossing property appeared; first as a property in Feynman graph perturbation theory and soon afterwards as a consequence of the same principles which already led to the dispersion relation. Bros Epstein Glaser and Martin **[2][3]** succeeded to proof the validity of crossing property by showing that the two particle elastic scattering amplitude is analytically connected to its crossed[1] version. The analytic connection between these processes establishes the existence of an analytic "masterfunction" which links all these different processes. The existence of such a masterfunction in turn suggested that the asymptotic high energy behavior of the different processes may not be independent, an idea which was confirmed in [4]. There exist also proofs of "asymptotic crossing" for $2 \to 3$ scattering and indications about how to generalize this to $2 \to n$ scattering [5]. Some comments on the ideas used in this derivation can be found in the next section.

Since causal localization is the only foundational property which distinguishes QFT within quantum theory (for this reason often referred to as LQP i.e. local quantum physics [14]), the fact the wealth of different models with their distinct physical manifestations are in some way related to localization is to be expected. What is however highly nontrivial is the chain of arguments and the richness of additional concepts which are needed in order to establish this connection. In the present work the crossing property is generalized to formfactors and general scattering amplitudes. The modular localization methods used in that derivation reveal that the conceptual setting is a *two-algebra generalization of the thermal KMS property* (section 5).

Continuing the S-matrix history, in the subsequent revival of S-matrix theory the newly discovered crossing played an essential role. It is the main distinctive feature with respect to Heisenberg's ill-fated prior S-matrix proposal of the 40's. The S-matrix bootstrap program attracted the attention of many particle theorists for almost a decade, before it disappeared from the journals and conferences[2]. The apparent reason was "physical

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[1]An incoming particle changes its position with an outgoing one and, as required by charge conservation, both particles become anti-particles.

[2]The fate which the S-matrix bootstrap community in conference publications predicted for QFT

anemia" i.e. its inability to produce any credible calculation from its underlying principles. There was certainly nothing wrong with its S-matrix principles of unitarity, Poincaré invariance and crossing, except that the "maximal analyticity" postulate resulted from a contemporary viewpoint from a misunderstanding of the role of analyticity in physics since it does not represent a physical principle but rather results from one. The connection between the physical causal locality principles and their analytic consequences are subtle and long winding but there is no way to sidestep these subtleties by turning the logic on its head.

What was however grossly misleading was the claim that the *nuclear democracy* behind the bootstrap principles have at most one solution (the possibility of having no solution was admitted) which describes the entire world of strong interaction. Such sweeping ultra-reductionist uniqueness claims arose occasionally in particle physics usually in connection with certain nonlinear structures[3] to which it was difficult to find any solution at all (e.g. the Schwinger-Dyson equation). The bootstrap unicity belief contained already germs of a new ideological thinking which in more recent times took the extreme form of a theory of everything (TOE).

Several years after the disappearance of the S-matrix bootstrap, the principles which underlie the construction of so-called factorizing two-dimensional models were discovered [6] which kick-started a still ongoing stream of results about a family of new interesting soluble models[4]. These rich results came from the observation that factorizing two-dimensional elastic S-matrices can indeed be classified and constructed by the those bootstrap principles of the meanwhile abandoned S-matrix bootstrap approach for strong interactions. Factorization in conjunction with dispersion theoretic analyticity led to meromorphy in terms of the rapidity variables as "maximal analyticity" and the physical reasons behind it in this special case. The protagonists of the bootstrap program of old never took notice of these astonishing new observations; in this way they spared themselves the confrontation with their earlier premature apodictic statements on this matter.

The two-dimensional bootstrap project has infinitely many solutions and serves as the starting point of a new infinitely large family of genuine nontrivial two-dimensional QFTs. These constructions did not only show that the claimed unicity was wishful imagination, but also revealed that the idea that all QFT can be described in a Lagrangian setting was too optimistic: the bootstrap classification of all two-dimensional factorizing S-matrices had infinitely many more solutions than those which can be described by Lagrangian couplings between free fields.

There were many ad hoc concepts invented in the wake of the S-matrix bootstrap, the most prominent (which was used in many later papers) was the Mandelstam spectral representation [8]. At that point the philosophy underlying physical research had significantly changed as compared to the era of dispersion relation[5]. For the latter it was

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namely "to fade away like a mortally wounded soldier on a battle field" but little did they know that this would become its own fate shortly after.

[3]Reasonable formulations as QFT "defuse" such structures (e.g. unitarity of the S-matrix) by showing that they result from linear asymptotic properties og fields.

[4]See the most recent one [7] and the references quoted therein.

[5]I recall warnings by Källén, Lehmann, Jost, Martin and others.

essential to be *rigorous* consequences of spectral representation (the Jost-Lehmann-Dyson representations) which in turn were derived from the locality and spectral principles of QFT. Without this strong connection with the underlying principles, the experimental verification of dispersion relations would have remained without much significance since they represented a check of the locality principles of QFT and not of a particular model.

The aim of the work of Mandelstam as well the later work of Veneziano leading up to string theory was very different; although they originated with a phenomenological entitlement, it soon turned into a rather freewheeling attempts to explore an imagined area beyond QFT with yet unknown principles. In other words these attempts were excursions into the "blue yonder" but certainly not from a firm platform of departure to which one could return in case of failure. As soon as the phenomenological basis was lost as a result of new experiments which turned out to be incompatible with the Regge trajectory phenomenology, the dual model and string theory became free-floating mathematical ideas without any conceptual basis to which they could safely return.

The main part of the paper will be concerned with a critical look at post S-matrix bootstrap ideas as the phenomenological dual model and the closely related string theory, which the protagonists of these models and others thought of as particular implementations of the crossing property. Following [30] it will be shown that the dual model properties are identical to the analytic properties of Mellin transforms of conformal correlation; they have nothing in common with the correctly understood crossing property of formfactors and scattering amplitudes which belong to a very different conceptual setting. Since the *crossing property is one of the most subtle relations between particles and fields*, part of our task consists in presenting an up to date account of a derivation of crossing from the causality and covariance principles of QFT.

The full depth of the crisis in contemporary particle physics cannot be perceived, and its causes cannot be understood without a careful conceptual and mathematical analysis based on a critical first hand historical knowledge. Commemorative articles as [8] are interesting and certainly contain a lot of important background material, but one should not expect to find a critical view in them.

If one asks a particle theorist of sufficient age to point at an important difference between the scientific discourse in the old days and the one in more recent decades, he will probably agree that, whereas the intellectual potential has remained the same or even increased, there has been a remarkable reduction of critical contributions and public controversies. The great conceptual discourse of the early years of QT gave way to a new style in which metaphorical arguments were allowed a more permanent position and in which the appreciation of the pivotal role of the delicate equilibrium between innovative speculations and their critical evaluation (which made particle physics such a success story) was declining.

At the time of Pauli, Lehmann, Källén, Feynman, Landau, Jost, Schwinger and others it was the critical analysis of new ideas which kept particle theory on a good track. Although controversies became sometimes abrasive for the persons directly involved, particle physics profited from them. Since Jost's criticism [9] of the S-matrix bootstrap idea in the 60s, there has not been any profound critical essay about the ideas leading from

S-matrix theory to string theory[6]. Nor was string theory itself subjected to critical evaluation about its conceptual-mathematical structure. Those prestigious physicists, who in previous times would have considered as their privilege, if not moral duty, to give a critical account, became string theories fiercest defenders, if not to say its propagandists.

For a historical and foundational interested researcher with textbook knowledge of QFT, the 40 year lasting dominance of this theory is surrounded by a nearly impenetrable mathematical conceptual cordon which makes it difficult to extract relevant foundational aspects. The present article can not change a situation which has been going on for 40 years and in this way became immunized against conceptual objections [7], but it does present some unknown facts which may become useful in a not so far future, when historians and philosophers finally become curious about what really went on in particle physics for almost half a century and in particular what happened to all those noisy promises of a TOE.

The content of the various sections is as follows. The next section explains the formal aspects of the crossing property. It contains in addition to mathematical facts also philosophical aspects. The third section presents the dual resonance model and explains why the absence of a critical evaluation of this interesting model prepared the ground which led into the metaphoric landscape on which string theory flourishes. The fourth section presents the arguments that string theory is, despite its name, not about objects which have a string-like spacetime localization; rather shows an "invisible" string of which only one point is visible (as some string theorists tried to reconcile the pointlike result of their calculation with their string metaphor) is nothing else than a *"dynamical" infinite component pointlike field* [8]; this section therefore constitutes the core of the critical part of the presentation. Section 5 and 6 present the modern view of the crossing property which to a certain extent explains why it led to so many misunderstandings and metaphoric ideas. Despite the highly mathematical level of these sections, the presentation of the mathematical state of art on crossing is not the principle motivation. But a critical exposition of ideas which historically emanated from an incompletely or even incorrectly understood crossing property would itself be incomplete without giving the modern viewpoint on this subtle property. The conclusions present a resumé and additional critical remarks.

2 The crossing property and the S-matrix bootstrap approach

In contrast to QM where particles play the role of stable quanta which keep their identity in the presence of interactions, QFT comes with a much more fleeting particle concept. Even in theories without interactions, where relativistic particles are synonymous with

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[6]By this I mean primarily an inner theoretical critical discourse clarifying the conceptual position with respect to the principles underlying previous successful theories.

[7]In the words of Feynman: "string theory has no arguments instead it uses excuses",

[8]The "dynamical" has been added in order to distinguish the intended meaning from the trivial case of an infinite direct sum of irreducible representations. In addition to such an infinite mass/spin tower there are also intertwiners between these representations without which one cannot generate a mass/spin spectrum.

free fields, composite operators as e.g. the important conserved currents exhibit the phenomenon of (finite) vacuum polarization, which makes such an object rather singular (an operator-valued distribution with no equal time restriction) and renders the definition of a *partial charge* corresponding to a finite volume a delicate problem with the help of which Heisenberg [10] discovered the property of vacuum polarization at the beginning of QFT.

The full subtlety of this problem only became manifest in the presence of interactions; this is the situation in which Furry and Oppenheimer [11] observed that even the basic Lagrangian fields, which without interactions were linear in the particle creation/annihilation operators, cannot create one-particle states without an admixed infinite cloud[9] of particle/antiparticle pairs. Re-interpreted in a modern setting, this observation permits the following generalization: *in an interacting QFT there exists no operator localized in a compact spacetime region which, if applied to the vacuum, creates a one-particle state without an infinite vacuum polarization cloud.* Or using recent terminology: a model which contains among its operators a compactly localized PFG (vacuum-**p**olarization-**f**ree **g**enerator) is generated by a free field [12][10]. The "shape" of the locally generated vacuum polarization cloud depends on the kind of interaction, but its infinite particle content is a characteristic property shared by all interacting theories; a finite number of particle-antiparticle polarization pairs created by "banging" on the vacuum with a local (composite) operator can only happen in a free theory. The sharpness of the localization boundary (horizon) accounts for the unboundedness of the energy content.

The subtlety of the particle/field problem (not to be confused with the particle/wave dualism of QM) was confirmed in the discovery of perturbative renormalization and the time-dependent scattering theory[11]. The main conceptual message was that in interacting QFT the notion of particles at finite spacetime had no intrinsic covariant (reference system-independent) meaning. Only the asymptotic particle states are intrinsic and unique, whereas the fields (basic or composites within the chosen description) form an infinite set of objects whose physical nature is somewhat fleeting since observationally they carry a large amount of redundancy in that infinitely many different fields lead to the same asymptotic particle and scattering amplitudes. The situation resembles the use of coordinates in geometry; the redundancy inherent in the use of different coordinate systems corresponds to the use of different field coordinatizations generating the same system of local operator algebras which correspond to the intrinsic (coordinate-free) way of doing geometry.

This view is reflected in the terminology of the 50s when fields were referred to as "interpolating" fields, thus highlighting that they should be considered as mediators of events involving particles. In fact the algebraic approach, which started shortly after the LSZ scattering theory, had as its main aim the establishment of a setting in which

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[9]In the sequel "cloud" is intended to automatically imply an infinite number of particles.

[10]The theorem is the algebraic version of the Jost-Schroer theorem, see [13]. The latter shows that the existence of a local covariant field which acts on the vacuum as PFG implies that it is a free field whereas the former replaces the pointlike covariance with the affiliation to a compact localized algebra.

[11]The elegant formulation leading to the well-known useful expressions in terms of correlation functions are due to Lehmann, Symanzik and Zimmermann (LSZ formalism) whereas the proof of the asymptotic convergence towards free fields was supplied by Haag and Ruelle [14].

the infinite plurality of fields is encoded into the infinite ways of coordinatizing a *unique* system of spacetime-localized algebras. In this way the setting of a spacetime-indexed net of operator algebras represents a compromise between an extreme on mass shell particle/S-matrix point of view and a formulation in terms of the infinitely many ways of generating a unique net of spacetime indexed algebras using covariant pointlike fields.

This particle-field problem has again become a controversially debated issue in the setting of QFT in curved space time (CST) [15] when the Poincaré symmetry including the notion of the vacuum and particle states is lost. There are many results of QFT which are consistent with the Lagrangian quantization setting (with which QFT is often incorrectly identified), but which cannot be derived by textbook Lagrangian methods but rather require operator algebraic methods. In this case it may be helpful for the reader to replace the standard terminology QFT by local quantum physics (LQP). The main difference is methodological and consists in the use of field-coordinatization independent algebraic methods wherever this is possible.

There exists an important area of QFT for which up to this day the use of pointlike covariant field coordinates cannot be completely avoided namely *renormalized perturbation theory*. But even there the causal perturbation theory a la Epstein-Glaser [16] in terms of an iterated lowest order input in the form of an invariant polynomial pointlike coupling between free fields contains some of the LQP spirit. The coupling of free fields to invariant interaction polynomals has hardly any direct relation to Lagrangian quantization[12]. The method is based on the iterative application of the causality and spectral principles of QFT; it does not follow the quantum mechanical logic of defining formal operator as e.g. Hamiltonians via momentum space cutoffs as unbounded non-covariant operators whose cutoff dependence must then be removed in order to be formally consistent with the principles. But even when the E-G formalism would reach its limit in the infrared divergencies in the perturbation theory of nonabelian gauge couplings there is still the possibility of a saving grace by separating the issue of states from operators and operator algebras and in this way arrive at an infrared finite local algebraic structure and leave the infrared problems to the construction of states [17]. Such a operator-state dichotomy is impossible in the Lagrangian or functional integral formulation.

There was however one seemingly mysterious property in the particle-field relation which, even using the advanced conceptional tool box of LQP, did not reveal its mystery. This is the *crossing property* (often called misleadingly "crossing symmetry"). Only recently this property has lifted some of its secrets (see last two sections). Since this property and other ideas which resulted from it constitute the central subject of the present essay, a clear definition is paramount. Fortunately this is not difficult since the problem is not in its presentation, but rather its connection with the principles of QFT.

Its formal aspects in Feynman's perturbative setting was obtained by combining two observations: the invariance of certain families of subgraphs in the same perturbative order under the conjugate interchange of incoming with outgoing lines (the graphical crossing), and the less trivial mass shell *projection* of the connecting analytical path resulting in an analytic relation *onto the complex mass shell* between amplitudes describing two different scattering processes. It is this second step of demonstrating the existence of an analytic

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[12]The covariantization of Wigner's unique representation theoretical classification leads to infinitely many spinorial fields (appendix), but most of them do not result from an Euler-Lagrange principle.

path on the complex mass shell linking the backward mass shell momentum accompanied by the interchange *in* ⟷ *out* and *particle* ⟷ *antiparticle* which (even in the setting of renormalized perturbation theory) remains somewhat nontrivial.

According to the LSZ scattering theory collision amplitudes can be obtained from formfactors, hence it is natural to formulate the crossing identity first in this context. A *formfactor* is a matrix elements of a field between "bra" states, consisting of say n-k outgoing particles, and k incoming particles in a "ket" state. Taking the simplest case of a scalar field $A(x)$ between spinless states of one species it reads

$$
\begin{aligned}
&{}^{out}\langle p_{k+1}, ...p_n \,|A(0)|\, p_1..., p_{k-1}, p_k\rangle^{in} \\
&= {}^{out}\langle -p_k^c, p_{k+1}, ...p_n \,|A(0)|\, p_1..., p_{k-1}\rangle^{in}_{c.o}
\end{aligned}
\tag{1}
$$

in words: the incoming momentum p_k is "crossed" into the outgoing $-p_k^c$, where the c over the momentum indicates that the particle has been crossed into its antiparticle. The subscript $c.o$ (contractions omitted) indicated that contraction terms of p_k and the other $p's$ (inner products) which are absent in the uncrossed configuration must be excluded after the crossing. Since their structure is different from the uncontracted leading terms, they can be easily separated from the main term. This notational complication can be avoided if one formulates the crossing relation in terms of free incoming/outgoing fields instead of particles (section 5).

The relation (1) would be physically void if it would not come with an assertion of analyticity which connects the unphysical backward mass shell momentum with its physical counterpart. The (still unphysical) crossing identity (1) together with the analyticity which connects backward to forward momenta constitute the crossing property. Their proof is provided by modular localization which will be the central issue of this essay.

The iterative application of the crossing relations permits to compute general formfactors from the vacuum polarization components of $A(x)$

$$
\langle 0\,|A(0)|\, p_1, p_2, ..., p_n\rangle^{in} = {}^{out}\langle -p_{k+1}^c, ... -p_n^c \,|A(0)|\, p_1..., p_{k-1}, p_k\rangle^{in}_{c.o}
\tag{2}
$$

where the charge conservation forces particles to be crossed into antiparticles. Only the vacuum polarization matrixelement does not need the subscript $c.o$ since contraction terms occur solely between bra and ket momenta. The identity only holds for unphysical momenta, but by analytic continuation one can get to any formfactor with the same total number of particles starting from the vacuum polarization component: all formfactors with the same total particle number are determined by the vacuum polarization components of a local "bang" on the vacuum $A\Omega$.

The S-matrix elements result from the formfactors by choosing for $A(x)$ the unit operator[13]; since the latter cannot absorb energy-momentum, the incoming momenta are bound to the outgoing by the energy-momentum conserving delta function which leads to some peculiarities. The analyticity in the momentum space representation can only be valid for the the function which remains after extracting the delta function. Hence by crossing on particle it is not possible to return to a physical scattering process. One

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[13]This could be achieved by cluster factorization of $A(x)$, assuming that A has a nontrivial vacuum component.

needs to cross simultaneously an incoming and an outgoing particle in order to preserve the energy-momentum delta function for physical momenta. This is particularly obvious if the crossing starts from a two-particle state so that a crossing only one particle will not lead to a physical process. The formfactor of the identity operator with the vacuum or with the one particle state on one side is trivial. In order to come to a relation whose analytic continuation has a nontrivial relation to elastic 2-particle scattering one must simultaneously cross a particle from the opposite side i.e. cross a pair in exactly the way in which crossing was first observed for two particle scattering in the setting of Feynman graphs. We will return to this case in section 5.

Crossing looks as being closely related to TCP. Although both properties are connected to localization in QFT, the derivation of crossing turns out to be much more subtle than that of the TCP theorem.

The main conceptual role of crossing is that it relates the various n-particle matrixelements of a local operator, which belong to different distributions of n particle momenta into incoming ket and outgoing bra states of an analytic *master function*. This is of course much more than the tautological statement that these matrix elements can be computed once a concrete model has been selected; it really means that once one process has been computed, the others are uniquely determined in a model independent way without doing another QFT computation.

Since this essay also addresses readers with interests in philosophical aspects, the occasional use of metaphoric arguments as a rapid vehicle to convey a mathematically difficult property which place LQP into sharp contrast with QM (even in its relativistic form [18][19][14]) should not cause problems. In any case this will be limited to solved problems whose mathematical presentation can be found in the existing literature.

The crossing properties of formfactors point at the most important consequence of causal localization in the presence of interactions: the ability to couple all particle channels with the same superselected quantum numbers with each other in particular the non-orthogonality of corresponding localized states. In the case of formfactors the analytic properties of crossing prevent that there are special matrix elements which vanish leading to the absence of certain processes. Crossing is a special illustration of a general property of LQP which often is expressed in an intuitive way as a kind of benign form of "Murphy's law": *particle states which (by charge superselection rules) are allowed to communicate (via formfactors), actually do communicate i.e. their coupling cannot be prevented it rather constitutes a structural property of any QFT.* It is this property which is behind the interaction-induced (infinite) vacuum polarization clouds resulting from "banging" with a local operator A on the vacuum; and it is certainly less metaphoric than the standard textbook presentation of the vacuum as a "broiling soup of virtual particles" which is allowed to violate the energy-momentum conservation for short times thanks to the uncertainty relation.

A special but important case of Murphy's law governing the coupling of channels is the principle of *nuclear democracy.* It states that QFT cannot distinguish between elementary and bound particles, the only hierarchy consistent with nuclear democracy is the one

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[14]The "direct particle interaction" (DPI) [18] is a relativistic particle setting which fulfills all properties of relativistic particles which one can formulate in terms of particles only including macro-causality (cluster factorization). Crossing can however not be implemented in such a setting.

between basic and fused *charges.* This means in particular that it is consistent to view any particle as the result of a fusion of a cluster of other particles whose fused joint charge is contained in the reduction of the fused charge spectrum of the cluster under consideration. Nuclear democracy is certainly a principle which contradicts the boundstate hierarchy of QM in a very radical way; if even the charge-carrying "elementary" particle can be interpreted as resulting from the collective fusion of its own charge with that of a local cluster of suitably chosen other local charges, then the strict hierarchy between elementary (fundamental) and composite certainly breaks down. Hence regarding the formation of "bound particles" i.e. eigenstates of the mass operator with a fused charge, the situation is *radically different from that in QM* because there is nothing which will prevent this particle from coupling in a formfactor to all other states which the superselection rules permit. The crossing property reduces the validity of Murphy's theorem and the resulting principle of nuclear democracy for formfactors to the phenomenon of vacuum polarization where there are theorems showing that no vacuum polarization component can vanish in a theory with nontrivial interaction[15].

Let us now sketch the ideas which was used in the original proof of crossing [2]. The elastic 2-particle amplitude is a function of the 3 Mandelstam variables s,t,u which are not independent but obey the relation $s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$. There are 3 physical processes (and their TCP conjugates) which can be reached if one knows the amplitude as a function of the full range of the Mandelstam variables s, t. Bros, Epstein and Glaser started from the LSZ representation in terms of Fourier transforms of time ordered functions and used known analytic properties of the latter in order to show that the physical region in terms of Mandelstam variables s>0, t<0 is connected with the two other possible physical regions by an analytic path. The proof is somewhat involved because it is not the primitive analyticity domain of the starting correlation function but rather its holomorphy envelop which leads to the desired result.

These papers are an illustration of the profound mathematical knowledge which physicists acquired in the pursuit of structural problems in QFT of the 60s. Although the proof of crossing and later generalizations only addressed special cases of scattering amplitudes and no formfactor was done in an entirely correct way, it did not reveal the physical context.

Only decades later it became clear that localization in QFT (restriction of the vacuum state to the local subalgebra) converts the vacuum state to a thermal KMS state

$$\Omega_{vac} \restriction \mathcal{A}(\mathcal{O}) \ is \ \Omega_{KMS} \tag{3}$$

where the Hamiltonian is canonically determined in terms of $(A(O), \Omega)$. The mathematical theory behind this is *modular theory.* This theory exists in two interconnected versions, the operator algebraic Tomita-Takesaki theory (of which important physical aspects were discovered independently by Haag, Hugenholtz and Winnink [14]) and the modular localization of relativistic wave functions and states [20][21]. These ideas led to a closer connection of the thermal aspects of event horizons in QFT in CST with thermal aspects caused by restricting the vacuum state of global QFTs to localized algebras. A much

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[15]The strongest result is a forthcoming theorem by Jens Mund (private communication) which generalizes the old Jost-Schroer theorem (see [13]).

discussed case is the restriction of the vacuum to the wedge-localized algebra $\mathcal{A}(W)$ as a wedge algebra $A(W)$ which leads to the Unruh effect and an interesting formula for the entropy near the horizon $H(W)$ (the entropy of a light-sheet [49]).

Two additional facts finally led to the somewhat surprising result that the crossing relation belongs to those phenomena which are related to thermal aspects of localization. The first was the observation that at least formally the KMS relation written for formfactors of free fields. For free fields and their composites restricted to a wedge region (with the test functions having support in W) one has[16]

$$\langle A(f_{l+1})..A(f_n)C(h)A(f_l)..A(f_1)\rangle = \langle A(f_1)\Delta A(f_{l+1})..A(f_n)C(h)A(f_l)..A(f_2)\rangle \quad (4)$$
$$\langle p_n..p_{l+1}|C(h)|p_l..p_1\rangle = \langle p_n..p_{l+1}, -p_1|C(h)|p_l..p_2\rangle_{c.o}$$

Here $C(h)$ is a h-smeared composite of a free field. For the validity the KMS relation with respect to the modular Hamiltonian $\Delta = e^{-2\pi K}$ with K the Lorentz boost the smearing functions must be localized in W. Since the mass-shell restriction of wedge-localized smearing functions form a dense set of wave functions, the momentum space relation in the second line is a consequence. The negative sign of the first momentum is a result of the analytic continuation implied by the imaginary 2π Lorentz rotation together with the Hermitian adjoint from passing from ket to bra states (7); for obvious reasons the backward mass shell momenta are referring to particles with the opposite charge i.e. "anti" with respect to the original one before the cyclic permutation. To obtain the particle states from field states one must Wick-order the A-field states on the left hand side and remember that the cyclic permuted $A(f_1)$ has no contractions with the fields on the right from where it was coming. In the transcription of this relation to particles the absence of left backward momentum states with right forwards ones is indicated by *c.o* (contractions omitted).

Hence the crossing relation in the interaction-free case is noting else than the thermal KMS relation of wedge localization (featuring in the Unruh effect) rewritten as a relation between particle matrix-elements (formfactor). In section 5 it will be shown that the interacting case is the particle transcription of a *new modular theory-based field relation which extends the KMS relation.*

The correlation functions have analyticity properties in the Lorentz boost parameter, they are analytic in the multi-strip [48]

$$0 \le \tau_1 \le \tau_2 \le .. \le \tau_l \le \tau \le \tau_{l+1}.. \le \tau_n \le 1 \quad (5)$$
$$A(f_i) \to e^{2\pi\tau_i K}A(f_i)e^{-2\pi\tau_i K}, \ C(h) \to e^{2\pi\tau K}A(f_i)e^{-2\pi\tau K}$$

The support properties in x space of wave functions are equivalent to analyticity properties. In particular they imply that certain complex Lorentz transformations which act on the Fourier transformed operators can be absorbed in the analytic continuation of test functions and vice versa.

Looking only at the contribution of (4) without contrations among the free fields and using the density of Fourier transformed wedge-supported smearing functions on-

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[16]The reader is asked to pay attention to the changes between the expectation value to the state notation.

massshell, one obtains the crossing relation for the free formfactor

$$\langle p_n..p_{l+1} \left| C(0) \right| p_l..p_1 \rangle = a.c._{q^c \to -p_1^c} \langle q, p_n..p_{l+1} \left| C(0) \right| p_l..p_1 \rangle_{c.o} \qquad (6)$$

where the subscript $c.o$ has the same meaning as before[17]. This identity between a particle matrixelement of C and an a crossed formfactor at an analytically continued momentum; (the notation $-p^c$ instead of simply $-p$ indicates that the momentum on the backward shell is that of an antiparticle to what it was on the ket side. The only somewhat tricky part of rewriting the KMS relation (4) into the crossing form (6) is taking the operator $A(f_1)\Delta$ as its conjugate to the bra vacuum and using modular theory to bring the resulting bra state into the desired form

$$\Delta A(f_1)^*\Omega = \Delta S A(f_1)\Omega = \Delta^{\frac{1}{2}} J A(f_1)\Omega = A^c(\check{f}_1)\Omega = \int \frac{d^3p}{2p_0} \left| -p^c \right\rangle \bar{f}(p) \qquad (7)$$

$$S = J\Delta^{\frac{1}{2}}, \ SA\Omega = A^*\Omega, \ A \in \mathcal{A}(W), \quad \check{f}(p) = \bar{f}(-p)$$

More details can be looked up in section V.4 of [14][18]. The application of the unbounded modular operators $\Delta^{\frac{1}{2}} = e^{-\pi K}, K = W$-associated Lorentz boost generator requires precisely that analytic continuability which is guarantied by the wedge localization. With respect to analyticity there is no difference between the KMS setting and its two-algebra generalization needed for the derivation of crossing in section 5.

Instead of invoking modular theory, the free field relation (6) can also be checked by explicit computation, but this privilege does not exist in the presence of interactions.

There is in fact a serious obstacle against applying this argument to interacting formfactors in order to establish the identity (1). The reason is obvious since there are 3 different algebras involved $A_{in}(W), A_{out}(W), A(W)$ and the modular operators of interacting operator algebras are different are different from those generated by their asymptotic free fields. But there is a fortunate circumstance which comes to one's rescue: at least the domains of the unbounded Tomita S operators S_{in}, S_{out}, S are identical i.e. the $\Delta's$ coalesce and hence the dense subspace of localized states are the same. The consequences of the identity of the domains are the subtle ingredients in the proof of crossing. We will return to this problem in section 5 and 6 and show that this suffices in order to derive crossing in the formfactor- as well as in the scattering- form.

It is interesting to compare the old derivation [2][3] which uses holomorphy properties of correlation functions in several variables, including the sophisticated tool of computing holomorphy envelopes (cutting of "noses"), with the present one. The wedge localization approach is quite different, even though both rely on analyticity properties coming from locality. Its analytic underpinning is that of Araki's KMS analyticity of correlation functions.

The modular approach is more economical in the sense that only the analyticity which is really necessary for the derivation of crossing is used and analytic completion techniques whose physical interpretation is not known. In this way the important role of crossing in

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[17]Instead of omitting certain contration terms one might as well use the unmodified formfactor and subtract terms of the form $c.t. = \sum_{r=l+1} \delta(p_1^c - p_r) \cdot lower \ formfactors$

[18]Especially recommended to philosophically motivated readers who prefer conceptual clarity over mathematical rigor.

the construction of factorizing models becomes clearer [21][22]. Finally crossing becomes part of a structural problem of wedge algebras whose thermal manifestations are important in the Unruh effect associated with a wedge and its causal horizon as well as in thermal aspects related to event horizons, including vacuum polarization induced entropy near null-horizons [23]. This connection between properties from the center of particle theory with properties which at least historically come from black hole physics is the real surprise.

At the time of the Bros-Epstein-Glaser work on crossing some quantum field theorists pinned high hopes on the use of new analytic methods for functions of several complex variables for a nonperturbative understanding of QFT. Källén and Wightman [24] tried for many years to construct a representation of the 3-point function which fulfilled all linear requirements of QFT. They never reached their goal, and this kind of technique fell out of favor. Whether is returns one day together with different problems, who knows?

3 The dual resonance model, superseded phenomenology or progenitor of a new fundamental theory?

The history of the crossing property starting in the early 60s is the key for understanding the direction into which a good part of particle physics research developed afterwards. It began by more or less accidentally stumbling across a property whose importance in particular for an S-matrix based approach to particle physics was apparent, but whose foundational aspects remained hidden. The necessary conceptual and mathematical tools for its understanding only appeared at the end of the century (appendix and sections 5,6).

Direct numerical attempts to find approximate solutions of the extreme nonlinear properties resulting from the S-matrix bootstrap "axioms" ended in failure and only strengthened the misleading belief of the existence of a unique non-Lagrangian theory of strong interactions. This was neither the first nor the last time that an ultra reductionist "theories of everything" (TOE) entered the particle theory discourse.

As mentioned before, after the completion of the dispersion theory project the underlying philosophy of research began to change. The new strategy was most clearly formulated by Mandelstam. In analogy to the rigorously established Jost-Lehmann-Dyson spectral constructions for matrixelements of field commutators [25] (generalizations of the simpler Källen-Lehmann representation for the two-point function) which became a seminal tool in the derivation of the dispersion relations, Mandelstam proposed an spectral representation for the two-particle scattering amplitude [8] in the hope that the crossing property may be simpler accessed in terms of spectral functions. This representation was never proven and the hope did not materialize, but taken together with ideas about the use of Regge pole trajectories in strong interaction phenomenology it led Veneziano to the mathematical construction of the dual resonance model for elastic two-particle scattering[19] [8] which was later generalized to an arbitrary number of particles.

In terms of Feynman graph terminology it represented the tree approximation for a process of two incoming particles which couple via trilinear interaction vertices to an infinite tower of intermediate particles with ever increasing masses and spins. The decrease

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[19]The added "resonance" expressed the wish to unitarize the model so that it could pass as an S-matrix Ansatz.

of the coupling strengths is carefully tuned in such a way that the sum of all these contributions from the infinite mass/spin tower of the interaction mediating particle poles not only converge in the s-channel (using the canonical terminology introduced by Mandelstam), but represents a function which allows a t-channel interpretation in terms of another sum of infinitely many exchanges via particles from the same mass/spin tower. To find such function in a pedestrian manner without an operational backup just by using known properties of gamma and beta functions, is an astonishing achievement which even nowadays commands respect [8].

In hindsight it is somewhat surprising that it was not realized that the dual model was the first nontrivial realization of an object which less than one decade earlier was looked for under the label *infinite component fields*. The motivation came from a completely different corner namely from the analogy to the $O(4, 2)$ "dynamic symmetry" of the hydrogen atom. Infinite component fields in the sense of Fronsdal, Barut, Kleinert and other authors [25] were not just infinitely many fields of varying mass and spin put together as a direct sum, but there was a "dynamic" content consisting in the existence of operators which "vertically" communicate between the different tower levels and set the mass/spin spectrum. This dynamic aspect was expected to arise from noncompact group representations which extend those of the Lorentz group, but this hope did not materialize and the cited authors remained empty handed. This dynamic requirement makes the construction of an infinite component field problem a very difficult problem. In fact up to date the 10-dimensional superstring field has remained the only infinite component pointlike solution.

String theory owes it success as an infinite component field theory only 6 years after the ill-fated infinite component program to the replacement of higher noncompact groups by the infinite degrees of freedom inherent in multicomponent chiral conformal currents.

It is one of the missed chances of history that even though the followers of the infinite component field program and the dual model community (which later became incorporated into the string community) had both strong phenomenological roots, they never noticed the proximity of their ideas. It certainly would have been very interesting to be informed that the duality requirement imposed on the vertices of a pole approximation for a scattering amplitude can be encoded into an infinite component field and operators which intertwine between the levels of the infinite mass/spin tower and there is no spacetime string which can be associated with this situation. And with a little bit of help from the infinite component camp the fateful step into misreading string theory as having something to do with string-like objects could have been avoided.

The duality idea arose from consistency arguments between the low energy resonance contributions and the expected high energy Regge behavior. Veneziano's first implementation led to several generalizations [8]. The formulation which is most suitable for an in-depth critical analysis is the operational setting of Fubini et al. [26] which uses multi-component conformal currents and their potential.

It may be helpful for the reader to recall at this point some results about conformal currents [8]. The simplest situation is that of a one component current which, similar to

a free field, is determined by its commutation relation

$$[j(x), j(y)] = -\delta'(x - y) \tag{8}$$

$$Q = \int j(x)dx, \quad \psi(x) = \text{"}e^{i\alpha\Phi(x)}\text{"}, \quad \Phi(x) = \int_{-\infty}^{x} j(x)dx \tag{9}$$

Despite its simplicity it leads to a very rich representation theory. There are continuously many representations (labeled by α) as a consequence of the continuous spectrum of the charge Q [23]. Formally such charged fields are written as exponentials of potentials i.e. half space integrals over the current. The quotation marks are meant to indicate that such formulas are conceptually not quite correct since the charge α carrying field ψ does not live in the vacuum sector as the naive reading of this formula would indicate[20]. This observation is inexorably linked with the infrared divergence of the integral representation which is the way in which the exponential announces that it is not a singular operator like the others in the Hilbert space generated by the currents. Unfortunately the extended algebra which incorporates *all* charge-carrying fields lives in an inseparable Hilbert space.

In order to use currents as a two-dimensional theoretical laboratory following the intrinsic logic of QFT, Buchholz, Mack and Todorov [23] introduced the concept of *maximal local extension* of the algebra of currents. The extension is done by adding certain fields of the form $\psi_\alpha(x)$ whose dimension $d_\alpha \sim \alpha^2$ is integer (and hence which for different localization points commute among each other) to the algebra of currents and view the resulting larger bosonic algebra as the new extended observable algebra. This reduces the number of charge sectors in a drastic way, their number is not only countable but even finite ("rational chiral theories"). It turns out that the denumerable set of maximal extension can be explicitly constructed. These do not commute among themselves or with each other but rather obey (abelian) braid group commutation relation.

The multi-component generalization of the representation theory of a current turned out to lead to a theory of remarkable richness [27][29]. In this case the maximal extensions are classified by *even lattices* L in $\mathbb{R}^n$, $L : (\alpha, \beta) = 2\mathbb{Z}$. The sectors are then classified by equivalence classes of the dual lattice L^*/L of which there exist finitely many. The cases with $L = L^*$ are particularly interesting. These constitute a finite number of models which only exist in their vacuum representation. They are related to finite exceptional groups among them the famous "moonshine model".

Besides this use of multicomponent current models following the intrinsic logic of LQP, these currents have also been used in an operational approach to the dual model in the work of Fubini at al. [26] which is somewhat different from the field theoretic logic. Their interest is in the direct use of the potentials Φ_i of the multi-component currents as some quantum mechanical objects

$$\Phi_i(x) = \int_{-\infty}^{x} j_i(x)dx \rightarrow X_i(z), \; i = 1....d \tag{10}$$

$$Q_i \rightarrow P_i, \; \alpha_i \rightarrow p_i, \; V(z, p) = e^{iP \cdot X(z)}$$

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[20]If one uses such formulas outside of the theory of superselected charges one must add the charge conservation by hand; only then does one obtain a Wightman theory in a Hilbert space.

this symbolic formulas are in need of some detailed explanation. The first line indicates a passage from the noncompact to the compact picture $x \to z$ and the notation $X_i(z)$ anticipates that the potentials are now going to be interpreted as quantum coordinates which classically would trace out a curve in a d-dimensional spacetime. The second line expresses the fact that one really wants to take this reinterpretation into a different direction by adding the identification of the d-component charge operator with the momentum operator and writing the charge-carrying exponential of an would-be n-component potential "spacetime" as a vertex (or in more recent generalizations *a chiral sigma field*) operator V which carries a noncompact spacetime symmetry (which from the chiral conformal viewpoint of the source theory would be called an inner symmetry). This is the famous source-target relation which later led to the notion of world sheets. But is this strange interpretation of multicomponent charge values as momenta with the operator dimensions of the charge-carrying operators passing to particle masses and the current potential Φ_i becoming a kind of position operator in an multicomponent internal symmetry space which by some magic defines the new "target" spacetime in a source-target relation in which the potential defines an embedding of the conformal light-line into spacetime? This picture would suggest that the conformal current theory defines an embedding of a line in spacetime which is the origin of the worldsheets (in analogy to Feynman's worldlines) in the duals model and its string theoretic extension. Admittedly the identification of an internal symmetry space with noncompact physical spacetime is one of the strangest ideas which entered particle physics [21], but is it consistent? We will show in the following that the worldsheet picture is incorrect and that instead the localization is as in standard QFT pointlike which leads to worldlines.

Fact is that one cannot embed a lower dimensional QFT into a higher dimensional, one can only restrict a higher dimensional QFT to lower dimensional part of spacetime. But physically this is (apart from some special cases) anyhow not a very useful procedure because the lower dimensional theory obtained in this way will inevitably have too many phasespace degrees of freedom for being a physical QFT in that lower dimension. The only exception to this rule is the holographic projection onto a null-surface resulting from a causal or event horizon.

In order to discuss problems of unitarity of Poincaré representations on inner symmetry space of chiral theories it is inconvenient to use the dual model setting. The reason is that even in case of the 10 dimensional superstring for which Hilbert space- as well as energy-positivity can be satisfied, the supersymmetric unitary representation is only obtained after passing to a subspace and dividing out zero norm-states. This blurs the picture of the target spacetime resulting from an inner symmetry of a multicomponent potential of a current and the presentation in terms of the bilinearized Nambu-Goto Lagrangian (see next section) is more convenient.

As indicated before one replaces the superselected charges by superposable momenta and the potentials by operators $X_i(z)$ whose lowest Fourier mode (which includes a logarithmic contribution) $X_i(z)_0 = x_i^{op} + icp_i^{op} \ln z$ defines quantum mechanical x^{op}, p^{op} operators[22]. In this way the inseparable Hilbert space which describes charged representations

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[21]In higher dimensions it has been shown from first principles that all inner symmetries are described by compact groups [28].

[22]The appearance of the logaithmic term is a mark of the formal infrared divergence of the potentials

for a continuum of charges is avoided and the continuous direct sum becomes a quantum mechanical direct integral in the sense of spectral decomposition theory. Although the presence of these quantum mechanical degrees of freedom prevent the conformal covariance of the zero dimensional $X_i(z)$ field, there is no problem with the covariance of the exponential vertex operators which carry an anomalous dimension proportional the square of charges which in the new reading corresponds to the square of momenta i.e. of masses

$$d_\psi \sim \alpha \cdot \alpha, \ d_V \sim p \cdot p = m^2 \tag{11}$$

So in the Fubini et al. formalism[26] Veneziano's rather involved gamma function setting is replaced by a formalism using the conformal invariant part (the part which depends only on the anharmonic ratios) of the 4-point function of the vertex operator. The higher point function dual model amplitude results from the invariant part of the higher correlations; in this way one arrives at a dual model representation for $n \to m$ particle scattering.

It is hard to criticize a proposal which is phenomenological in nature, apart from expressing some unease about putting together raw phenomenology ideas (which were later contradicted by new experiments) with subtle mathematical concepts which already have a different very precise conceptual position. It is probably the attractive mathematical aspect which explains why this proposal did not disappear completely together with the Regge phenomenology when the latter came to an end. Being a somewhat too ambitious setting for a mere phenomenological description, the theory had its later comeback in the form of string theory; but whereas its mathematical entitlement was natural, the same cannot be said about its physical interpretation. It finally became acclaimed as the millennium TOE which, different from the S-matrix bootstrap, allegedly also includes gravity.

The conceptual distinction resulting from the of apparent uniqueness of mathematically ambitious projects as the implementation of the highly nonlinear duality structure has often mislead people[23]. In the beginning there was only Veneziano's version of the dual model which was constructed by a clever use of properties of gamma functions. But now we know that there are myriads of functions of the Mandelstam variables s_{ij} which are meromorphic with an infinite tower of particle poles in the position of duality. They are constructed by starting from any conformal theory in any spacetime dimension. As explained in detail in a beautiful paper of Mack [30], one only has to write the connected part of a conformal n-point function as a Mellin transform M

$$G^c(x_1, ...x_n) = \left(\frac{1}{2\pi i}\right)^{n/2} \int .. \int d\delta M^c(\{\delta_{ij}\}) \prod_{ij} \Gamma(\delta_{ij}) \left(-\frac{1}{2}x_{ij}\right)^{-\delta_{ij}} \tag{12}$$

There are as many integration variables as there are independent conformal invariant anharmonic ratios. The aim is to show that by identifying the operator dimensions of the conformal fields with the masses of particles and the Mellin variables δ_{ij} to the Mandelstam variables s_{ij} one obtains a meromorphic Mellin transform which has the correct poles as

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which by themselves (outside their exponential form) are not conformal fields. The label op distinguishes quantum mechanical operators from the numerical momentua (alias chage values)

[23]The nonlinear S-matrix bootstrap and the Schwinger-Dyson illustrate such misconceptions.

required by the duality property. The reduced Mellin transform M^c can be defined in such a way that the spacetime dimensionality does not enter[24] i.e. one can obtain dual models in a fixed spacetime dimension from conformal theories in any dimension, not only from chiral conformal theories. The systematic construction of dual model amplitudes via conformal QFTs has nothing to do with the physical picture of a one-particle saturation (resonance approximation) of the conjectured Mandelstam representation.

The convergence of the infinite sums over poles as well as certain positivity properties of the associated residues follow from the established validity of global operator expansions in conformal theories [30]. At this level, there is however no claim that the Mandelstam variables are related to momenta on which a unitary representation of the Poincaré group acts. This problem was not part of the dual model program since the only positivity requirement in the Mandelstam setting of scattering amplitudes are conditions on the correct sign of residua of poles. It however became a pressing problem after the original phenomenological purpose of the formalism was abandoned and the setting was allowed to become the driving force of a free-roaming TOE under mathematical (but practically no) conceptual control. The rallying point for this development was the observation that the only unitary positive energy representation of a Poincaré group which can act on the index space of a multi-component current and its potentials is the 10-dimensional superstring representation. In this case the Mandelstam invariants result from a unitary momentum space representation of the Poincaré group.

In the present context the Mellin formalism demystifies Veneziano's observation to some extent in that it shows that the duality structure, far from being a lucky discovery of a special way to implement (an approximated form of) the crossing property, is in reality a kinematical aspect of a certain transformation property of conformal correlation functions. Unlike the Fourier transform of correlation functions it cannot be expressed in terms of single operators but needs the entire correlation function for its definition[25]. The operator version of the Veneziano dual model [26], which starts from a chiral conformal current model, turns out to be a special case of Mack's conformal Mellin transformation formalism. But whereas in the former the momenta enter explicitly via the continuous charge spectrum, the appearance of momenta in Mack's setting is less overt; they only enter in parametrizing a relation which links the anomalous dimension of the conformal theory to the independent variable in the Mellin transform [26].

As mentioned before the existence of a unitary positive energy representation of the Poincaré group behind the Mandelstam variables is not part of the Mellin transformation formalism. The verification of its existence in d=10 (the superstring theory) is certainly an unexpected curiosity since there was no reason at the beginning to expect a chiral conformal theory to support a noncompact inner symmetry as a Lorentz group representation

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[24]The properly reduced Mellin amplitudes are independent of spacetime dimensions; this is similar (actually closely related) to the invariant part of conformal correlation which only depends on dimension-independent conformally invariant harmonic ratios.

[25]One needs the conformally invariant part of the correlation, a step which permits no operator formulation.

[26]The interpretation of the (appropriately defined) Mellin transform as a 4-dimensional dual model is idependent of the spacetime dimensionality of the associated conformal model. For the Fubini et al. [26] model it is a multi-component abelian chiral current.

start[27]. But to take such a property of a two-dimensional conformal theory as a hint of having a new understanding about spacetime is far-fetched if not a step into mysticism.

As mentioned before, the infinite component field of superstring theory in d=10 is the first and only nontrivial realization of a dynamic pointlike irreducible infinite component theory in the before explained sense [25]. The protagonists of the infinite component field idea (if some of them are still around) would perhaps notice with satisfaction that by allowing quantum mechanical oscillators to connect the levels and to generate the mass/spin spectrum one obtains the first illustration of what they had in mind; perhaps they would have been less than happy about the high spacetime dimension of this unique realization and its resulting metaphoric epiphenomenon.

A relation between masses and operator dimensions which is not related to Mellin transformation occurs in a more intrinsic physical context of the AdS-CFT correspondence. This correspondence will appear in a different context in the concluding remarks.

Whatever one wants to make out of the operator setting of the dual model or the Mellin formalism, there is certainly no intrinsic physical reason why one should re-interpret charges as momenta and inner symmetry spaces of chiral theories as spacetime arenas for physical events. And why should one follow somebody who claims that the generating objects of ST are stringlike (in blatant contradiction to the pointlike computational results) leading to worldsheets on such an incorrect metaphoric path? Why mystify the different 10 dimensional superstrings and their presumed connection via M-theory as revealing deep secrets of physical spacetime when there is the autonomous possibility of explaining these surprising properties as peculiarities of inner symmetries of chiral models which are known not to have to follow the inner symmetry pattern in terms of compact group representation of higher dimensional symmetries? Behind all this is the general question: is particle physics only interesting after, following the modern Zeitgeist, it has been sexed up or mystified?

4 String theory, a TOE or a tower of Babel within particle theory?

String theory addresses some of the questions which the dual model left open or could not handle convincingly as: can one really obtain a unitary representation of the Poincaré group on the internal symmetry space of a chiral current theory and if yes, what is the covariant localization concept in such a source-target relation and in particular does it really lead, as claimed, to a notion of world sheets? Last not least one would like to know whether the use of special exponentials of potentials (in the operator duality approach) can be replaced by a more general setting in which, similar to the Wigner approach to particles, a representation space is defined in terms of generating wave functions with clear localization properties, which are then used to pass to an (interaction-free) operator field formalism. For this purpose it has turned out to be covenient to start from a slightly

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[27]The use of inner symmetry indices of a QFT as an arena for representations of spacetime symmetries is one of the strangest proposals ever made in particle physics. Once accepted, it opened the flood gates for other metaphoric ideas as e.g. the conversion of unwanted spacetime dimensions via "compactification" into inner inner symmetries.

more general point of view which prepares the desired unitary representation theory more directly in terms of the current potentials $X^\mu(z)$.

But before going into these technicalities some general remarks are in order. There exist operator algebras and state spaces which have no pointlike but rather semiinfinite string-like generators; Wigner's massless infinite spin representation family presents the only noninteracting illustration [31] and it shows that string localization is incompatible with a Lagrangian description. In this case one may speak of world sheets being traced out in spacetime. But the generating wave function of string theory and their second quantized counterparts are pointlike generated. Originally the string world sheets were not part of the dual model of old, they appeared in a later stage when it was incorrectly claimed that the source-target relation can be understood as an embedding of the one-dimensional chiral theory as a one-dimensional submanifold into a 10-component target space representing spacetime. To support such a picture string theorists invented a Lagrangian description of relativistic particles [32]. Compared with Wigner's clear representation-theoretical classification, the functional integral representation in terms of relativistic particle mechanics falls short of a convincing attempt to support string theory; it is mathematically ill-defined[28], falls short of describing all irreducible positive energy representations, and was never used by particle physicists outside string theory who characterize particles following Wigner. Such ad hoc inventions of analogies sometimes backfire instead of lending support.

Before going into ST details, it is helpful to start with a theorem from unitary representation theory which limits the localization of states (appendix).

Theorem 1 *The causal localization (modular localization, see appendix) inherent in unitary positive energy representations of the covering of the Poincaré group is pointlike generated apart from Wigner's massless infinite spin representation whose optimally localized generators are semiinfinite spacelike strings [31].*

Some comments are in order.

Unitary positive energy representations are canonically related to free fields or (in case of reducible representations) to direct sums of free fields. One only has to show the absense of the Wigner infinite spin representation from the positive energy unitary 10-dimensional superstring representation in order to secure that it is pointlike generated.

This theorem also covers the localization in string theory, since the Lagrangian which underlies the quantum string is bilinear and hence the graded commutator must be a c-number. This Lagrangian supplies the operator formalism acting in the Hilbert space of the string wave functions. This one-string representation space is an analog of the Wigner one particle space apart from the fact that there is a severe restriction from the unitarity of the action of the Poincaré group. This is because the central issue is the quantization of a Lagrangian and the unitarity problem is an additional restriction The situation resembles vaguely that of the vector potentials in QED in that one has to form sub- and factor- spaces in order to get rid of the negative and zero norm states. But

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[28]It requires to pass through apparently unavoidable infinite intermediate steps resulting from the necessity to extract infinite factors coming from reparametrization invariance which have nothing to do with intrinsic properties of particles.

whereas in QED this idea is independent of the spacetime dimension and certainly does not effect the noninteracting theory (where it only appears if one uses potentials instead of field strengths), the origin in string theory is quite different. It can be traced back to the unmotivated (i.e. not physically justifiable) demand that one wants a *unitary representation of the covering of the Poincaré group on the internal symmetry space of a chiral current.*

Nature could have answered this extravagant requirement by providing the same negative response which has been known in higher dimensional QFT namely: any inner symmetry is necessarily described by a compact group; noncompact groups as spacetime symmetries would be in contradiction with the localization principles of LQP [14]. But surprisingly there are exceptions in chiral QFT where besides "rational" models (which are in many ways similar to the inner symmetry structure of higher dimensional models) and models with countably many superselection sectors, there are also quite different "irrational" internal symmetries. Models in which the observable algebras are defined by multicomponent abelian currents belong to the latter. They have a continuum of charged representations and there is indeed the possibility to have (in an appropriate sense) a positive energy representation of the covering of the Poincaré group on a 10 dimensional internal symmetry space of a chiral current model. But from the context in which this somewhat surprising observation arises it is clear that it has nothing to do with a new mysterious insight into foundational problems of spacetime but rather with an unexpected property of the particular chiral model (other surprising properties of maximal extended current algebras were mentioned in the previous section).

Whatever one's position is towards spacetime symmetries appearing on the inner symmetry space of chiral currents, there can be no doubt about the fact that the one string space (or the uniquely associated string string field theory) is pointlike generated. This is the unavoidable conclusion from the previously stated theorem as well as from the below mentioned concrete calculations.

At this point it is very important not to equate the localization of states with that of operators beyond the setting of free fields. Whereas only the family of massless infinite spin Wigner representations is semiinfinite string-like generated [20][31], the absence of pointlike algebraic generators in certain charged subalgebras is quite common. The best known case is that of electrically charged fields in QED [33], it is impossible to localize a charge-carrying operator in a compact spacetime region. Within massive theories the possibility of such a situation was investigated by Buchholz and Fredenhagen [34], but since in this case there would be no infrared manifestation of string localization in Lagrangian perturbation theory, there are no known illustrative models[29]. A B-F stringlike or an electrically charged field applied to the vacuum decomposes into pointlike generating wave functions, but this decomposition process has no counterpart in the local algebras.

By leaving the issue of localization in string theory to be settled as a special consequence of a powerful structural theorem in local quantum physics as above, one deprives oneself of some interesting insight into one of the most fascinating episodes in 20th cen-

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[29]The presence of zero mass photons with an infrared-strong coupling to charged particles results in a weakening in localization of the latter. The optimal (sharpest) localization of the latter is semiinfinite stringlike as described by the well-known Dirac-Jordan-Mandelstam line integral representations. Charged fields interpolate "infraparticles" instead of Wigner particles

tury particle physics namely a more detailed understanding of where did the arguments leading up to string theory fail. For this reason we will now follow this more interesting path.

The formal starting point is the bilinear Lagranian form in which the Nambu-Goto Lagrangian [35][36] is used in string theory

$$L = \iint (\partial_\tau X_\mu \partial_\tau X^\mu - \partial_\sigma X_\mu \partial_\sigma X^\mu) d\tau d\sigma \tag{13}$$
$$\left(\partial_\tau^2 - \partial_\sigma^2\right) X^\mu(z) = 0$$

In the simplest case the τ, σ dependent "zero dimensional position field" $X_\mu(\tau, \sigma)$ (the string analog of the Fubini... potential) is considered to be defined on $R \times (0, \pi)$ with appropriate (Neumann) boundary conditions. The equation of motion is a two-dim- wave equation which together with the boundary conditions leads to the Fourier representation

$$X^\mu(\tau, \sigma) = x^\mu + p^\mu \tau + i \sum_{n \neq 0} \alpha_n^\mu e^{-in\tau} \frac{\cos n\sigma}{n} \tag{14}$$

The α_n^μ are oscillator-type creation and annihilation operators which by Lorentz covariance are forced to act in an indefinite metric space. Denoting these chiral current potentials by X^μ may create the delusion that we are describing a path in target space; with the conservative notation Φ^μ such an association is less automatic.

In the present form there is yet no free go for a unitary Poincaré on target space, such a move must be more carefully prepared. Imposing subsidary conditions

$$(\partial_\sigma X \pm \partial_\tau X)^2 = 0 \tag{15}$$

does the job, after they have been adjusted to the quantum setting (valid only on states). The Klein Gordon equation on target space with a mass operator of an integral spaced spectrum is among them. These conditions express reparametrization invariance and they would have been a consequence of the Nambu-Goto Lagrangian which is a nonlinear expression in the ∂X. Clasically the true N-G Lagrangian is equivalent to its bilinearized version plus constraints. The reparametrization invariance trivializes part of the infinite dimensional conformal covariance. All these aspects are subordinated to the construction of a unitary Poincaré group representation on the appropriately defined target space of a multicomponent current potential; they do not have any intrinsic physical meaning.

Any unitary representation of the Poincaré group acts in a Hilbert space can be obtained by a two-step process from a formal covariant representation in linear negative metric covariant space of the form

$$H_{sub} \subset L^2(\mathbb{R}^n, \rho(\kappa) d\kappa d\mu(p, \kappa)) \otimes H_{QM} \tag{16}$$

where the first factor is a spinless relativistic particle representation space with a continuous mass distribution and H_{QM} contains vector-valued or spinor-valued quantum mechanical variables (as the α_n^μ) which strictly speaking are prevented by Lorentz covariance to be genuine "quantum" (acting in a Hilbert space).

In the simplest case of finite dimensional massive representations, the "quantum mechanical space" is the n-dimensional (nonunitary) vector representation space of the

Lorentz group $H_{QM} = V^{(n)}$. To get e.g. to a unitary massive $s = 1$ representation of the Poincaré group one uses Wigner's idea of the little group and obtains a *unitary p-dependent Lorentz transformation law* which results from the original non-unitary co-variant law through an *intertwiner* (a 4-component function on the forward mass shell) between the original n=4 vector representation with and its manifestly unitary form which acts covariantly on a positive metric subspace $H_{phys} = H_{sub} \subset L^2(R^n) \times V^{(n)}$.

In the N-G case at hand the selection of the mass specrum is done by imposing the Klein-Gordon equation with the mass operator, its spectrum then leads to a direct sum over equally spaced mass eigenstates including a "lowest" tachyonic contribution

$$\sum_{\kappa=-2,0,2,\dots} L^2(\mathbb{R}^n, d\mu(p,\kappa)) \otimes H_{Osc} \qquad (17)$$

And one has to still implement the complete set of subsidery conditions. For this purpose one uses the vector-valued oscillators belonging to the higher Fourier components of the current potential whose Lorentz invariant inner product is indefinite. There is no chance to find a subspace through subsidiary conditions which is positive semidefinite with one exception. Only for the multicurrent model with 26 components does one arrive at a semidefinite metric [37]. The last step is canonical, having arrived at a semidefinite situation, the positive definite situation is gratis. Details can be found in many articles [39]. The obtained 26 dimensional representation is not of positive energy as a result of the presence of a tachyon. However admitting spinorial-valued chiral current components (which would require a spinorial change of the N-G Lagrangian) one arrives at the 10 dimensional positive energy superstring representation.

The transition from unitary to covariant representations is done with the help of so called u, v intertwiners. This step is in complete analogy to what Weinberg presented in a group theoretic setting in the first volume of his well-known textbook [38]. It also can be obtained by applying modular localization to the Wigner representation theory [31] (see appendix). Although for each irreducible unitary representation there is only one wave function space, there are infinitely many different looking covariant wave functions and free fields (see appendix).

Since a unitary representation of (necessarily noncompact) spacetime symmetry group on an internal symmetry space of a current algebra is a strange requirement from a viewpoint of local quantum physics[30], it would be very natural to have received a negative answer to the target space issue. But inner symmetries in low dimensional QFT are different from their standard realization and lo and behold there is precisely one exception namely the positive energy "superstring" representation in 10 spacetime dimension.

But does the existence of this exception indicate some mysterious new insight into spacetime? Certainly not, but it does reveal some unexpected property of the potentials Φ^μ (and their charge-carrying exponentials) of multicomponent chiral currents. Actually the solution is not completely unique since there is a finite number of 10 dimensional superstrings and there exists even a conjecture (M-theory) about their possible relations.

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[30]The unresisted acceptance of identifying inner symmetries of conformal symmetries with actual space-time and its opposite of mutating spacetime dimension into inner symmetries by "rolling them up" (com-pactification) is an indicator for how much the conceptual framework of QFT principles has been lost and replaced by a collection of computational recipes.

It would be interesting to present these observations (in analogy to Mack's Mellin formalism) solely in terms of multicomponent currents and their potentials, leaving spacetime metaphors aside.

The correct reading of the string as a dynamic infinite component field in a way[31] shares the inner symmetry $\rightarrow$ spacetime symmetry reinterpretation with that of string theory. But there is less temptation to elevate the construction of a (possibly unique) dynamic infinite component field to a new foundational insight into spacetime or to interpret its near unicity as the indicating a TOE.

Every correct investigation of localization by string theorists led to the pointlike result. The safest calculation is that via the commutator of two string fields. All these calculations led to one result: a pointlike localized spacelike (graded) c-number commutator, whose explicit form still depends on the choice of the internal part (the vertically acting oscillators) of the smearing function [39]. With other words the infinite mass/spin tower spectrum is a general characteristic property of the theory, but the strength with which they contribute to a particular point-localized wave function or second quantized field analog can be manipulated with operators acting between the levels.

But being ideology driven, the pointlike character of the generating wave function/field is never clearly spelled out. What string theorist describe [40][41] is not the pointlike result of their calculation, but some sort of extended but at the same time hidden object, a kind of nearly invisible string of which only the c.m. point is visible. The actual calculation remains in strange contrast to the imagined string-like extension.

Remembering that the conquest of quantum theory is inexorably linked with a clear exposition of quantum reality and localization in particular, one wonders why string theory leads people to mystical regressions. The cited papers constitute an interesting historical document for a time in which clear calculations could not prevent their metaphoric interpretation. The tower of Babel in particle theory is erected on the difference between computations and prevailing ideology. It is of course important that the calculations are correct, and it is not plausibe that the interpretation which fails to match the calculation was distorted on purpose. The tower of Babel effect is rather the result of the Zeitgeist of domination of a TOE.

Perhaps the path into a self-defeating metaphoric world started already with such innocent looking choice of notation which feigns target space localization as writing X^μ for the current potentials Φ^μ. With the loss of conceptual knowledge about local quantum physics, the idea of a stringlike target space localization may have received a helping hand from an unlucky notation which could have exacerbated an already present misunderstandings.

String theory unlike QFT has no built-in operational way of introducing interactions. Whereas the spacetime principles underlying QFT are strong enough to not only determine the form of interactions consistent with the locality principle but also to rigorously derive scattering theory, all these ideas of deriving global properties from local principles are lost in a pure S-matrix approach. Its principles of unitarity, Poincaré invariance and possibly crossing are the only guides and every additionally imposed structure has to justify itself a posteriori by its phenomenological success. Hence it is not surprising

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[31]One has less problems with looking at the source $->$ target embedding as a purely formal device.

that interactions are defined by hand via highlighting certain operators which act in the Hilbert space of the string oscillators as in the dual model.

Being deprived of large time asymptots which relates the S-matrix with a Lagrangian via interpolating fields, string theorists simply define the lowest order (tree approximation) of the string S-matrix by functional formulas which are equivalent to the the Fubini-Virasoros exponential expressions. Already in the setting of the dual model, attempts were made to find reasonably looking recipes to imitate the loop corrections of QFT by adapting Feynman's rules for world lines to world sheets. String theorists introduced computational recipes in form of graphical descriptions in terms of rules for combining and splitting tubes which are supposed to represent the world sheet traced out by strings, but what does this mean for pointlike objects whose spacetime string extension is metaphoric and not real? Whereas such recipes in QFT can be shown to be a graphical illustration of operator relations, their quantum meaning in string theory remain unclear. The characteristic feature of a relation or formula in quantum theory is that it can be expressed in terms of operators and states. Despite a search over more than 4 decades for an operator formulation behind those recipes for perturbative string S-matrix amplitudes by the best minds in the string community, no such quantum theoretical formulation was ever found.

In this context it is interesting to remind oneself that Stuekelberg discovered Feynman rules precisely in this graphical recipe form. In his studies of macrocausality properties of an S-matrix he realized that, whereas the spacelike macrocausality amounts to the cluster factorization of the S-matrix, there was a finer macrocausality property for asymptotic timelike separation. A 3→3 particle scattering for example should contain the possibility that first 2 particles interact in form of a 2-particle scattering and afterwards the third particle enters the causal future of the first process and meets and interacts with one of the outgoing particles. He showed that the timelike trajectory between the two local scattering centers is a propagator with (what later became known as) Feynman's ε-prescription expressing the fact that the second interaction happened later. By assuming that interaction regions can be idealized as pointlike vertices he obtained the Feynman rules. Of course nobody, including himself, paid much attention to such an ad hoc recipe. The general acceptance came only with the derivation in terms of operators and states which started with Feynman and found its most concise expression in the work of Dyson.

In the string case not only is there no operator formulation for the word sheet picture, such a formulation would create a clash with the pointlike nature of the free string. There remains of course the possibility that an infinite collection of pointlike fields offers a new kind of pointlike interaction which has no counterpart in the standard setting of polynomial (possibly infinite degree) interactions. But even if such a possibility exists, any quantum interaction must allow a formulation beyond recipes and prescriptions in terms of the quantum setting of operators and states.

String theory, either in its factual infinite component pointlike setting, or its metaphoric guise of a "invisible string" is markedly different from (finite component) QFT if it comes to the notion of *degrees of freedom*. QFT has more phase space degrees of freedom than QM; whereas in QM there is a finite number of degrees of freedom in a finite phase space volume, the cardinality in QFT is described by a mild form of infinity (the compactness or nuclearity property of QFT [14]). This is precisely what guaranties the existence of thermal states at any temperature and the causal shadow property which states that

the algebra of a spacetime region equals that of its causal completion [71] (the quantum counterpart of the Cauchy wave propagation). Both properties are lost in string theory. In view of its importance for the problem of holographic relations of QFTs in different dimensions this issue will reappear (last section) in a different context.

5   Relation between modular localization and the crossing property

In order to become aware of the significant conceptual differences between the crossing property and duality it is necessary to have a profound understanding of crossing. In the sequel I will for the first time present some recent insight on this problem within the setting of modular localization theory (appendix).

The important concept from modular theory which relates to the crossing property is *localization equivalence with respect to the wedge W spacetime region*[32] *(*which will be denoted denote by $\overset{W}{\sim}$) between operators affiliated to ($\prec$) different wedge algebras $\mathcal{A}(W)$ and $\mathcal{B}(W)$ which live in the same Hilbert space and share the same positive energy representation of the Poincaré group.

$$B \overset{W}{\sim} A : B\Omega = A\Omega, \quad B \prec \mathcal{B}(W), \ A \prec \mathcal{A}(W) \tag{18}$$

Since under such conditions modular theory identifies the dense subspaces generated by applying the two wedge algebras[33] to the vacuum, it brings about a one to one relation between generally unbounded operators which does not respect the algebraic multiplication structure. Hence the $\overset{W}{\sim}$ relation is a *bijection* between the individual operators affiliated to two wedge localized operator algebras which both live in the same Hilbert space and share the same unitary representation of the Poincaré group but may be very nonlocal relative to each other. The situation which is relevant for the derivation of crossing is that in which $\mathcal{B}(W)$ is the wedge-localized algebra from an interacting net of local algebras which admits a complete asymptotic interpretation and $\mathcal{A}(W) = \mathcal{A}_{in}(W)$ is the wedge algebra generated by its incoming fields.

The underlying idea resembles in some sense the algebraic notion of *relatively local fields* which led to the concept of Borchers equivalence class [13]. But since there is no direct algebraic connection beyond the bijection between the operators of two algebras sharing the same localized states, the same representation of the Poincare group and as a consequence, the same Reeh-Schlieder subspace [14], the two algebras may be quite different in the algebraic sense.

The existence of this bijection is a straightforward generalization of an argument about modular theory in [12]. In that work the interacting representation of a wedge-

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[32]Although localization equivalence can be defined between operator algebras which share the same Poincaré representation theory in the same Hilbert space, only the wedge situation leads to the crossing relation.

[33]More precisely modular theory identifies the range of the two algebras after closing it in the graph norm of shared $\Delta^{\frac{1}{2}}$ which defines the same dense subspace. This domain of Wightman fields is believed to include that subspace but the range of those $B(f,..)$ which are l.e. in the expained sense is smaller (see later).

localized one-particle state[34] was considered. Such vacuum polarization-free objects are not available in interacting theories for compact localization region, in fact the wedge region is the only kind of region for which such interacting one-particle generators exist or in a more intuitive formulation: *wedge regions lead to the best compromise between particles and fields* apart from nets generated by free fields which have vacuum polarization free generators for any localization region. Hence vacuum polarization cloud aspects offer a completely intrinsic criterion for interactions. The arguments are easily generalized to the interacting representation of a wedge-localized n-particle state; in fact another way of stating the result is through the equality of the modular-closed Reeh-Schlieder spaces which are dense subspaces of the full Hilbert space H.

Since this a journal on foundations of physics it may be appropriate to mention that these dense subspaces have attracted the attention of renown philosophical and foundational motivated physicists. [43][44][45][46][47]. Their existence is surprising to anybody who obtained her/his physical intuition from experience with QM and constitutes one of the most characteristic features of QFT. Although the domains *domS* are identical the way how the different S act on this shared domain is different (appendix).

In the case the algebras generated by the cyclically acting fields are identical $\mathcal{A}(W) = \mathcal{B}(W)$, the bijection $\overset{W}{\sim}$ leads back to the trivial relation $A = B$. Hence the bijection is a generalization of the algebraic notion of local equivalence which is closely related to the notion of the *Borchers class* of relative local fields. Both concepts are also related (but not identical) to *weak locality [13]*.

The $\overset{W}{\sim}$ bijection concept comes with a prize. If the operator $A(f) \prec \mathcal{A}_{in}(W)$ is a f - smeared covariant field with $suppf \subset W$, having the standard Wightman domain properties, the existence of $B's$ is paid for by unwieldy domain properties. Although acting on the vacuum they do induce the same dense space of states; their domain properties are weaker than those of smeared Wightman fields. Their generally smaller domain is not translational invariant i.e. the translated domain of an operator $B \prec \mathcal{B}(W)$ is outside $domB$ [12]. The translation invariance of the domain would imply $S = I$ if $d > 1 + 1$ or in case d=1+1 the model has only elastic scattering. This shows that modular theory does not only show deep connections between spacetime geometry and the mathematics of operator algebras, but also reveals deep connections between domain properties of unbounded operators and the presence of interactions.

For $S_{scat} = 1$ to occur it is enough if such a Poincaré invariant dense domain exists for a particular

$$B(f) \overset{W}{\sim} A_{in}(f) \tag{19}$$

i.e. B which **g**enerates a vacuum **p**olarization **f**ree *one particle state* (such a B is called a **PFG**[35]) [12]. The triviality of the scattering matrix $S_{scat} = 1$, and therefore the equality of the Tomita operator $S_{Tomita} = S_{free}$ with that of a free field follows (as long as one

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[34]Such operators were called PFGs (vacuum-polarization-free-generators). They allow to generalize the Jost-Schroer theorem (saying essentially that interacting theories cannot have compact localizable PFGs) and play a crucial role in the modular construction of factorizing models (see next section).

[35]PFGs do not exist for causally complete subwedge regions unless the theory is generated by a free field. (stronger than the triviality of scattering). The wedge is the "smallest" causally closed region for which PFGs exist, though generally only at the prize of nontranslational invariant domains. Well behaved ("temperate") PFGs for Ws only exist in d=1+1.

avoids low dimensions ($d > 1 + 1$) and and 3-dimensional models with plektonic statistic) The interesting question to what extend this implies the absence of interaction in the stronger sense of $\mathcal{A} = \mathcal{B}$ will be commented on later.

The case of factorizing models, for which the S-matrix is nontrivial but has a rather simple structure, will be presented in detail in the next section.

The important relation which leads to the derivation of the crossing property is [12]

$$B\Omega = \Phi, \ \Phi \in domS_B, \ B \in B(W) \tag{20}$$
$$\curvearrowright B^*\Omega = S_B\Phi$$

Here Φ may result from applying a product of fields from different wedge-localized algebras to the vacuum; as long as the different algebras share the same Poincare representation this will lead to a Φ with the required domain properties.

The crossing relation in its simplest formulation (selfconjugate spinless fields, only incoming fields in the uncrossed configuration) reads

$$\left\langle BA_{in}^{(1)}A_{in}^{(2)} \right\rangle = \left\langle A_{out}^{(2)\,*}\Delta B A_{in}^{(1)} \right\rangle \tag{21}$$

Here $A_{in}^{(1)}$ and $A_{in}^{(2)}$ may be products of smeared fields

$$A_{in}^{(1)} = A_{in}(g_1), .. A_{in}(g_k), \ \ A_{in}^{(2)} = A_{in}(f_1)...A_{in}(f_l) \tag{22}$$

The proof of this relation is reminiscent of the modular derivation of the KMS relation. In that case all operators are taken from the same wedge algebra whereas in the present case there are not only operators from different algebras to start with, but the crossing procedure brings a third algebra into the game namely $\mathcal{A}_{out}(W)$. As in the case of KMS, the presence of the unbounded analytically continued operator Δ leads to the same analytic properties.

The formfactor crossing

$$\langle B|p_1..p_kq_1, .q_l\rangle_{in} = \ _{out}^{a.c.}\langle -q_1.. - q_l|B|p_1...p_k\rangle_{in}^{c.o} \tag{23}$$

results if one takes Wick products of incoming fields and uses the density of wedge-localized wave functions in order to obtain an on-shell identity in momentum space[36]. The notation is as follows, the $a.c.$ refers to the analytic continuation from the positive mass shell to the backward shell (using the momentum space analyticity of wedge localized mass-shell reduced test functions) and the $c.o.$ indicates the omission of contractions between the p's and q's which reflects the fact that the l+k particle state on the left hand results from a Wick-ordered product of in-fields; since there are no contrations between in-particle on the left hand side, there can be none after crossing on the right hand side either. The negative momenta $-q$ come about from the imaginary 2π Lorentz rotation $\Delta = e^{-2\pi iK}$ apart from the fact that these particles are outgoing. The absence of such contraction on the left hand side which in terms of the original relation (21) means that we Wick-ordered

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[36]In the case of non selfconjugate particles the q-momenta refer to antiparticles and it would be better to use the notation $\bar{q}$.

the entire product of incoming operators. For particles which are not selfconjugate, the backward mass shell momenta are belonging to outgoing anti-particles (opposite charge).

For the proof one uses the formula (20) (first line, second equation)

$$\left\langle BA_{in}^{(1)}A_{in}^{(2)} \right\rangle = ((BA_{in}^{(1)})^*\Omega, A_{in}^{(2)}\Omega) = (S_B(BA_{in}^{(1)})\Omega, S_B A_{out}^{(2)*}\Omega) = \tag{24}$$

$$= \left\langle A_{out}^{(2)*}\Delta B A_{in}^{(1)} \right\rangle$$

where in the the last line the antilinearity of S as well as the relation $S^*S = \Delta$ was used. Apart from the involvment of different algebras, the derivation of the crossing relation resembles strongly the modular derivation of the KMS property of localized algebras may be seen as a *generalization of the KMS setting.* In fact the relation (24) is a KMS relation for two different wedge-localized algebras $\mathcal{B}(W), \mathcal{A}(W)$ which share the same representation of the modular group (the Lorentz boost) which in this particular case results from the sharing of the Poincaré group between the interacting theory and its asymptotes. It is a special case of an extended KMS relation for two algebras which are standard with respect to the same state and have the same modular group but different modular reflections $J_A \neq J_B$

$$\langle BA \rangle = \langle A_B\Delta B \rangle, \ \ A_B^*\Omega \equiv S_B A\Omega \tag{25}$$

with S_B being the modular Tomita operator for the algebra $\mathcal{B}(W)$[37] the two-algebra generalization of the KMS situation evidently reduces to the one algebra case for $\mathcal{B}(W) = \mathcal{A}(W)$ and hence $A_A = A$. This extension of KMS offers a wealth of new application, but in this article the main use is the above crossing property.

Since the shared analyticity properties result from the domain properties of Δ, it is helpful to remind the readers of the standard analytic KMS properties as Araki [48] first established them

Definition 2 *Let $\mathfrak{C}$ be a C^*algebra on which α_i acts as a one parameter automorphism group. Then ω is called a KMS state with respect to α_t at temperature $\beta > 0$ if for each pair of operators $A, B \in \mathfrak{C}$ there exists a function $F_{A,B}(z)$, analytic on the open strip $\{z \in C, \ 0 < \mathrm{Im}\, z < \beta\}$, continuous and bounded on its closure, such that*

$$F_{A,B}(t) = F_{A,B}(Aa_t(B)), \ \ F_{A,B}(t + i\beta) = F(a_t(B)A) \tag{26}$$

Araki showed that the correlation functions in a KMS state inherit analytic properties from this definition; they are boundary values of function analytic in certain tube regions of the product of complex plain

$$\omega_\beta(\alpha_{t_1}(B_1)....\alpha_{t_n}(B_n)) = \lim_{\mathrm{Im}\, z \to 0} \omega_\beta(\alpha_{z_1}(B_1)....\alpha_{z_n}(B_n)) \tag{27}$$

$$C_{\beta,<}^{(n)} : 0 < \mathrm{Im}\, z_1 < ..... < \mathrm{Im}\, z_n < \beta$$

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[37]In general the computation of the action of a modular S- operator on a state generated by a Δ-related operator algebra onto the vacuum is a difficult problem.

where the second line specifies the analyticity region from whose inside the boundary limit has to be taken. In thermal QFT α_t is defined in terms of the time translation and ω_β exists under rather general conditions for all $\beta > 0$. There are also analytic properties of KMS states which come with half the analyticity region [48]

$$\alpha_{t_1}(B_1)....\alpha_{t_n}(B_n)\Omega_{KMS} = \lim_{\mathrm{Im}\, z \to 0} \alpha_{z_1}(B_1)....\alpha_{z_n}(B_n)\Omega_{KMS} \tag{28}$$

$$C^{(n)}_{\beta/2,<} : 0 < \mathrm{Im}\, z_1 < ..... < \mathrm{Im}\, z_n < \beta/2$$

There is no statement about whether different orderings can be related by analytic continuation. In the case of Wightman functions this follows from spacelike (graded) commutativity and for the so called temperate PFGs of d=1+1 factorizing theories this is a consequence of the Zamolodchikov-Faddeev commutation relations for generators of wedge localized algebras (next section).

In the case at hand the thermal aspect does not come about in the standard way i.e. by subjecting a global algebra of QFT to a heat bath which converts its ground state into a KMS state, but rather by restricting a global vacuum state to a wedge-localized subalgebra. With the conventions from modular theory we have

$$\Delta^{it} = e^{-2\pi itK}, \ K = generator \ of \ W - Lorentz \ boost \tag{29}$$

$$\beta_{\mathrm{mod}} = -1 \ corresponds \ to \ \beta_K = 2\pi$$

Whereas the modular temperature and the modular Hamiltonian is dimensionless, the boost generator K and its conjugate hyperbolic Lorentz parameter $\chi \sim 2\pi t$ have reciprocal dimensions and the Araki tube is a region in the product plane of complex Lorentz angles.

As stated before the analytic properties associated with crossing are the same as for KMS; the only difference to the free field case (4) in section 2 is that the cyclic permutation within the thermal expectation is accompanied by a change of "in" into "out".

The question how the basic quantities of a heat bath situation, as energy and entropy, are related to their counterparts arising from localization is a fundamental problem of quantum theory. In the heat bath case the KMS state, which is associated to the time translation Hamiltonian, brings the dimensionful Boltzmann constant together with the standard temperature into the calculation, whereas in the thermal formalism associated with modular localization there is no such dimensionful concepts, so that e.g. in case of conformal invariant quantum matter there would be no obvious way to obtain a dimensionless entropy which is proportional to the area of the causal horizon of a spacetime localization region[38]. So the problem is how to reconcile an area proportionality with a dimensionless entropy. In the case of the heat bath volume law this is achieved through appropriate kT factors but the localization thermality is a pure geometric notion and there would be no dimensionful parameter in the quantum matter if we restrict to conformal quantum matter.

To unravel this enigma I first looked at chiral conformal models i.e. at a theory living on a lightray. In that case one can rigorously show that the one-dimensional "volume"

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[38]Since localization entropy is a vacuum polarization related concept closely related (but not part of) to holographic projection onto the horizon, the area proportionality is intuitively expected and confirmed by computations [49].

law of the heat bath situation passes to a dimensionless logarithmic heat bath behavior (which depends on ε, a conformally invariant ratio of 4 points) by a certain conformal transformation (the "inverse Unruh effect" [49]). The result of an educated guess which generalizes this rigorous result is that the contribution from the higher dimensions is a dimensionless area factor

$$Entr \simeq \ln \varepsilon \cdot a(\varepsilon), \quad a(\varepsilon) = \frac{Area}{\varepsilon^{d-2}} \tag{30}$$

where ε is interpreted as the thickness of a light-sheet around the causal (or event) horizon. For d=1+1 this entropy law can be proven [49].

Having arrived at such a logarithmically modified area formula, it is consequential to expect a kind of multi-dimensional extension of the chiral inverse Unruh effect, namely one common abstract archetype hyperfinite type III$_1$ algebraic structure which is behind both kind of thermal manifestations, so that only the parametrization of this phenomenon is contextual, for more see [49].

It is interesting to compare the modular derivation of crossing relation with earlier derivation of special cases based on the use of LSZ scattering theory in combination with analytic multivariable properties of vaccum expectation values [2].

The rigorous basis of its derivation is the time-dependent Haag-Ruelle scattering theory which is a consequence of the principles of local quantum physics [50]. The better known LSZ scattering theory and its useful stationary scattering formulas is in turn a rigorous consequence of that Haag-Ruelle scattering formalism. The one time application of the LSZ reduction reads

$$^{out}\langle ..q_1 \,|\mathcal{O}|\, p_1.. \rangle^{in} = \,^{out}\langle ..q_1 \,\big|a^{out*}(p_1)\mathcal{O}\big|\, .. \rangle^{in} + \tag{31}$$

$$+ \, i \int d^2x K_x \,^{out}\langle ..q_1 \,\big|T\left\{\mathcal{O}B^\#(x)\right\}\big|\, .. \rangle^{in} e^{-ip_1 x}$$

$$^{out}\langle ..q_1 \,|\mathcal{O}|\, p_1.. \rangle^{in} = \,^{out}\langle ... \,\big|\mathcal{O}a^{in}(q_1)\big|\, p_1.. \rangle^{in} +$$

$$+ \, i \int d^2x^{out} K_x \,^{out}\langle ..q_1 \,\big|T\left\{\mathcal{O}B^\#(x)\right\}\big|\, .. \rangle^{in} e^{iq_1 x}$$

$$where \quad K_x = (\partial^2 + m^2), \,\, B^\# = B \,\, or \,\, B^c \,\, (antiparticles)$$

Iterating this reduction for all particles in the bra and ket states until one arrives at a vacuum expectation results in the Fourier transforms of an n-fold time ordered correlation function with the momenta of the bra particles being on the backward mass shell modified by contact terms which have the structure of the first term in (31) i.e. they consist of contractions between bra and ket particle states multiplied by matrix element of time ordered products involving a lower number of bra and ket states. Omitting contraction terms (indicated by a subscript $c.o$) the result of the iteration leads to the well-known expression of the connected scattering amplitude in terms of mass-shell restricted amputated Fourier transforms of time ordered correlation functions

$$^{out}\langle ..q_1 \,|\mathcal{O}|\, p_1.. \rangle^{in}_{c.o} = i(p_1^2 + m^2)..i(q_1^2 + m^2).. \left\langle T\mathcal{O}B^\#(p_1)..B^\#(-q_1).. \right\rangle \tag{32}$$

Since the LSZ formalism on its own does not lead to an analytic continuation properties which could connect the backward outgoing bra momentum with a physical outgoing bra momentum on the forward mass shell, the crossing statement remains a formal observation; the two crossed expression are quantities in the same theory but without any intrinsic structural model-independent connection between them. The method which worked for certain scattering amplitudes (i.e. $\mathcal{O} = \mathbf{1}$) consisted, as mentioned before, in using multi-variable analytic properties of time-ordered and retarded functions; this is a difficult problem in multi-variable analytic function spaces. Unlike functions in the complex plane (where any region comes with a space of analytic functions whose natural analyticity region coincides precisely with the given region) there are higher dimensional analyticity regions which are not natural i.e. every function which is analytic on such a region admits a continuation into a bigger natural region (cutting of "noses").

It is to be expected that such a method, even if ingeniously applied as in [2], is too bulky for a general solution of the problem. Indeed its exploration came to an end already in the 70s after Källen Wightman tried for many years in vain to derive a general representation of a 3-point function on the basis of computation of natural muti-variable analyticity domains.

The only aspect in common of the BEG method with the modular approach is that both methods rely on analyticity from locality; but this is a nearly empty statement in a theory for which *any* property must be deducible from causal locality and the closely interwoven spectral positivity. The biggest drawback of the old method is that it reveals nothing about the relation with wedge localization and its thermal aspects.

Historically thermal properties of localization entered QFT through the Hawking radiation of quantum matter behind an event horizon. For some time this was thought of as a separate issue of QFT in curved spacetime. But the main difference between event horizons in curved spacetime and causal horizons in Minkowski spacetime QFT are that the former are objective locations given by the external metric, whereas the latter are fleeting Gedanken-objects whose physical realization depends on non-inertial observers (viz. the Unruh effect); the thermal aspects are the same. Their fleeting existence of causal horizons does not at all mean that they are unimportant for a structural comprehension of QFT. The fact that the insufficiently understood crossing property of particle physics reveals it full physical significance in the setting of thermal manifestations of modular localization confirms this. This confluence of particle physics concepts with concepts coming from black hole physics is a very exciting process of ongoing conceptual unification which promises to bring a wealth of new insights. It is the most fundamental property which I ever met during my professional career and as a result of its fundamental nature I submitted it to a journal dedicated to the foundations of physics instead of a particle physics journal

There are interesting structural consequences of the crossing property, e.g. the Aks theorem [51] stating that d>1+1 quantum fields cannot lead to elastic scattering without the presence of inelastic scattering processes. The factorizing models in d=1+1 are an exceptional case; such models have the full infinite vacuum polarization, but its S-matrices are certain combinatorial products of two-particle $S_{scat}^{(2)}(\theta_1 - \theta_2)$. Another consequence of localization equivalence and crossing is that $S_{scat} = 1$ implies that the theory is that of a

free field[39], an old conjecture in AQFT [52].

Crossing is a consequence of the specific field theoretic (modular) localization and not a general property of relativistic QT. There exists a relativistic particle quantum mechanics, the DPI (direct particle interaction theory) [19] which is based on the non-covariant Born-Newton-Wigner localization [53] resulting from the spectral decomposition of the selfadjoint position operator. The DPI Hilbert space carries an interacting multiparticle representation of the Poincaré group which fulfills the cluster factorization property. However it contains no covariantly localized objects at finite times, the only such object is the (global) S matrix which is invariant and has the cluster decomposition property for spacelike directions (macrocausality). In fact it fulfills all properties which one is able to formulate in terms of particles [19].

On the other hand the properties presented in this section need the causal relativistic localization which, although leading to important consequences for particle scattering (as crossing), cannot be understood in a pure particle setting. The velocity of light in DPI setting, similar to the velocity of acoustic waves, comes about through quantum mechanical state averaging at large times, whereas in QFT it is a microscopic property of the observable algebra which is not related to the c.m. movements of wave packets. The good news is however that in QFT the BNW localization becomes asymptotically covariant and thus consistent with modular localization. In particular the asymptotic interpretation of QFT inherits the BNW probability.

6   An exceptional case of localization equivalence: d=1+1 factorizing models

In the generic setting, the use of the l. e. and the crossing property is limited to structural arguments of QFT, but in context of the d=1+1 factorizing models these properties turn into powerful tools of model constructions. From the modular viewpoint these models are distinguished by the fact that their algebra contains what has been referred to as "temperate PFGs" (vacuum **p**olarization-**f**ree **g**enerators). PFGs are operators operators which applied to the vacuum have translation invariant domains and well behaved Fourier transforms. With other words the d=1+1 B-fields which are W-localization equivalent to incoming/outgoing free fields have translational invariant domains and well defined Fourier transforms and describe some form of interactions. It turns out that all so called factorizing models [57] are in this class and it appears that temperate PFG always lead to factorizing models. The covariant domain properties result in the existence of a wedge-independent Fourier transformation leading to a free field like representation [12] which for the simplest family of models (the Sinh-Gordon model) [55][22] are of the form

$$\Phi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \left(Z^*(\theta)e^{ipx} + h.c. \right) \frac{d\theta}{2}, \quad p = m(ch\theta, sh\theta) \tag{33}$$

$$Z(\theta_1)Z(\theta_2) = s(\theta_1 - \theta_2)Z(\theta_2)Z(\theta_1), \ Z(\theta_1)Z^*(\theta_2) = s(\theta_1 - \theta_2)Z^*(\theta_2)Z(\theta_2) + \delta(\theta_1 - \theta_2)$$

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[39]I am indepted to Jens Mund who informed me about a forthcoming paper by Jens Mund on this generalization of the Jost-Schroer theorem.

where it is convenient to use the mass shell rapidity instead of the mass shell momentum. Here s is the two-particle scattering function of the Sinh-Gordon model; in the general case of factorizing models the Z-operators are multi-component creation/annihilation operators and the scattering function becomes a scattering matrix. The Z-commutation relations are often referred to as the Zamolodchikov-Faddeev algebra but in contrast to their original use as pure algebraic calculational devices, the Z's in the present wedge localized approach have a spacetime interpretation, since although the affiliated field Φ for $s \neq 1$ lacks pointlike localization, it can be shown to be at least wedge-like localized [21]. When I began to realize this connection during the 90s, I had the idea the the more spacetime one has at once's disposal, the better the chance to tame vacuum polarization and the simpler to find generators of these algebras with mathematically simple properties; at least simpler than pointlike interacting fields which always have infinite polarization clouds and correspondingly infinite series representation in terms of particle creation/annihilation operators.

The best localization region below the full algebra associated with the Minkowski spacetime which still admits a particle structure is the (noncompact) wedge region. this algebra is the "smallest" which contains for the first time PFG operators i.e. operators which once applied to the vacuum behave like a free field but have a complicated action on other states; i.e. although far more involved than free fields, applied once to the vacuum they behave precisely like a free field. This was the beginning of a new construction principle which I first applied to factorizing models [21] before Lechner [22] used it to proof the first existence theorem of the strictly renormalizable (short distance singularities worse than those of free fields), but not superrenormalizable factorizing models. The Fourier transforms of the wedge generating fields were the Z-F operators of the above form.

In the standard terminology Φ is a *nonlocal* on-mass-shell covariant field, but an application of modular theory shows that it is far from being completely nonlocal since it is wedge localized [22] in the sense that smeared with W-supported test functions $\Phi(f) \prec \mathcal{B}(W)$. Contrary to free fields for which the localization is entirely governed by the support of the test function, the use of compact localized test function inside W does not improve the situation.

The possibility of "localizing in momentum space" in d=1+1 i.e. to work with operators $Z(\theta)$ (33) with Wightman-like domain simplifies the discussion and permits to arrive at more detailed results than the crossing of the previous section where algebraic properties of the operators $B(f, ..)$, which are comparable to those of the temperate PFG generators Z, are not available.

There exists a very simple-minded almost kinematical argument why in d=1+1 the temperateness of wedge localized PFGs does not exclude interactions. It so happens that the two-dimensional energy-momentum conserving delta function coalesces with the tensor product of two particle mass shell delta functions which appear in the inner product of a two-particle state. This has as a consequence that the *cluster factorization argument* for the S-matrix cannot distinguish between an elastic $S^{(2)}$ and the identity $S^{(2)} = \mathbf{1}$ i.e. clustering in d=1+1 cannot remove a two particle interaction and arrive at a trivial scattering amplitude. In this sense the models are close to non-interacting situations. Nevertheless the *off-shell structure of these models is surprisingly rich*, in particular they possess the full vacuum polarization struture for compact spacetime localization although

these model have no on-shell particle creation through scattering. Their mathematical and conceptual structure has been the object of many studies and they continue to play the role of a theoretical laboratory in which quantum field theoretical ideas can be tested and studied under full mathematical control.

The states obtained by the iterative application of the Z have a very simple structure

$$TZ^+(\theta_1)....Z^+(\theta_n)\Omega = a_{in}^*(\theta_1)...a_{in}^*(\theta_n)\Omega \tag{34}$$

$$\bar{T}Z^+(\theta_1)....Z^+(\theta_n) = a_{out}^*(\theta_1)...a_{out}^*(\theta_n)\Omega$$

where T is the θ-ordering (same symbol as for time-ordering) and $\bar{T}$ denotes the opposite ordering and the right hand side only involves symmetric Bose operators. The analytic properties of the vacuum polarization component for a fixed order

$$F(\mathcal{O}, \theta_1...\theta_n) = \langle 0 \, | \mathcal{O} \, | Z^*(\theta_1)...Z^*(\theta_n)\rangle^{in}, \quad \theta_1 > ... > \theta_n \tag{35}$$

are those expected from the the previous section. But now the analytic properties go beyond those coming from the cyclic KMS property since the Z commutation relations also algebraize what happens when the order is changed. This is similar to the extension of the primitive tube domain of Wightman functions by the use of locality. On should not confuse this commutation with (graded) bosonic statistics. The latter has been already absorbed by encoding statistics equivalent states into one ordered masterstate $\theta_{i_1} > .. > \theta_{i_n}$ written as $|Z^*(\theta_{i_1})...Z^*(\theta_{i_n})\rangle$ by using graded Bose statistics; this couples the θ-order with the operator order in products. Without this coupling it is not possible to understand the algebraic aspects of the work on the bootstrap formfactor construction.

The knowledge about commutation properties is not availabe in the general case; in the derivation of the crossing in the previous section we only used the Araki KMS analyticity. Crossing does not tell anything about an analytic exchange of two $\theta's$, the analyticity which permits to change the order of rapidities comes from the algebraic commutation structure of the Z generators.

The factorizing models confirm again that crossing has no conceptual relation to duality. One-particle bound states which are poles in scattering processes have no special place in crossing; models without or with bound state fulfill crossing and in case there are bound states present, they are mixed via crossing with the scattering continuum in a complicated way. Even in the perturbative crossing relations one-particle direct or exchange contributions do not play any special role, there is no crossing in which only one-particle states contribute. 40 years of research on S-matrix based particle theory (duality, ST) have been founded on misunderstandings of the crossing property.

The full analytic setting of the so-called bootstrap-formfactor program (which resulted from a correct understanding of crossing) was already formulated at the late 70s [56]; since that time there has been a steady stream of novel model and new insights based on the analytic properties of their formfactors [57]. In all cases the calculated formfactors were not only meromorphic functions in the multi-strip regions (where their poles has a direct interpretation in terms of bound states), but they were even meromorphic in the full complex θ-plane (the infinitely many different sheets in the Mandelstam variables).

The conceptual basis of this approach received a significant boost when it was observed that the analytic rules for the construction of formfactors permit a formal algebraic encoding. What was first introduced as a trick without any apparent intrinsic physical

meaning [54] in the 90s acquired the spacetime meaning of being closely related to wedge localization [55] which finally led to the first existence proof for factorizing models [22].

The interesting problem is to find an higher dimensional counterpart of these observation. In the present context one certainly does not expect a simple analog of $Z^{\#}$ operators which relate the different θ orderings in the sense that the connection between the different θ-orders in the vacuum polarization formfactor (35) can be encoded into the operator positions.

To look for an algebraic interpretation of analytic continuation in terms of an auxiliary QFT is not so absurd as it appears at first sight. The analogy with Wightman theory is worth exploring. Wightman functions are distributions whose analytic properties come from the energy positivity. The analytic tube regions for different spacetime orderings are related by the algebraic properties of covariance and local commutativity. This gets quite complicaed in case of d=1+2 braid group commutation structures where the analytic continuation leads to multivalued functions. The formfactors in factorizing theories are also multi-valued in the Mandelstam variables and by rewriting this in case of temperate into the uniformizing θ variables one finds an algebraic structure. The crucial question is whether the analyticity properties of formfactors in the general case also permits to interchange θ-orders via analytic continuation; such a property would go beyond crossing and probably be of invaluable help for nonperturbative model constructions.

There are many more d=1+1 unitary elastic S-matrices satisfying crossing than there are pointlike Lagrangian couplings i.e. most of the existing factorizing models do not have a Lagrangian name. There is no reason to believe that this is in any way different in higher dimensions so there is a strong suggestion that even outside factorizing models the Lagrangian formalism only covers a tiny area.

As often in the history of physical ideas, the best inside into their inner workings has little resemblance with the history which led to the present understanding. Indeed the original observation leading eventually to factorizing models had little to do with what was presented in this section, in fact it was not even related with factorizing S-matrices but rather with integrable looking quasiclassical mass spectra of certain field theories (notable Sine-Gordon). In analogy to integrable systems of QM as the hydrogen atom, it was natural to look for higher conservation laws. But historically the first hints came from mass shell restriction of perturbative correlation functions leading to scattering amplitudes which were expected to show the absence of on-shell creation. As a curiosity I remember how one of my PhD students (Bernd Berg) in the beginning of the 70s demonstrated such statements numerically on one of the old Hewlett-Packard pocket calculators. From such confidence-building calculations sprung the first suspicion that behind these observation there was the S-matrix bootstrap, but this time without the old ideological bombast [58][6]. The first structural arguments pointing into the direction of the S-matrix bootstrap approach set off a frenzy of model classifications and construction according to the bootstrap S-matrix program. It soon became part of a new *bootstrap-formfactor approach to factorizing models.*

7   Resumé, some personal observations and a somewhat downbeat outlook

The era of post renormalization QFT began at the end of the 50s with a return of the incompletely understood age-old *particle-field problem.* The formulation of the LSZ scattering theory, its rigorous derivation by Haag, Ruelle and Hepp are important landmarks in this conquest. Another more recent important step is the partial resolution of the apparent contradiction between the noncovariant Born-Newton-Wigner localization, which brings the indispensable probabilistic concept of QM[40] into QFT, and on the other hand the modular localization, which is intrinsic to QFT and does not lead to the probability of finding a particle in a specified spacetime region [19]. It is deeply satisfying that in the large time scattering limit both localizations match; hence in particular the noncovariant BNW localization becomes covariant[41] and the modular localization becomes consistent with a probability concept which in turn is the prerequisite for an invariance S-matrix and the probabilistic interpretation of the associated cross sections.

This large time asymptotic coexistence between particles and fields or their generated localized operator algebras is crucial for our understanding of QFT and the crossing property is the most subtle manifestation of the particle-field relation.

The first successful test of scattering theory consisted in the derivation of the experimentally verified Kramers-Kronig dispersion relations from analytic properties of locality. This was important for strengthening the confidence in the locality and spectrum principes of QFT.

It was in this context that the crossing relation arose for the first time in form of the existence of an analytic masterfunction which connects different processes with different distribution between incoming and outgoing particles. This was a crossing identity in which the crossed in/out particles were in an unphysical position. One still needed analytic continuation properties which the LSZ property by its own theory did not provide. For certain scattering configurations this analytic argument was supplied in [2]. In the S-matrix bootstrap approach the crossing analyticity was simply assumed under the heading "maximal analyticity", it was treated as a basic postulate together with the other physical principles as Poincaré invariance and unitarity. This unphysical way of looking at a problem foreclosed the chance to understand crossing in terms of localization and ensuing thermal KMS properties; in particular the KMS-like cyclic permutation property (24) of scattering amplitudes and formfactors remained unnoticed.

Historically the next step was the understanding of the crossing relations within the bootsstrap-formfactor program [72] for factorizing models. But this did not involve a structural proof either; rather crossing was one of the assumptions in the constrution of these models. The fact that at the end one had constructed a nontrivial model meant that crossing is really a property of this particular class of models. Again no connection to modular localization properties and their thermal manifestations was noticed. This only changed with the realization that behind the Zamolodchikov Faddeev algebraic re-

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[40]Born [59] introduced this probability concept first in the setting of scattering theory (the Born approximation for the cross section) before it was extended to x-space wave functions.

[41]In the literature one sometimes encounters an "effective" version stating that covariance is attained for distances beyond the Compton wave length..

formulation factorizing models there are nonlocal wedge-localized generators [21]. Only then the construction in terms of recipes for formfactors finally became a classification and construction of factorizing models according to the underlying principles of QFT [22].

The derivation of crossing for formfactors presented in this paper is according to my best knowledge the first one outside the narrow setting of two-dimensional factorizing models. Since the context of this paper is a rather broad one, a more detailed specific account of the derivation of crossing from modular localization theory and implications thereof will be given in a separate publication.

The history of crossing shows also that an early flare-up of ideas, before their conceptual-mathematical understanding is available, may under certain sociological conditions cause disarray[42]. The dual model and string theory and with it that strange idea of a millennium TOE would not have come about without a certain amount of conceptual confusion. As we know nowadays the properties of Mellin transform of conformal QFTs permits the "mass production" of dual models, including the one discovered by Veneziano and others; for their construction one does not have to know anything about the crossing property. Historically their ingeneous construction by hand using properties of Gamma function and thinking of one-particle approximations of the conjectured Mandelstam representation for scattering amplitudes led to the belief that one discovered a deep and still mysterious new area of particle physics, whereas in reality it was the entrance into a physical no man's land.

The string theoretic extension of the dual model aggrevated the problem, in particular since its pointlike localized nature was overlooked as the result of confounding the localization of QM with its covariant relativistic counterpart. The decisive factor to get to to the present state of confusion was however the sociological impact of the enthusiastic support by renown members of the physics community. Who can deny the impact of statements about string theory as "a present of the 21st century to the 20th", "there is no other game in town" or the citation of Churchills famous die-hard slogan "never, never,...never give up" will stand accused of living in an ivory tower.

At this point the difference to particle physics before the 80s becomes clear: the fragile equilibrium between the innovative and speculative side of particle physics and the critical counterweight had broken down[43]. The worst aspect is the evaporation of historically grown pre-electronic basic knowledge about QFT which, in the presence of a millennium TOE, appeared now irrelevant. This is accompanied by an increasing split between applied QFT, where the main aim is to find computational recipes about a subject which has not been properly understood, and LQP, which is dedicated to making profound structural gains according to the inner logic of the theory at the prize of loosing contact with the actual reality of particle physics. There is hardly any cross fertilization; the one side fails to penetrate the conceptual-mathematical barrier in order to understand new structural insights into QFT (and often thinks it is not even worth a try), whereas

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[42]Usually premature observations disappear and return often in a different context when the understanding of their conceptual-mathematical struture is in place [33].

[43]The first version of the present paper was uploaded to arXiev:hep-th when a moderator placed it to the general physics setion with a built-in barrier to prevent any crosslisting of the paper. There is no more fitting description of the present sociological state of particle theory, any commentary about this episode is superfluous.

the other side has distanced itself so much from the phenomena that even when one of their findings can be connected to an application, it would go unnoticed.

Speculative proposals with little conceptual support but lot of public attraction were made at all times; particle theory is by its own nature a highly speculative science where it is sometimes necessary to take a dive into the "blue yonder". What was however different during the last 4 decades of dominance of string theory is that the critical counterweight, which through physicists as Pauli, Landau, Källén, Lehmann, Jost and many others had quite a tradition on the old continent, was not available when it was most needed. The sucessive leading figures in mathematical physics and (algebraic) quantum field theory who have the conceptual insight to play this indispensible critical role did not enter the fray and thus discontinued the old "Streitkultur". In the beginning the phenomenonological proposals of Regge-trajectories were far removed from any structure which one could relate with known principles of relativistic quantum theory, but when the sudden transition to a fundamental TOE took place[44], the uncritical acceptance of the new string theory as a TOE happened with such a speed that a critical discourse was hardly possible. The string protagonists occupied research and university positions within a short time and often their only credentials were that they are working on the most important millennium theory. After some leading state laboratories began to hire string theorists, it was a matter of national and scientific pride to have a signboard of the new millennium physics in form a representative for string theory.

In order to avoid misunderstandings, the derailment of parts of particle physics caused by string theory did not come about because it is mathematically nonsensical. As an infinite component QFT which contains operators which communicate between the different floors of an infinite particle/spin tower it is well-defined. The point where the conceptual confusion starts is that in order to introduce interactions one uses pictures *as if the pointlike localized infinite component field*[45] *would be stringlike,* since the perturbative calculations are based on tube rules in analogy to Feynman rules. It is important to note that, different from Feynman rules, these tube (worldsheet) rules, despite an intense search by the creme of string theorists over many decades, did not permit a presentation in terms of operators and states, so that their connection with quantum theory continues to remain questionable to say the least.

Perhaps the most spectacular episode is the fray which developed around the anti-De Sitter–conformal field theory (AdS-CFT) correspondence, an issue although not directly related to string theory, but which suddenly obtained prominence in its conceptual setting. Within a short time string theorists managed to convert this issue into something mystical if not to say surreal.

The subtle point of this correspondence is the radical change of the spacetime localization involved in the *spacetime reordering of quantum matter* passing from AdS spacetime

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[44]The begin of modern string theory has a date, it is the week in Paris in 1974 when Scherk and Schwarz [42] wrote up their famous paper. Underlining the rapidity of change one may call it the Bartholomew-like massacre of the old string theory which started with phenomenology of Regge trajectories.

[45]The attempts to construct infinite component irreducible (in the described sense) pointlike fields based on higher noncompact group representations (similar to the O(4,2) hydrogen spectrum) are described in [25]. Unfortunately there was no communication between the two groups of which only the string construction was successful (a success certainly not appreciated by string theorists).

to a lower dimensional CFT. Since physics is not only determined by the abstract quantum matter (e.g. CCR or CAR or any other matter characterized by its abstract spacetime independent properties) but also by its spacetime ordering, some physical properties change with the spacetime reordering in passing from AdS_5 to CFT_4; the relevant question is how much can they change if the abstract matter which is ordered according to the causal locality in different spacetimes with different dimensions remains the same? The answer is, that although there is no correspondence (isomorphism) between pointlike fields, there is one between operator algebras which are generated by pointlike fields. This coarser than pointlike correspondence is sufficient to fix one side of the correspondence in terms of the other [62].

The naive expectation about any isomorphismus (correspondence) is that when one starts from a theory with a *physically acceptable cardinality of degrees of freedom* (intuitively speaking, one coming from Lagrangian quantization) and spatially reorders them in such a way that there remains a local algebraic isomorphism for certain regions[46], then there will be *too many degrees of freedom* in case that the reordering leads to a spacetime of lower dimension as in the AdS_5–>CFT_4correspondence. Although perfectly consistent from a mathematical viewpoint, this causes serious physical pathologies (Hagedorn temperature or no thermal states at all, anomalies in the causal propagation etc.) [71]. In the opposite direction CFT_4–>AdS_5 the resulting AdS theory obtained from a physical CFT model will be too "anemic" concerning its degrees of freedom in order to be of any direct physical interest (the degrees of freedom hover near the boundary). This is the content of a rigorous mathematical theorem [62] and can be explicitly illustrated in terms of a free field AdS model [63].

The Maldacena conjecture [64] is more specific than the above structural theorem in that it places a concrete model on both sides of the correspondence; on the AdS side a certain 5-dimensional supersymmetric gravity model (suggested by string theory) and on the CFT side a conformal N=4 supersymmetric Yang-Mills theory[47]. These are two physical models with physical phase space degree of freedoms with respect to their spacetime dimensionality. The above theorem says that this is structurally impossible; if one side is physical the other is a purely mathematical chimera which may be useful in order to understand better certain physical properties of the physical side, but itself has no intrinsic physical existence.

Since the Maldacena statement is only a conjecture, the logical resolution of this paradox is obvious. What renders the whole situation delicate from a sociological viewpoint however is the fact that meanwhile more than 6000 papers have been written in support of Maldacena's conjecture (but, as expected, without any conclusion about its validity) and the saying that so many people cannot err is, as well-known, one of the most accepted vernaculars. It is hard to think of a more convincing illustration about the loss of solid

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[46]Neither in the case of the AdS-CFT correspondence, nor in the case of holographic projections on the horizon (a nullsurface) of a bulk region, the dimension-changing holographic map can be expressed in terms of pointlike fields.

[47]Rigorous proofs for the vanishing of the Beta functions and the absence of radiative corrections in certain anomalies where given in the 70s by combining Callen-Symanzik equations with Ward identities [60][61]. Apparently the knowledge about these techniques has been lost since the new order by order or lightcone quantization attempts applied to the supsersymmetric N=4 Yang Mills model are unconvincing.

scientific knowledge than this episode concerning the Maldacena correspondence.

The discovery that instead of the finite phase space degrees of freedom in QM (one per unit phase space cell of size $\hbar$), the cardinality of degrees of freedom in QFT is different, namely "mildly infinite" (compact, nuclear) was made in the 60s [65][71]. In the spirit of this article it is important to emphasize that this difference is a consequence of the different concepts of localization [19]. If one compresses the O(4,2) symmetric degrees of freedom from a physical density in a five-dimensional spacetime into four dimensions, then there are "too many phase space degrees" in order to maintain the causal propagation property which is the LQP version of the classical causal Cauchy propagation. With too many phase space degrees of freedom the quantum causal shadow property $\mathcal{A}(\mathcal{O}) = \mathcal{A}(\mathcal{O}'')$, where $\mathcal{O}''$ is the causal completion of the spacetime region $\mathcal{O}$ (the causal complement taken twice), is being violated; the right hand side is bigger.

¿From the viewpoint of somebody whose intuitive understanding of QFT comes from Lagrangian quantization which formally obey this property, the violation is very strange and mysterious. The only way he can uphold his picture of propagation is by using a metaphor that some degrees of freedom enter "sideways" from an extra dimension or from another universe ("poltergeist degrees of freedom"). In the present Zeitgeist inspired by string theory, where extra dimensions and multiverses are daily consequences on which articles are written, this only sounds like a yoke. The problem is that deep concepts as the cardinality of degrees of freedom [65][66] and their preservation in correspondences between QFT in different spacetimes have vanished from the conceptual screen in the 80s so that especially those younger people who work on holographic problems are not aware of their existence. A more detailed recent presentation of this phase space degrees of freedom issue can be found in [71].

Part of the problem of holographic spacetime reordering of quantum matter is that it is too radical in order to allow a formulation in terms of the standard setting of QFT in terms of individual pointlike fields; there is however no problem to express this in terms of operator algebras associated with suitable causally closed regions [62].

The only kind of holography which complies with the thinning out of phase space degrees of freedom is the holography onto nullsurfaces i.e. the holographic projection of bulk QFT onto causal or event horizons. In that case the reduction of degrees of freedom goes hand in hand with a reduction of symmetry: the symmetry of a lightfront is a 7-parameter subgroup of the Poincaré group and the problem of "filling up" the degrees of freedom to their orginal strength is equivalent to knowing the action of the remaining Poincaré transformations on the lightfront degrees of freedom. Equivalently it would suffice to know the lightfront theory in a GPS manner in different positions; in d=1+3 three different positions are certainly enough [19].

The problems which led to a derailment of a large part of particle theory (which to many appeared very important) can however not fully explain why the comperativly healthy standard model, after impressive initial gains, entered a period of stagnation. For almost 4 decades there has been not a single conceptual addition on the age-old central problems of gluon and quark confinement and the Schwinger-Higgs screening mechanism. Such a situation is certainly unique in the more then 8 decades lasting history of particle physics. In some cases there was even a regress in that earlier ideas have been lost in the maelstrom of time [71].

If there was any influence of S-matrix approach on the standard model research, it certainly was not of a hepful kind. Rather the perilous charm which a TOE supported by prominent community members exerts on intelligent and zealous newcomers may have been one reason why the standard model research may not have attracted the brightest minds; not to mention the considerable material support enjoyed by string-related research; a closely related argument is the promise of the leading string theorists that the standard model has to appear anyhow as a "low energy effective theory" of a TOE. Finally there is a widespread but misleading opinion that the only remaining theoretical problems of the standard model are of a computational nature; this is strengthened by the credo that QFT is a low energy footnote of string theory.

All these miscarried ideas have eroded the enthusiasm for new conceptual investments. A serious obstacle against a conceptual renewal is the fact that the teaching of QFT has fallen back behind what can be found in books written before 1980 e.g. in the book of Itzykson and Zuber. More recent books often appear as a kind of QFT filtered through string theory glasses. It is nearly impossible to start research on important conceptual problems (as the problem of the crossing property in this paper) on the basis of contemporary books on QFT; their content only suffices for doing similar computations following the given recipes in different settings. This has led to a situation in which the number of people who know QFT sufficiently well in order to contribute to a conceptual progress of QFT has shrunk to a few individuals in an advanced age.

Speculative proposals with little conceptual support but a lot of public attraction were of course made at all times; particle theory by its very nature is a highly speculative science where it is normal (at least once in a while) to take a dive into the "blue yonder". What was however different during the last 4 decades of dominance of string theory, is that the critical counterweight, which through physicists as Pauli, Landau, Källén, Lehmann, Jost and others had quite a tradition on the old continent, was not available when it was most needed. The leading figures in mathematical physics and (algebraic) quantum field theory who are in the possession of the necessary conceptual insight to play this indispensible critical role do not enter the fray.

At the beginning the phenomenonological proposals (the Regge trajectory setting) were far removed from any structure which one could relate with known principles of relativistic quantum theory, and when the sudden transition to a pretended fundamental TOE took place[48], the uncritical spread of the new string theory was that rapid, that there was hardly time for a critical discourse. The string protagonists occupied research and university positions within a short time, and often their only credentials were that they are working on the most important millennium theory.

With Res Jost and Harry Lehmann gone, as the last exponent of a critical engaging and highly knowledgable particle theory culture, there was hardly anybody left who could continue the traditional European "Streitkultur" and seriously challenge the basic tenets of string theory. There are of course others besides myself who know more or less the causes behind the derailment. In some of their articles one even finds the statements that strings are against their name really point-localized objects. But since no critical

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[48]The begin of modern string theory has a date, it is the week in Paris in 1974 when Scherk and Schwarz [42] wrote up their famous paper. Underlining the rapidity of change one may call it the Bartholomew-like massacre of the old string theory which started with phenomenology of Regge trajectories.

conclusions are drawn; such articles not only do not create frictions with their string theory colleagues (even when they contribute jointly to the same book [77]), these are tolerated as the kind of critical remarks which is part of a living science. The fruitful Streitkultur belonged to the bygone golden age of engagement in particle theory.

Similar arguments apply to the sociological [75][74] and philosophical [76] critique of string theory. Whereas scientific critique may have the power to erode metaphoric constructs, sociological and philosophical arguments do not constitute any danger to the popularity of string theory and certainly have nothing in common with a critical engagement within a Streitkultur; to the contrary they lead to a profitable symbiosis between string propagandists and its critics, with the latter running the risk of loosing their subject without the former. Why can't a theory which has strong conceptual credentials be explored for whatever time is necessary to get to its limits, and isn't a consistent theory which incorporates the existing one as a limiting case an interesting goal even if it does not describe reality? And is observational agreement the only criterion? The old (pre-oxigen) phlogiston theory of burning which dominated for many decades shows that a wrong theory may be able to live for a long time in reasonable agreement with observations especially if it explains sufficienty many observed phenomena. The only kind of critique which must be taken seriously in the long run is one which establishes that a theory is conceptually flawed.

All these observations show that the adventurous journey that started more than 4 decades ago with some misunderstandings in the particle-field relation around the crossing property, has grown into a profound crisis of particle physics. The resulting metaphoric discourse of placing superficial conclusions based on calculations done outside any conceptual control above profound critical evaluations is not any more confined to ST; the concommittant sociological phenomenon around the AdS-CFT issue is a clear indication of the spread of the crisis beyond the borders of string theory.

The disappearance of criticism has led to a new culture of establishing a scientific truth starting from a conjecture and ending after several reformulations with a with the acceptance within a community at the level of a theorem. This process has been insightfully described in a series of essays by a young string theorist [73]. The author, Oswaldo Zapata, has an ambivalent position with respect to string theory; having been raised with string theory and being aware about his limitations with respect to QFT, he knows that he cannot confront it on its *scientific* truth content. Instead he carefully analyzes the sociological aspects of its discourse and comes to remarkable conclusions. His aim is to understand how his fellow string theorists, having disposed of classical methods of establishing theorems, arrive at what they consider as truths, and how they present their results without becoming subjectively dishonest within the community and to the outside world. He does this by studying changes in the string communities discourse over a larger laps of time, during which there was no change in the facts.

Interestingly enough he gives the strongest argument for his thesis about the relation of the string community to facts involuntary by not referring to the aforementioned rigorous theorems about AdS-CFT. They are all in the public domain, but their conceptual mathematical content [62] is not known by the community members because most of them are not on a level on which they can understand structural theorems on local quantum physics. This shows that the control of the community over facts does not end at what

is coming from the inside (which Zapata as an insider of this community is well aware of), but it extends also to shielding inconvenient theorems from the outside in the most possible honest manner, namely by ignorance about large parts of QFT. In this way even Zapata remains uninformed that his critical sociological observations about the discourse of the string community have a profound scientific counterpart.

Reading Zapata's essay may not help to learn about conceptual errors of string theory. But his method is very successful in exposing the surreal aspect which accompanies the string community's almost messianic "end of the millennium belief" in a TOE. His account of how a metaphoric conjecture ends after several sweeps through the community as a community-accepted fact is truely remarkable. It shows that some individuals of the string generation, having been deprived of a critical conceptual scientific basis, can still make fascinating critical observations about the logic and sociology of the discourse within the string community.

It is quite revealing that Zapata takes a dim view on some missing arguments in two books by Lee Smolin and Peter Woit [74][75]. These authors take a critical look at the dominant position of string theory and explain very well the sociological reasons why younger people uncritically internalize the catechism of string theory. *But they never explain why respectable older people, who are under no such career pressures (especially those who are the main string proselytizers mentioned before) believe in the validity of the theory.* It is of course common practice to blame the foot-soldiers (in the present context, the young partisans of string theory) and the propaganda division (Brian Greens and others), but spare the generals; there should be no place for this attitude in particle physics.

It would be wishful thinking that articles as the present one or the essay of Zapata could have an influence on the tide of events. But they provide a valuable help for historians and philosophers of science to analyze what went on in particle theory during a substantial part of the 20th and the beginning of the 21st century.

Since readers need some encouragement at the end, the present essay should not end in a downbeat mood. There are some interesting new developments around higher spin field, in particular massless fields. They start from the observation made in the appendix in (42) where it was mentioned that the reduced possibilities for $(m = 0, s)$ with $s = 1, 2$ which exclude covariant vector potentials and $g_{\mu\nu}$ tensors can be fully complemented to all possibilities which occur in the massive case if one permits modular semiinfinite string localization. This leads to a new way of looking at the problems behind gauge theory. Already in the abelian case of QED for which it has been known for a long time that electrically charged states are semiinfinite string-localized (associated to infraparticles), the new setting for the first time incorporates the perturbative aspects of these physical charge-carrying fields into the formalism i.e. they do not have to be defined by hand as in the famous stringlike formulas of Dirac-Jordan-Mandelstam [71][33]. The Higgs model results as a Schwinger-Higgs screening of the electric charge of a scalar fields and leads to a theory in which the massive matter field is neutral (real) and pointlike localized [71]. The new conceptual frame of modular localization promises to lead to a conceptual renovation also beyond the issue of crossing.

8   Appendix: a sketch of modular localization

8.1   Modular localization of states

The simplest context for a presentation of the idea of modular localization is the Wigner representation theory of the Poincaré group. It has been realized by Brunetti, Guido and Longo [20] [49] there is a natural localization structure on the Wigner representation space for any positive energy representation of the proper Poincaré group. Upon second quantization this representation theoretically determined localization theory gives rise to a local net of operator algebras on the Wigner-Fock space over the Wigner representation space.

The starting point is an irreducible representation U_1 of the Poincaré group on a Hilbert space H_1 that after "second quantization" becomes the single-particle subspace of the Hilbert space (Wigner-Fock-space) H_{WF} of the field [50]. The construction then proceeds according to the following steps [20][78][31]. To maintain simplicity, we limit our presentation to the spinless bosonic situation.

One first fixes a reference wedge region, e.g. $W_0 = \{x \in \mathbb{R}^d, x^{d-1} > |x^0|\}$ and considers the one-parametric L-boost group (the hyperbolic rotation by χ in the $x^{d-1} - x^0$ plane) which leaves W_0 invariant; one also needs the reflection j_{W_0} across the edge of the wedge which is apart from a π-rotation in the transverse plane identical to the TCP transformation. The Wigner representation is then used to define two commuting wedge-affiliated operators

$$\delta_{W_0}^{it} = \mathfrak{u}(0, \Lambda_{W_0}(\chi = -2\pi t)), \ \mathfrak{j}_{W_0} = \mathfrak{u}(0, j_{W_0}) \tag{36}$$

where attention should be paid to the fact that in a positive energy representation any operator which inverts time is necessarily antilinear[51]. A unitary one-parametric strongly continuous subgroup as $\delta_{W_0}^{it}$ can be written in terms of a selfadjoint generator as $\delta_{W_0}^{it} = e^{-itK_{W_0}}$ and therefore permits an "analytic continuation" in t to an unbounded densely defined positive operators $\delta_{W_0}^s$. Poincaré covariance permits to extend these definitions to wedges in general position, and intersections of wedges lead to the definitions for general localization regions (see later). Since the localization is clear from the context, a generic notation without subscripts will be used. With the help of this operator one defines the unbounded antilinear operator $\mathfrak{s}$ which has the same dense domain.

$$\mathfrak{s} = \mathfrak{j}\delta^{\frac{1}{2}}, \ dom\,\mathfrak{s} = dom\,\delta^{\frac{1}{2}} \tag{37}$$

$$\mathfrak{j}\delta^{\frac{1}{2}}\mathfrak{j} = \delta^{-\frac{1}{2}} \tag{38}$$

Whereas the unitary operator δ^{it} commutes with the reflection, the antiunitarity of the reflection causes a change of sign in the analytic continuation as written in the second line. This leads to the involutivity of the s-operator as well as the identity of its range with its domain

$$\mathfrak{s}^2 \subset \mathbf{1}$$

$$dom \ \mathfrak{s} = ran \ \mathfrak{s}$$

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[49] With somewhat different motivations and lesser mathematical rigor see also [21].

[50] The construction works for arbitrary positive energy representations, not only irreducible ones.

[51] The wedge reflection $\mathfrak{j}_{W_0}$ differs from the TCP operator only by a π-rotation around the W_0 axis.

. Such operators which are unbounded and yet involutive on their domain are quite unusual; according to my best knowledge they only appear in modular theory and it is precisely these unusual properties which are capable to encode geometric localization properties into domain properties of abstract quantum operators. The more general algebraic context in which Tomita discovered modular theory will be mentioned later.

The idempotency means that the s-operator has ± 1 eigenspaces; since it is antilinear the +space multiplied with i changes the sign and becomes the - space; hence it suffices to introduce a notation for just one of the two eigenspaces

$$\mathfrak{K}(W) = \{domain \ of \ \Delta_W^{\frac{1}{2}}, \ \mathfrak{s}_W \psi = \psi\} \tag{39}$$
$$\mathfrak{j}_W \mathfrak{K}(W) = \mathfrak{K}(W') = \mathfrak{K}(W)', \ duality$$
$$\overline{\mathfrak{K}(W) + i\mathfrak{K}(W)} = H_1, \ \mathfrak{K}(W) \cap i\mathfrak{K}(W) = 0$$

It is important to be aware that, unlike QM, we are here dealing with real (closed) subspaces $\mathfrak{K}$ of the complex one-particle Wigner representation space H_1.

An alternative which avoids the use of real subspaces is to directly work with complex dense subspaces as in the third line. Introducing the graph norm of the dense space, the complex subspace in the third line becomes a Hilbert space in its own right. The upper dash on regions in the second line denotes the causal disjoint (which is the opposite wedge) whereas the dash on real subspaces means the simplectic complement with respect to the simplectic form $Im(\cdot, \cdot)$ on H_1.

The two equations in the third line are the defining property of what is called the *standardness property* of a subspace[52]; any standard K-space permits to define an abstract s-operator as follows

$$\mathfrak{s}(\psi + i\varphi) = \psi - i\varphi \tag{40}$$
$$\mathfrak{s} = \mathfrak{j}\delta^{\frac{1}{2}}$$

whose polar decomposition (written in the second line) returns the two modular objects δ^{it} and $\mathfrak{j}$ which outside the context of the Poincaré group has in general no geometric significance. The domain of the Tomita s-operator is the same as the domain of $\delta^{\frac{1}{2}}$ namely the real sum of the K space and its imaginary multiple. Note that in the present context this domain is determined solely by Wigner's group representation theory.

It is easy to obtain a net of K-spaces by $U(a, \Lambda)$-transforming the K-space for the distinguished W_0. A bit more tricky is the construction of sharper localized subspaces via intersections

$$\mathfrak{K}(\mathcal{O}) = \bigcap_{W \supset \mathcal{O}} \mathfrak{K}(W) \tag{41}$$

where $\mathcal{O}$ denotes a causally complete smaller region (noncompact spacelike cone, compact double cone). Intersection may not be standard, in fact they may be zero in which case

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[52]According to the Reeh-Schlieder theorem a local algebra $\mathcal{A}(\mathcal{O})$ in QFT is in standard position with respect to the vacuum i.e. it acts on the vacuum in a cyclic and separating manner. The spatial standardness, which follows directly from Wigner representation theory, is just the one-particle projection of the Reeh-Schlieder property.

the theory allows localization in W (it always does) but not in $\mathcal{O}$. Such a theory is still causal but not local in the sense that its associated free fields are pointlike.

There are three classes of irreducible positive energy representation, the family of massive representations ($m > 0, s$) with half-integer spin s and the family of massless representation which consists really of two subfamilies with quite different properties namely the $(0, h)$, h half-integer class (the neutrino, photon class), and the rather large class of $(0, \kappa > 0)$ infinite helicity representations parametrized by a continuous-valued Casimir invariant κ [31].

For the first two classes the $\mathfrak{K}$-space is standard for arbitrarily small $\mathcal{O}$ but this is definitely not the case for the infinite helicity family for which the compact localization spaces turn out to be trivial[53]. Their tightest localization, which still permits nontrivial (in fact standard) $\mathfrak{K}$-spaces for *all* positive energy representations, is that of a *spacelike cone* [20] with an arbitrary small opening angle whose core is a *semiinfinite string* [31]; after "second quantization (see next subsection) these strings become the localization region of string-like localized covariant generating fields[54]. The modular localization of states, which is governed by the unitary representation theory of the Poincaré group, has only two kind of generators: pointlike state and semiinfinite stringlike states; generating states of higher dimensionality ("brane states") are not needed.

Although the observation that the third Wigner representation class is not pointlike generated was made many decades ago, the statement that it is semiinfinite string-generated and that this is the worst possible case of state localization is of a more recent vintage [20] since it needed the support of the modular theory.

There is a very subtle aspect of modular localization which one encounters in the second Wigner representation class of *massless finite helicity representations* (the photon, graviton..class). Whereas in the massive case all spinorial fields $\Psi^{(A,\dot{B})}$ the relation of the physical spin s with the two spinorial indices follows the naive angular momentum composition rules [38]

$$\left| A - \dot{B} \right| \leq s \leq \left| A + \dot{B} \right|, \ m > 0 \tag{42}$$

$$s = \left| A - \dot{B} \right|, \ m = 0$$

the second line contains the considerably reduced number of spinorial descriptions for zero mass and finite helicity although in both cases the number of pointlike generators which are linear in the Wigner creation and annihilation operators [31].

By using the recourse of string-localized generators $\Psi^{(A,\dot{B})}(x, e)$ one can *restore the full spinorial spectrum* for a given s i. e. one can move from the second line to the first line in (42) by relaxing the localization. Even in the massive situation where pointlike generators exist but have short distance singularities which increase with spin. there may be good reasons (lowering of short distance dimension down to sdd=1) to use string-like generators. In all cases these generators are covariant and "string-local"

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[53]It is quite easy to prove the standardness for spacelike cone localization (leading to singular stringlike generating fields) just from the positive energy property which is shared by all three families [20].

[54]The epithet "generating" refers to the tightest localized singular field (operator-valued distribution) which generates the spacetime-indexed net of algebras in a QFT. In the case of localization of states the generators are state-valued distributions.

$$U(\Lambda)\Psi^{(A,\dot{B})}(x,e)U(\Lambda) = D^{(A,\dot{B})}(\Lambda^{-1})\Psi^{(A,\dot{B})}(\Lambda x,\Lambda e) \tag{43}$$

$$\left[\Psi^{(A,\dot{B})}(x,e),\,\Psi^{(A',\dot{B}')}(x',e')\right]_{\pm} = 0,\ \ x+\mathbb{R}_+e >< x'+\mathbb{R}_+e'$$

Here the unit vector e is the spacelike direction of the semiinfinite string and the last line expresses the spacelike fermionic/bosonic spacelike commutation. The best known illustration is the $(m=0,s=1)$ representation; in this case it is well-known that although a generating pointlike field strength exists, there is no pointlike vectorpotential. The modular localization approach offers as a substitute a stringlike vector potential $A_\mu(x,e)$. In the case $(m=0,s=2)$ the "field strength" is a fourth degree tensor which has the symmetry properties of the Riemann tensor; in fact it is often referred to as the linearized Riemann tensor. In this case the string-localized potential is of the form $g_{\mu\nu}(x,e)$ i.e. resembles the metric tensor of general relativity. The consequences of this localization for a reformulation of gauge theory will be mentioned in a separate subsection.

The most radical form of string localization occurs in the massless infinite spin representation family. In that case the representation space does not contain any pointlike localized generators which play the role of field strength hence such a theory is without any local observables.

A different kind of spacelike string-localization arises in d=1+2 Wigner representations with anomalous spin [79]. The amazing power of this modular localization approach is that it preempts the spin-statistics connection in the one-particle setting, namely if s is the spin of the particle (which in d=1+2 may take on any real value) then one finds for the connection of the simplectic complement with the causal complement the generalized duality relation

$$\mathfrak{K}(\mathcal{O}') = Z\mathfrak{K}(\mathcal{O})'$$

where the square of the twist operator $Z = e^{\pi i s}$ is easily seen (by the connection of Wigner representation theory with the two-point function) to lead to the statistics phase: $Z^2 = $ statistics phase [79]. The one-particle modular theory also leads to a relation which may be considered as the proto-form of crossing in the one-particle space

$$\mathfrak{d}^{\frac{1}{2}}\mathfrak{j}\psi(p) = \overline{\psi(-p)} \tag{44}$$

in words the $\mathfrak{d}^{\frac{1}{2}}\mathfrak{j} = \mathfrak{s}^*$ transformed wave function is equal to the complex conjugate (antiparticle) of the from forward to backward mass shell analytically continued (through the connecting complex mass shell) wave function.

The fact that one never has to go beyond string localized wave functions (and fact, apart from those mentioned cases, even never beyond point localization) in order to obtain the generating fields for a QFT is remarkable in view of the many attempts to introduce extended objects into QFT.

It should be clear that modular localization, which is formulated in terms of either real or dense complex subspaces, cannot be connected with probabilities and projectors. It is rather related to causal localization aspects and the standardness of the K-space for a compact region is nothing else then the one-particle version of the Reeh-Schlieder property. Fortunately one needs the probability carrying BNW localization only for asymptotic timelike scattering distances where it becomes covariant.

8.2 Localized subalgebras

A net of real subspaces $\mathfrak{K}(\mathcal{O}) \subset H_1$ for an finite spin (helicity) Wigner representation can be "second quantized"[55] via the CCR (Weyl) respectively CAR quantization functor; in this way one obtains a covariant $\mathcal{O}$-indexed net of von Neumann algebras $\mathcal{A}(\mathcal{O})$ acting on the bosonic or fermionic Fock space $H = Fock(H_1)$ built over the one-particle Wigner space H_1. For integer spin/helicity values the modular localization in Wigner space implies the identification of the simplectic complement with the geometric complement in the sense of relativistic causality, i.e. $\mathfrak{K}(\mathcal{O})' = \mathfrak{K}(\mathcal{O}')$ (spatial Haag duality in H_1). The Weyl functor takes the spatial version of Haag duality into its algebraic counterpart. One proceeds as follows: for each Wigner wave function $\varphi \in H_1$ the associated (unitary) Weyl operator is defined as

$$Weyl(\varphi) := expi\{a^*(\varphi) + a(\varphi)\}, Weyl(\varphi) \in B(H) \tag{45}$$
$$\mathcal{A}(\mathcal{O}) := alg\{Weyl(\varphi)|\varphi \in \mathfrak{K}(\mathcal{O})\}'', \quad \mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}')$$

where $a^{\#}(\varphi)$ are the usual Fock space creation and annihilation operators of a Wigner particle in the wave function φ. We then define the von Neumann algebra corresponding to the localization region $\mathcal{O}$ in terms of the operator algebra generated by the functorial image of the modular constructed localized subspace $\mathfrak{K}(\mathcal{O})$ as written in the second line. By the von Neumann double commutant theorem, our generated operator algebra is weakly closed by definition.

The functorial relation between real subspaces and von Neumann algebras via the Weyl functor preserves the causal localization structure and hence the spatial duality passes to its algebraic counterpart. The functor also commutes with the process of sharpening localization through intersections $\cap$ according to $K(\mathcal{O}) = \cap_{W \supset O} K(W)$, $\mathcal{A}(\mathcal{O}) = \cap_{W \supset O} \mathcal{A}(W)$ as expressed in the commuting diagram

$$\begin{array}{ccc} \{K(W)\}_W & \longrightarrow & \{\mathcal{A}(W)\}_W \\ \downarrow \cap & & \downarrow \cap \\ K(\mathcal{O}) & \longrightarrow & \mathcal{A}(\mathcal{O}) \end{array} \tag{46}$$

Here the vertical arrows denote the tightening of localization by intersection, whereas the horizontal ones denote the action of the Weyl functor.

The case of half-integer spin representations is analogous [78], apart from the fact that there is a mismatch between the causal and simplectic complements to be taken care of by a *twist operator* $\mathcal{Z}$ and as a result one arrives at the CAR functor instead of the Weyl functor.

In case of the large family of irreducible zero mass infinite spin representations for which the lightlike little group, different from the finite helicity representations, is faithfully represented, the finitely localized K-spaces are trivial $\mathfrak{K}(\mathcal{O}) = \{0\}$ and the *most tightly localized nontrivial spaces are of the form* $\mathfrak{K}(\mathcal{C})$ for $\mathcal{C}$ *a spacelike cone*. As a double

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[55]The terminology 2nd quantization is a misdemeanor since one is dealing with a rigorously defined functor within QT which has little in common with the artful use of that parallellism to classical theory called "quantization". In Edward Nelson's words: (first) quantization is a mystery, but second quantization is a functor.

cone contracts to its pointlike core, the core of a spacelike cone $\mathcal{C}$ is a *covariant spacelike semiinfinite string*. The above functorial construction works the same way for the Wigner infinite spin representation except that there are no nontrivial compactly localized algebras with a smaller localization than $\mathcal{A}(\mathcal{C})$ and there are no generating fields which are sharper localized than a semiinfinite spacelike string. Point- (or string-) like covariant fields are singular generators of these algebras i.e. they are operator-valued distributions. Stringlike generators, which are also available in the pointlike case, turn out to have an improved short distance behavior; whereas e.g. the short distance dimension of a free pointlike vectorfield is $sddA_\mu(x) = 2$, its stringlike counterpart has $sddA_\mu(x, e) = 1$ [31] thanks to the fact that the vacuum fluctuations are spread into e as well. Covariant representations are constructed from the unique Wigner representation by so called intertwiners between the canonical and the many possible covariant (dotted-undotted spinor finite representations of the L-group) representations. Whereas for pointlike generators this is done by group theoretic methods as in [38], the construction of string-like intertwiners require the use of modular localization. The Euler-Lagrange formalism plays no role in these construction since the causal aspect of hyperbolic differential propagation are fully taken care of by modular localization.

A basis of local covariant field coordinatizations is defined by Wick composites of the free fields. The string-like fields do not follow the classical behavior; already before introducing composites one has a continuous family of non-classical intertwiners between the unique Wigner infinite spin representation and the *continuously many covariant string interwiners*. These non-classical aspects, in particular the absence of a Lagrangian, are the reason why their spacetime description in terms of semiinfinite string fields has been discovered only recently and not at the time of Jordan's field quantization nor at the time of Wigner's representation theory.

Using the standard notation Γ for the second quantization functor which maps real localized (one-particle) subspaces into localized von Neumann algebras and extending this functor in a natural way to include the functorial images of the $\mathfrak{K}(\mathcal{O})$-associated s, δ, j (denoted by S, Δ, J) one arrives at the Tomita Takesaki theory of the interaction-free local algebra $(\mathcal{A}(\mathcal{O}), \Omega)$ in standard position[56]

$$H_{Fock} = \Gamma(H_1) = e^{H_1}, \;\; \left(e^h, e^k\right) = e^{(h,k)} \tag{47}$$
$$\Delta = \Gamma(\delta), \;\; J = \Gamma(j), \;\; S = \Gamma(s)$$
$$SA\Omega = A^*\Omega, \;\; A \in \mathcal{A}(O), \;\; S = J\Delta^{\frac{1}{2}}$$

With this result we got to the core statement of the Tomita-Takesaki theorem which is a statement about the two modular objects Δ^{it} and J on the algebra

$$\sigma_t(\mathcal{A}(\mathcal{O})) \equiv \Delta^{it}\mathcal{A}(\mathcal{O})\Delta^{-it} = \mathcal{A}(\mathcal{O}) \tag{48}$$
$$J\mathcal{A}(\mathcal{O})J = \mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}')$$

in words: the reflection J maps an algebra (in standard position) into its von Neumann commutant and the unitary group Δ^{it} defines an one-parametric automorphism-group

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[56]The functor Γ preserves the standardness i.e. maps the spatial one-particle standardness into its algebraic counterpart.

σ_t of the algebra. In this form (but without the last geometric statement involving the geometrical causal complement $\mathcal{O}'$) the theorem hold in complete mathematical generality for standard pairs $(\mathcal{A}, \Omega)$. The free fields and their Wick composites are "coordinatizing" singular generators of this $\mathcal{O}$-indexed net of algebras in the sense that the smeared fields $A(f)$ with $supp f \subset \mathcal{O}$ are (unbounded operators) affiliated with $\mathcal{A}(\mathcal{O})$.

In the above second quantization context the origin of the T-T theorem and its proof is clear: the symplectic disjoint passes via the functorial operation to the operator algebra isimplectic and the spatial one-particle modular automorphism goes into its algebraic counterpart. The definition of the Tomita involution S through its action on the dense set of states (guarantied by the standardness of $\mathcal{A}$) as $SA\Omega = A^*\Omega$ and the action of the two modular objects Δ, J (47) is part of the general setting of the modular Tomita-Takesaki theory; standardness is the mathematical terminology for the Reeh-Schlieder property [14] i.e. the existence[57] of a vector $\Omega \in H$ with respect to which the algebra acts cyclic and has no "annihilators" of Ω. Naturally the proof of the abstract T-T theorem in the general setting of operator algebras or in the more restricted context of interacting QFT is more involved [14].

The important property which renders this useful beyond free fields as a new constructive tool in the presence of interactions, is that for $(\mathcal{A}(W), \Omega)$ the antiunitary involution J depends on the interaction, whereas Δ^{it} continues to be uniquely fixed by the representation of the Poincaré group i.e. by the particle content. In fact it has been known for some [21] time that J is related with its free counterpart J_0 through the scattering matrix

$$J = J_0 S_{scat} \tag{49}$$

This modular role of the scattering matrix as a relative modular invariant between an interacting theory and its free counterpart comes as a surprise. It is precisely this role which opens the way for an inverse scattering construction [52] and the constructive approach to factorizing models [22]. Closely related to this observation is the realization that the wedge region leads to a coexistence of one particle states in interacting theories (section 6) with modular localization; namely there is a dense set of wedge-localized one particle states and their multiparticle in/out extensions in the interacting theory. With other words the wedge region is the "smallest" region for which PFGs (vacuum **p**olarization **f**ree **g**enerators) and their multiparticle generalizations are available. This is the origin of the crossing property as explained in section 5.

For the construction of a QFT it suffices to specify wedge algebra $\mathcal{A}(W)$ for one particular wedge W as well as the action of the Poincaré group on $\mathcal{A}(W)$ which results in a net of wedge algebras $\{\mathcal{A}(W)\}_{W \in \mathfrak{W}}$. Knowing a wedge algebra means knowing its position in the global algebra $\mathcal{A}(W) \subset B(H)$; in practice this is achieved by describing $\mathcal{A}(W)$ in terms of generators as explained before in the special case of factorizing models. By taking suitable intersections of wedge algebras one obtains (in case the double cone intersections are nontrivial) a net of local observables i.e. a nontrivial local QFT or (if they are trivial) there is no local QFT associated with the system of wedge algebras. In this

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[57]In QFT any finite energy vector (which of course includes the vacuum) has this property as well as any nondegenerated KMS state. In the mathematical setting it is shown that standard vectors are "$\delta-$dense" in H.

way one is able to separate the existence proof for a local QFT from the harder problem of the construction of its pointlike fields[58] via their correlation functions or formfactors. Hence the construction of a QFT may be seen as a generalization of those ideas which lead to a proof of the crossing property.

An "observable net" is a spacetime-indexed family of operator algebras consisting of chargeless operators. By definition these operators fulfill spacelike commutativity and have, as the vacuum, vanishing charge. There exists a very deep theory which automatically constructs all charged sectors and combines them to a generally quite large "field-algebra" which in a way defines the maximal extension of the observable algebra; this is the famous Doplicher-Haag-Roberts (DHR) superselection theory [14]. It explains statistics and inner symmetries in terms of spacetime localization properties of the observable net[59]. From a point of view of principles of QFT one can show that in more than 3 dimensions all compact groups can appear. What does not appear in this classification is supersymmetry.

There is a slight reformulation of this algebraic setting which leads to a (philosophically) quite spectacular new view of the core nature of local quantum physics. Namely it is possible to encode the entire content of QFT i.e. the net of local observables *as well* as all its superselected charge sectors and their interpolating charged fields including the representation of the Poincaré group acting on it, into a finite set of copies of the monads (physically interpreted as $\mathcal{A}(W)s$) carefully positioned in a joint Hilbert space with the help of modular theory, using concepts of "modular inclusion" and "modular intersection " within a joint Hilbert space [19]. The representation theory of the Poincaré group and therefore spacetime itself arises from the joint action of the individual modular groups in the form of unitary operators in the shared Hilbert space. This is as close as one can get to how Leibniz envisaged reality as emerging from *relations* between monads, the monads (here copies of the unique hyperfinite Type III$_1$ factor algebra) themselves being structureless[60].

It is an interesting open question whether a characterization of a QFT in terms of positioning of a finite number of monads can be extended to curved spacetime. The recent successful quantum formulation of the principle of local covariance [80] nourishes some hope that this may be the case.

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[58]The necessarily singular pointlike fields are universal generators for algebras of arbitrary (small) localization.

[59]There is a complication in low-dimensional theories in which braid group statistics may occur in which case there is no sharp separation between inner and spacetime symmetries. Nevertheless these representations appear in the DHR theory [14].

[60]In this sense they are like points in geometry except that monads can be mutually included and intersected.

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