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UNITARY SYMMETRY AND THE HIGH-ENERGY PHOTONUCLEAR REACTIONS

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II, III and IV.

As the photon has a vanishing U-spin, the initial state in a photonuclear reaction has the U-spin of the nuclear target. Its possible final states are then obtained by combining baryon multiplets of table I or table III with meson multiplets of table II or table IV, in such a way that - among other variables - their electric and baryonic charge, strangeness and U-spin be equal to the corresponding quantum number of the initial state. The relations among the amplitudes of the possible reactions are then a straightforward consequence of the U-spin conservation.

In §1 the matrix element relationships for single meson production in a photoproton reaction are reviewed. They have been obtained previously by several authors [3]. One can allow for transitions induced [4] by the mass-splitting interaction - which breaks the unitary symmetry - but no experimentally significant information on the corresponding amplitude seems to be obtainable even in these simplest reactions.

In §2, the relations among the amplitudes for the photoproduction of meson pairs on proton are deduced. They are given in equations (21) to (25) and (27)[5]. The equation (29) in §3 constitutes an example for the case of the photoproduction of three mesons on proton.

The photoproduction of mesons by a nucleus is accomplished through a very large number of channels. It is possible, however, to obtain relatively simple amplitude relations for some of these channels. The study of the correlations among the products of these reactions may yield information on short-lived hypernuclei. The unitary symmetry affords a systematics of the products obtainable; this is illustrated in §5, where we consider some channels of the photodisintegration of helium.

In §4, the equations (30) and (30a) relate amplitudes for the production of single mesons in the photodisintegration of the deuteron. In particular, the relations (31) and (32) correspond to the production of baryon resonances in this disintegration.

1. Single Photomeson Production On Proton

The initial state of a photoproton reaction is $|U = 1/2, U_z = 1/2\rangle$. The following are therefore the possible final-state multiplet combinations for a single photomeson production on proton:

$$\gamma + p \longrightarrow \begin{pmatrix} p \\ \Sigma^+ \end{pmatrix} + \frac{1}{2}\left(\sqrt{3}\,\pi^0 - \eta\right), \tag{1}$$

$$\longrightarrow \begin{pmatrix} p \\ \Sigma^+ \end{pmatrix} + \begin{pmatrix} & K^0 & \\ \frac{1}{2}\bigl(\pi^0 & + & \sqrt{3}\,\eta\bigr) \\ & \bar{K}^0 & \end{pmatrix}, \tag{2}$$

$$\longrightarrow \begin{pmatrix} & n & \\ \frac{1}{2}\bigl(\Sigma^0 & + & \sqrt{3}\,\Lambda\bigr) \\ & \Xi^0 & \end{pmatrix} + \begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix}, \tag{3}$$

$$\longrightarrow \frac{1}{2}\left(\sqrt{3}\,\Sigma^0 - \Lambda\right) + \begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix}, \tag{4}$$

$$\longrightarrow N^{*++} + \begin{pmatrix} \pi^- \\ K^- \end{pmatrix}, \qquad (5)$$

plus those which follow from the above ones by the substitutions:

$$\begin{pmatrix} p \\ \Sigma^+ \end{pmatrix} \longleftrightarrow \begin{pmatrix} N^{*+} \\ Y_1^{*+} \end{pmatrix}, \quad \begin{pmatrix} n \\ \frac{1}{2}(\Sigma^0 + \sqrt{3}\ ) \\ \Xi^0 \end{pmatrix} \longleftrightarrow \begin{pmatrix} N^{*0} \\ Y^{*0} \\ \Xi^{*0} \end{pmatrix};$$

$$\frac{1}{2}(\sqrt{3}\,\pi^0 - \eta) \longleftrightarrow \frac{1}{2}(\sqrt{3}\,\rho^0 - \omega_8), \begin{pmatrix} K^0 \\ \frac{1}{2}(\pi^0 + \sqrt{3}\,\eta) \\ \overline{K}^0 \end{pmatrix} \longleftrightarrow \begin{pmatrix} K^{*0} \\ \frac{1}{2}(\rho^0 + \sqrt{3}\,\omega_8) \\ \overline{K}^{*0} \end{pmatrix} \qquad (6)$$

$$\begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix} \longleftrightarrow \begin{pmatrix} K^{*+} \\ \rho^+ \end{pmatrix}, \quad \begin{pmatrix} \pi^- \\ K^- \end{pmatrix} \longleftrightarrow \begin{pmatrix} \rho^- \\ K^{*-} \end{pmatrix}.$$

The multiplets in (2) give the following channels:

$$\gamma + p \longrightarrow p + \frac{1}{2}(\pi^0 + \sqrt{3}\,\eta), \qquad (7)$$
$$\longrightarrow \Sigma^+ + K^0,$$

which are mixed in the final state as a result of the Glebsch-Gordan coupling of the partial U-spins $U^{(1)} = 1/2$, $U^{(2)} = 1$. Call this partial final state $|U^{(1)} = 1/2, U^{(2)} = 1; U = 1/2, U_z = 1/2\rangle$ and let R be the operator which transforms the initial into the final state:

$R|\gamma p\rangle = f|U^{(1)} = 1/2, U^{(2)} = 1; U = 1/2, U_z = 1/2\rangle$ + all other partial final states. (8)

The Clebsch-Gordan composition of the final state indicated in (8) is:

$|U^{(1)} = 1/2, U^{(2)} = 1; U = 1/2, U_z = 1/2\rangle = -(1/\sqrt{3})|U^{(1)} = 1/2, U^{(2)} = 1; U_z^{(1)} = 1/2, U_z^{(2)} = 0\rangle + \sqrt{\frac{2}{3}}\,|U^{(1)} = 1/2, U^{(2)} = 1; U_z^{(1)} = -1/2, U_z^{(2)} = 1\rangle$.

We therefore obtain (considering the minus sign in the state $|1,1\rangle$ in the tables) for the amplitudes of the reactions (7):

$$\langle p\,\pi^{0}|R|\gamma\,p\rangle + \sqrt{3}\,\langle p\,\eta\,|R|\gamma\,p\rangle = \sqrt{2}\,\langle \Sigma^{+}\,K^{0}|R|\gamma\,p\rangle . \quad (9)$$

A comparison of this relation with the one derived for the possible reactions in (3) gives:

$$\frac{\langle \Sigma^{0}\,K^{+}|R|\gamma\,p\rangle + \sqrt{3}\,\langle \Lambda K^{+}|R|\gamma\,p\rangle}{\langle n\,\pi^{+}|R|\gamma\,p\rangle} = \frac{\langle p\pi^{0}|R|\gamma p\rangle + \sqrt{3}\,\langle p\,\eta|R|\gamma p\rangle}{\langle \Sigma^{+}\,K^{0}|R|\,\gamma\,p\rangle} .$$

This equality - and those which result from it by the substitutions (6) - are presumably valid only for photon energies large in comparison with the mass differences within the multiplets and for which, therefore, the unitary symmetry is good.

At low energies, one may expect that the effect of the mass-splitting interaction will not be negligible. As this interaction behaves like the sum of a U-scalar and a U-vector - its expectation value giving the multiplet mass relations - it will lead to transitions into a final state which may contain components with $U = 3/2$, $U_z = 1/2$ (if one takes the U-vector parallel to the z-axis in the U-space). In this case, one may write for the reactions (7) (in lieu of (8)):

$$R|\gamma p\rangle = f\,|U^{(1)} = \tfrac{1}{2},\ U^{(2)} = 1;\ U = \tfrac{1}{2},\ U_z = \tfrac{1}{2}\rangle + g\,|U^{(1)} = \tfrac{1}{2},\ U^{(2)}=1;$$

$$U = \tfrac{3}{2},\ U_z = \tfrac{1}{2}\rangle + \text{all other partial final states}$$

whence:

$$\langle p\,\tfrac{1}{2}\,(\pi^{0}+\sqrt{3}\,\eta)|R|\gamma p\rangle = -\frac{1}{\sqrt{3}}\,f + \sqrt{\frac{2}{3}}\,g ,$$

$$-\langle \Sigma^{+}\,K^{0}|R|\,\gamma\,p\rangle = \sqrt{\frac{2}{3}}\,f + \frac{1}{\sqrt{3}}\,g .$$

The amplitude g, which arises from the violation of the SU_3-symmetry by the mass-difference terms, is thus related to the reaction amplitudes by the equation:

$$\langle p\,\pi^0|R|\gamma p\rangle + \sqrt{3}\,\langle p\eta|R|\gamma p\rangle - \sqrt{2}\,\langle \Sigma^+ K^0|R|\gamma\, p\rangle = \sqrt{6}\ g(E)$$

At high energies we expect that g will decrease to zero. At low energies, however, below the threshold for the production of kaons, one should have:

$$\sqrt{2}\ f\ (E) = -\ g(E), \quad \text{for } E < K - \text{production threshold.}$$

The behaviour of the amplitude g(E) is therefore of the form:

$$g(E'') \cong \frac{1}{\sqrt{6}} \langle p\ \pi^0|R|\ \gamma\, p\rangle_{E''} \quad \text{for } E'' < K - \text{production threshold;}$$

$$g(E') \cong \frac{1}{\sqrt{6}} \left\{ \langle p\ \pi^0|R|\ \gamma\, p\rangle_{E'} - \sqrt{2}\,\langle \Sigma^+ K^0|R|\gamma p\rangle_{E'} \right\},$$

for

$$K - \text{production threshold} < E'' < \eta - \text{production threshold;}$$

$$g(E) = \frac{1}{\sqrt{6}} \left\{ \langle p\ \pi^0|R|\gamma\ p\rangle + \sqrt{3}\ \langle p\,\eta|R|\gamma p\rangle - \sqrt{2}\,\langle \Sigma^+\ K^0|R|\gamma p\rangle \right\}$$

for $E > \eta$ - production threshold; and:

$$\lim_{E \to \infty} g(E) = 0$$

2. Photoproduction of Meson Pairs on Proton

The following are the possible final-state multiplet combinations corresponding to the production of meson pairs on proton:

$$\gamma + p \longrightarrow \frac{1}{2}\,(\sqrt{3}\,\Sigma^0 - \Lambda) + \begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix} + \frac{1}{2}\,(\sqrt{3}\,\pi^0 - \eta)\ , \qquad (11)$$

$$\longrightarrow \frac{1}{2}\,(\sqrt{3}\,\Sigma^0 - \Lambda) + \begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix} + \begin{pmatrix} K^0 \\ \frac{1}{2}\left(\pi^0 + \sqrt{3}\,\eta\right) \\ \bar{K}^0 \end{pmatrix}, \qquad (12)$$

$$\rightarrow \begin{pmatrix} p \\ \Sigma^{+} \end{pmatrix} + \frac{1}{2}(\sqrt{3}\,\pi^{0} - \eta) + \frac{1}{2}(\sqrt{3}\,\pi^{0} - \eta)\,, \tag{13}$$

$$\rightarrow \begin{pmatrix} p \\ \Sigma^{+} \end{pmatrix} + \frac{1}{2}(\sqrt{3}\,\pi^{0} - \eta) + \begin{pmatrix} K^{0} \\ \frac{1}{2}(\pi^{0} + \sqrt{3}\,\eta) \\ \overline{K}^{0} \end{pmatrix} \tag{14}$$

$$\rightarrow \begin{pmatrix} p \\ \Sigma^{+} \end{pmatrix} + \begin{pmatrix} K^{0} \\ \frac{1}{2}(\pi^{0} + \sqrt{3}\,\eta) \\ \overline{K}^{0} \end{pmatrix} + \begin{pmatrix} K^{0} \\ \frac{1}{2}(\pi^{0} + \sqrt{3}\,\eta) \\ \overline{K}^{0} \end{pmatrix}\,, \tag{15}$$

$$\rightarrow \begin{pmatrix} p \\ \Sigma^{+} \end{pmatrix} + \begin{pmatrix} K^{+} \\ \pi^{+} \end{pmatrix} + \begin{pmatrix} \pi^{-} \\ K^{-} \end{pmatrix}\,, \tag{16}$$

$$\rightarrow \begin{pmatrix} \Sigma^{-} \\ \Xi^{-} \end{pmatrix} + \begin{pmatrix} K^{+} \\ \pi^{+} \end{pmatrix} + \begin{pmatrix} K^{+} \\ \pi^{+} \end{pmatrix}\,, \tag{17}$$

$$\rightarrow \begin{pmatrix} n \\ \frac{1}{2}(\Sigma^{0} + \sqrt{3}\,\Lambda) \\ \Xi^{0} \end{pmatrix} + \begin{pmatrix} K^{+} \\ \pi^{+} \end{pmatrix} + \frac{1}{2}(\sqrt{3}\,\pi^{0} - \eta)\,, \tag{18}$$

$$\rightarrow \begin{pmatrix} n \\ \frac{1}{2}(\Sigma^{0} + \sqrt{3}\,\Lambda) \\ \Xi^{0} \end{pmatrix} + \begin{pmatrix} K^{+} \\ \pi^{+} \end{pmatrix} + \begin{pmatrix} K^{0} \\ \frac{1}{2}(\pi^{0} + \sqrt{3}\,\eta) \\ \overline{K}^{0} \end{pmatrix}. \tag{19}$$

To these one must add the multiplet combinations which follow from the above ones by the substitutions (6).

We now write in explicit form the contribution of these partial channels to the final state, in the case of exact unitary symmetry:

$$R|\gamma p\rangle = \sum_{\alpha} \sum_{U^{(i)}} f\left(U^{(1)}, U^{(2)}, U^{(3)}; \alpha\right) |U^{(1)}, U^{(2)}, U^{(3)}; U, U_{z}; \alpha\rangle + \ldots =$$

$$= \sum_{\alpha} \Big\{ f_{1}(\alpha) \left|\tfrac{1}{2}, 0, 0; \tfrac{1}{2}, \tfrac{1}{2}; \alpha\right\rangle + f_{2}(\alpha) \left|\tfrac{1}{2}, 1, 0; \tfrac{1}{2}, \tfrac{1}{2}; \alpha\right\rangle +$$

$$+ f_{3}(\alpha) |(U^{(1)} = \tfrac{1}{2}, U^{(2)} = 1)\; U_{12} = \tfrac{1}{2};\; U^{(3)} = 1; \tfrac{1}{2}, \tfrac{1}{2}; \alpha\rangle +$$

$$+ f_4(\alpha)\left|\left(U^{(1)}=\frac{1}{2},\ U^{(2)}=1\right)U_{12}=\frac{3}{2};\ U^{(3)}=1;\ \frac{1}{2},\ \frac{1}{2};\ \alpha\right\rangle +$$

$$+ f_5(\alpha)\left|\left(U^{(1)}=\frac{1}{2},\ U^{(2)}=\frac{1}{2}\right)U_{12}=1;\ U^{(3)}=\frac{1}{2};\ \frac{1}{2},\frac{1}{2};\alpha\right\rangle + \qquad (20)$$

$$+ f_6(\alpha)\left|\left(U^{(1)}=\frac{1}{2},\ U^{(2)}=\frac{1}{2}\right)U_{12}=0;\ U^{(3)}=\frac{1}{2};\ \frac{1}{2},\ \frac{1}{2};\ \alpha\right\rangle + \dots$$

α stands for the variables which distinguish reactions with different particles but the same U-spin multiplets, such as (11) and (13). Each of the f-terms represents a partial state which results from the coupling of three U-spins to give a state with $U = 1/2$, $U_z = 1/2$. U_{12} is the resultant of the coupling of $U^{(1)}$ and $U^{(2)}$. The pertinent Clebsch-Gordan developments are:

$$\left|U^{(1)}=\frac{1}{2},\ U^{(2)}=0,\ U^{(3)}=0;\ U=\frac{1}{2},\ U_z=\frac{1}{2}\right\rangle = \left|U^{(1)}=\frac{1}{2},\ U^{(2)}=0,\ U^{(3)}=0;\ U_z^{(1)}=\frac{1}{2},\ U_z^{(2)}=U_z^{(3)}=0\right\rangle;$$

$$\left|U^{(1)}=\frac{1}{2},\ U^{(2)}=1,\ U^{(3)}=0;\ U=\frac{1}{2},\ U_z=\frac{1}{2}\right\rangle = -\frac{1}{\sqrt{3}}\left|U^{(1)}=\frac{1}{2},\ U^{(2)}=1,\ U^{(3)}=0;\ U_z^{(1)}=\frac{1}{2},\ U_z^{(2)}=U_z^{(3)}=0\right\rangle + \sqrt{\frac{2}{3}}\left|U^{(1)}=\frac{1}{2},\ U^{(2)}=1,\ U^{(3)}=0;\ U_z^{(1)}=-\frac{1}{2},\ U_z^{(2)}=1,\ U_z^{(3)}=0\right\rangle;$$

$$\left|\left(U^{(1)}=\frac{1}{2},\ U^{(2)}=1\right)U_{12}=\frac{1}{2},\ U^{(3)}=1;\ U=\frac{1}{2},\ U_z=\frac{1}{2}\right\rangle = -\frac{2}{3}\left|U^{(1)}=\frac{1}{2},\ U^{(2)}=1,\ U^{(3)}=1;\ U_z^{(1)}=\frac{1}{2},\ U_z^{(2)}=-1,\ U_z^{(3)}=1\right\rangle + \frac{\sqrt{2}}{3}\left|U^{(1)}=\frac{1}{2},\ U^{(2)}=1,\ U^{(3)}=1;\ U_z^{(1)}=-\frac{1}{2},\ U_z^{(2)}=0,\ U_z^{(3)}=1\right\rangle + \frac{1}{3}\left|U^{(1)}=\frac{1}{2},\ U^{(2)}=1,\ U^{(3)}=1;\ U_z^{(1)}=\frac{1}{2},\ U_z^{(2)}=0,\ U_z^{(3)}=0\right\rangle - \frac{\sqrt{2}}{3}\left|U^{(1)}=\frac{1}{2},\ U^{(2)}=1,\ U^{(3)}=1;\ U_z^{(1)}=-\frac{1}{2},\ U_z^{(2)}=1,\ U_z^{(3)}=0\right\rangle;$$

$$\left|\left(U^{(1)}=\tfrac{1}{2}, U^{(2)}=1\right)U_{12}=\tfrac{3}{2}, U^{(3)}=1; U=\tfrac{1}{2}, U_z=\tfrac{1}{2}\right\rangle=\frac{\sqrt{2}}{6}\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=1, U^{(3)}=1;\right.$$

$$\left.U_z^{(1)}=\tfrac{1}{2}, U_z^{(2)}=-1, U_z^{(3)}=1\right\rangle+$$

$$+\frac{1}{3}\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=1, U^{(3)}=1; U_z^{(1)}=-\tfrac{1}{2}, U_z^{(2)}=0, U_z^{(3)}=1\right\rangle-$$

$$-\frac{\sqrt{2}}{3}\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=1; U_z^{(1)}=\tfrac{1}{2}, U_z^{(2)}=0, U_z^{(3)}=0\right\rangle- \qquad (20a)$$

$$-\frac{1}{3}\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=1, U^{(3)}=1; U_z^{(1)}=\tfrac{1}{2}, U_z^{(2)}=1, U_z^{(3)}=0\right\rangle+$$

$$+\frac{\sqrt{2}}{2}\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=1, U^{(3)}=1; U_z^{(1)}=\tfrac{1}{2}, U_z^{(2)}=1, U_z^{(3)}=-1\right\rangle;$$

$$\left|\left(U^{(1)}=\tfrac{1}{2}, U^{(2)}=\tfrac{1}{2}\right)U_{12}=1, U^{(3)}=\tfrac{1}{2}; U=\tfrac{1}{2}, U_z=\tfrac{1}{2}\right\rangle=$$

$$=\sqrt{\frac{2}{3}}\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=\tfrac{1}{2}, U^{(3)}=\tfrac{1}{2}; U_z^{(1)}=U_z^{(2)}=\tfrac{1}{2}, U_z^{(3)}=-\tfrac{1}{2}\right\rangle-$$

$$-\frac{\sqrt{6}}{6}\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=\tfrac{1}{2}, U^{(3)}=\tfrac{1}{2}; U_z^{(1)}=\tfrac{1}{2}, U_z^{(2)}=-\tfrac{1}{2}, U_z^{(3)}=\tfrac{1}{2}\right\rangle-$$

$$-\frac{\sqrt{6}}{6}\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=\tfrac{1}{2}, U^{(3)}=\tfrac{1}{2}; U_z^{(1)}=-\tfrac{1}{2}, U_z^{(2)}=\tfrac{1}{2}, U_z^{(3)}=\tfrac{1}{2}\right\rangle;$$

$$\left|\left(U^{(1)}=\tfrac{1}{2}, U^{(2)}=\tfrac{1}{2}\right)U_{12}=0, U^{(3)}=\tfrac{1}{2}; U=\tfrac{1}{2}, U_z=\tfrac{1}{2}\right\rangle=$$

$$=\frac{1}{\sqrt{2}}\left\{\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=\tfrac{1}{2}, U^{(3)}=\tfrac{1}{2}, U_z^{(1)}=\tfrac{1}{2}, U_z^{(2)}=-\tfrac{1}{2}, U_z^{(3)}=\tfrac{1}{2}\right\rangle-\right.$$

$$\left.-\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=\tfrac{1}{2}, U^{(3)}=\tfrac{1}{2}; U_z^{(1)}=-\tfrac{1}{2}, U_z^{(2)}=\tfrac{1}{2}, U_z^{(3)}=\tfrac{1}{2}\right\rangle\right. .$$

The identification of the possible final states of the channels

(11) to (19) is now straightforward. We list as an example the states resulting from the coupling of the U-spins 1/2, 1 and 1:

$$\left|U^{(1)}=\tfrac{1}{2},\ U^{(2)}=1,\ U^{(3)}=1;\ U_z^{(1)}=\tfrac{1}{2}, U_z^{(2)}=-1, U_z^{(3)}=1;\alpha_1\right\rangle = -\left|K^+\bar{K}^0\ n\right\rangle ,$$

$$\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=1,\ U^{(3)}=1; U_z^{(1)}=-\tfrac{1}{2},\ U_z^{(2)}=0, U_z^{(3)}=1;\alpha_1\right\rangle = -\left|\pi^+\ \tfrac{1}{2}(\pi^0+\sqrt{3}\eta)n\right\rangle ,$$

$$\left|U^{(1)}=\tfrac{1}{2},\ U^{(2)}=1,\ U^{(3)}=1;\ U_z^{(1)}=\tfrac{1}{2}, U_z^{(2)}=0,\ U_z^{(3)}=0;\ \alpha_1\right\rangle = \left|K^+\ \tfrac{1}{2}(\pi^0+\sqrt{3}\eta)\ \tfrac{1}{2}\ (\Sigma^0+\sqrt{3}\ \Lambda)\right\rangle ,$$

$$\left|U^{(1)}=\tfrac{1}{2},\ U^{(2)}=1,\ U^{(3)}=1; U_z^{(1)}=-\tfrac{1}{2}, U_z^{(2)}=1,\ U_z^{(3)}=0;\ \alpha_1\right\rangle = - \left|\pi^+\ K^0\ \tfrac{1}{2}\ (\Sigma^0+\sqrt{3}\Lambda)\right\rangle ,$$

$$\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=1, U^{(3)}=1; U_z^{(1)}=\tfrac{1}{2}\ U_z^{(2)}=1, U_z^{(3)}=-1;\alpha_1\right\rangle = -\left|K^+K^0\ \Xi^0\right\rangle ,$$

$$\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=1, U^{(3)}=1; U_z^{(1)}=\tfrac{1}{2}\ ,\ U_z^{(2)}=-1,\ U_z^{(3)}=1;\alpha_2\right\rangle = -\left|p\ \bar{K}^0K^0\right\rangle ,$$

$$\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=1, U^{(3)}=1; U_z^{(1)}=-\tfrac{1}{2}, U_z^{(2)}=0, U_z^{(3)}=1, \alpha_2\right\rangle = -\left|\Sigma^+\ \tfrac{1}{2}(\pi^0+\sqrt{3}\eta)K^0\right\rangle ,$$

$$\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=1,\ U^{(3)}=1;\ U_z^{(1)}=+\tfrac{1}{2},\ \ U_z^{(2)}=0,\ U_z^{(3)}=0,\ \alpha_2\right\rangle = \left|p\ \tfrac{1}{2}\ (\pi^0+\sqrt{3}\eta)\ \tfrac{1}{2}\ (\pi^0+\sqrt{3}\ \eta\ )\right\rangle ,$$

$$\left|U^{(1)}=\tfrac{1}{2}, U^{(2)}=1,\ U^{(3)}=1;\ U_z^{(1)}=-\tfrac{1}{2}, U_z^{(2)}=1,\ U_z^{(3)}=0,\ \alpha_2\right\rangle = -\left|\Sigma^+\ K^0\ \tfrac{1}{2}\ (\pi^0+\sqrt{3}\ \eta\ )\right\rangle ,$$

$$\left| U^{(1)}=\tfrac{1}{2}, U^{(2)}=1, U^{(3)}=1; U_z^{(1)}=\tfrac{1}{2}, U_z^{(2)}=1, U_z^{(3)}=-1, \alpha_2 \right\rangle = -|p\, K^0\bar{K}^0\rangle.$$

The substitution of these formulae into equation (20) gives the following relations among reactions amplitudes:

$$\langle\pi^-\pi^+p|R|\gamma p\rangle + \langle K^-K^+p|R|\gamma p\rangle = -\langle\pi^-K^+\Sigma^+|R|\gamma p\rangle\,,$$
$$\langle\pi^+K^+\Sigma^-|R|\gamma p\rangle + \langle K^+\pi^+\Sigma^-|R|\gamma p\rangle = -\langle\Xi^-K^+K^+|R|\gamma p\rangle\,; \qquad (21)$$

$$\frac{\sqrt{2}}{2}\langle(\sqrt{3}\,\pi^0-\eta)(\pi^0+\sqrt{3}\,\eta)p|R|\gamma p\rangle = \langle(\sqrt{3}\,\pi^0-\eta)K^0\Sigma^+|R|\gamma p\rangle\,,$$
$$\frac{\sqrt{2}}{2}\langle(\sqrt{3}\,\pi^0-\eta)(\Sigma^0+\sqrt{3}\Lambda)K^+|R|\gamma p\rangle = \langle(\sqrt{3}\,\pi^0-\eta)n\,\pi^+|R|\gamma p\rangle\,, \qquad (22)$$
$$\frac{\sqrt{2}}{2}\langle(\sqrt{3}\,\Sigma^0-\Lambda)(\pi^0+\sqrt{3}\eta)K^+|R|\gamma p\rangle = \langle(\sqrt{3}\,\Sigma^0-\Lambda)K^0\,\pi^+|R|\gamma p\rangle\,;$$

$$\sqrt{2}\,\langle n\,\bar{K}^0K^+|R|\gamma p\rangle - \tfrac{1}{2}\langle n(\pi^0+\sqrt{3}\,\eta)\,\pi^+|R|\gamma p\rangle =$$
$$= \frac{\sqrt{2}}{4}\,\langle(\Sigma^0+\sqrt{3}\Lambda)(\pi^0+\sqrt{3}\,\eta)K^+|R|\gamma p\rangle + \langle(\Sigma^0+\sqrt{3}\,\Lambda)K^0\pi^+|R|\gamma p\rangle\,,$$
$$\sqrt{2}\,\langle n\,\bar{K}^0K^+|R|\gamma p\rangle + \langle n(\pi^0+\sqrt{3}\,\eta)\,\pi^+|R|\gamma p\rangle =$$
$$= \frac{\sqrt{2}}{4}(\Sigma^0+\sqrt{3}\Lambda)(\pi^0+\sqrt{3}\,\eta)K^+|R|\gamma p\rangle - \tfrac{1}{2}\langle(\Sigma^0+\sqrt{3}\,\Lambda)K^0\,\pi^+|R|\gamma p\rangle =$$
$$= \sqrt{2}\,\langle\Xi^0\,K^0K^+|R|\gamma p\rangle\,; \qquad (23)$$

$$\sqrt{2}\,\langle K^0\bar{K}^0p|R|\gamma p\rangle - \tfrac{1}{2}\langle K^0(\pi^0+\sqrt{3}\,\eta)\,\Sigma^+|R|\gamma p\rangle =$$
$$= \frac{\sqrt{2}}{4}\,\langle(\pi^0+\sqrt{3}\eta)(\pi^0+\sqrt{3}\eta)p|R|\gamma p\rangle + \langle(\pi^0+\sqrt{3}\,\eta)K^0\Sigma^+|R|\gamma p\rangle\,,$$
$$\sqrt{2}\,\langle K^0\bar{K}^0p|R|\gamma p\rangle + \langle K^0(\pi^0+\sqrt{3}\,\eta)\,\Sigma^+|R|\gamma p\rangle = \qquad (24)$$
$$= \frac{\sqrt{2}}{4}\,\langle(\pi^0+\sqrt{3}\,\eta)(\pi^0+\sqrt{3}\,\eta)p|R|\gamma p\rangle - \tfrac{1}{2}\langle(\pi^0+\sqrt{3}\eta)K^0\Sigma^+|R|\gamma p\rangle =$$
$$= \sqrt{2}\,\langle\bar{K}^0K^0p|R|\gamma p \quad .$$

As a consequence of these equations one has:

$$\langle n(\pi^0+\sqrt{3}\,\eta)\pi^+|R|\gamma p\rangle = -\langle(\Sigma^0+\sqrt{3}\,\Lambda)K^0\,\pi^+|R|\gamma p\rangle\,,$$

$$\langle K^0(\pi^0+\sqrt{3}\,\eta)\,\Sigma^+|R|\gamma p\rangle = -\langle(\pi^0+\sqrt{3}\,\eta)\,K^0\,\Sigma^+|R|\gamma p\rangle,$$

$$\sqrt{2}\,\langle(\bar{K}^0K^0 - K^0\bar{K}^0)p|R|\gamma p\rangle = \langle K^0(\pi^0+\sqrt{3}\eta)\,\Sigma^+\,|R|\gamma p\rangle\,, \qquad (25)$$

$$\langle\bar{K}^0K^0 + K^0\bar{K}^0)p|R|\gamma p\rangle = \frac{1}{2}\langle(\pi^0+\sqrt{3}\,\eta)(\pi^0+\sqrt{3}\,\eta)p|R|\gamma p\rangle\,.$$

If ones defines the fictitious spin-zero particle $b_o = \frac{1}{2}(\pi^0+\sqrt{3}\eta)$, the second of relations (25) states that the reaction which produces the mesons K^0 and b_o can only lead to final states with odd orbital angular momenta of these particles relative to their center of mass: indeed, K^0 and b_o must form a wave function with $U = 1$, $U_z = 1$, which is antisymmetric in the U-spin variables; the principle that a two boson wave function must be totally symmetric in space, spin and U-spin variables leads to space antisymmetric states in the reaction under consideration.

Besides the remaining equations which are obtained from (21), (22), (23), (24) by the substitutions (6), the quartet of baryons of table III gives rise, when produced in a photoproton reaction, to other amplitudes relationships. The reactions we have in mind are:

$$\gamma + p \longrightarrow \begin{pmatrix} N^{*-} \\ Y_1^{*-} \\ \Xi^{*-} \\ \Omega^- \end{pmatrix} + \begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix} + \begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix} \qquad (26)$$

together with those which result from the substitution of the pseudoscalar doublet by the positive charge vector meson doublet of table IV. The final state in (26) will be, in the case of exact unitary symmetry, the result of the composition of the baryon quartet, U = 3/2, with the triplet, U = 1, formed by the two meson doublets. It will thus be described by a wave function of the form of (20a):

$$\frac{\sqrt{6}}{6}\left|U^{(1)}=\tfrac{3}{2}, U^{(2)}=1; U_z^{(1)}=-\tfrac{1}{2}, U_z^{(2)}=1\right\rangle - \frac{\sqrt{3}}{3}\left|U^{(1)}=\tfrac{3}{2}, U^{(2)}=1;\right.$$

$$\left.U_z^{(1)}=\tfrac{1}{2}, U_z^{(2)}=0\right\rangle + \frac{\sqrt{2}}{2}\left|U^{(1)}=\tfrac{3}{2}, U^{(2)}=1; U_z^{(1)}=\tfrac{3}{2}, U_z^{(2)}=-1\right\rangle$$

One therefore obtains:

$$\langle N^{*-}\pi^+\pi^+|R|\gamma p\rangle|\langle Y_1^{*-}\frac{1}{\sqrt{2}}(K^+\pi^+ + \pi^+K^+)|R|\gamma p\rangle|\langle \Xi^{*-}K^+K^+|R|\gamma p\rangle =$$
$$= \frac{\sqrt{2}}{2}\left|-\frac{\sqrt{3}}{3}\right|\frac{\sqrt{6}}{6}. \quad (27)$$

We can compare the production of K and π mesons accompanied of the resonances Y^* and Ξ^* with the production of the same mesons accompanied of the baryons Σ and Ξ. The relations (27) give:

$$\frac{\langle Y_1^{*-}\frac{1}{\sqrt{2}}(K^+\pi^+ + \pi^+K^+)|R|\gamma p\rangle}{\langle \Xi^{*-}K^+K^+|R|\gamma p\rangle} = -\sqrt{2} \qquad (27a)$$

whereas the equations (21) lead to:

$$\frac{\langle \Sigma^-\frac{1}{\sqrt{2}}(K^+\pi^+ + \pi^+K^+)|R|\gamma p\rangle}{\langle \Xi^- K^+K^+|R|\gamma p\rangle} = -\frac{1}{\sqrt{2}} \qquad (21a)$$

The wave function $\frac{1}{\sqrt{2}}|K^+\pi^+ + \pi^+K^+\rangle$ is symmetric in the U-spin variables since it is identical to $|U = 1, U_z = 0\rangle$. Here again, the symmetry of the wave function of two bosons under the permutation of the space, spin and U-spin variables, implies the conclusion that both reactions:

$$\gamma + p \longrightarrow \Sigma^{-} + K^{+} + \pi^{+}$$
$$\gamma + p \longrightarrow Y_1^{*-} + K^{+} + \pi^{+}$$

give rise to final states in which the two mesons K^+ and π^+ can only have even orbital angular momenta relative to their center of mass.

3. Photoproduction of Three Mesons on Proton

In this case, the relations among amplitudes afforded by invariance under the unitary U-spin group, are still more complicated than the ones derived above. Consider, for example the reaction:

$$\gamma + p \longrightarrow \begin{pmatrix} p \\ \Xi^{+} \end{pmatrix} + \begin{pmatrix} K^{+} \\ \pi^{+} \end{pmatrix} + \begin{pmatrix} \pi^{-} \\ K^{-} \end{pmatrix} + \begin{pmatrix} \frac{1}{2}(\begin{matrix} K^0 \\ \pi^0 \\ \bar{K}^0 \end{matrix} + \sqrt{3}\,\eta) \end{pmatrix} . \qquad (28)$$

In the right-hand side we may combine the last two doublets to give a singlet and a triplet, $U^{(3)} = 0, 1$; these will then couple with the doublet and quartet, $U_{12} = 1/2, 3/2$, resulting from the sum of the remaining triplet and doublet, $U^{(1)} = 1$, $U^{(2)} = 1/2$. We thus have:

$$R|\gamma p\rangle = \ldots\ldots + \sum_{\alpha}\Big\{ f_1^{\frac{1}{2}}(\alpha) | (U^{(1)} = \tfrac{1}{2},\ U^{(2)} = 1)\, U_{12} = \tfrac{1}{2},\ U^{(3)} = 1;$$
$$U = \tfrac{1}{2},\ U_z = \tfrac{1}{2}\rangle + f_0^{\frac{1}{2}}(\alpha) | (U^{(1)} = \tfrac{1}{2},\ U^{(2)} = 1) U_{12} = \tfrac{1}{2},\ U^{(3)} = 0; U = \tfrac{1}{2},$$
$$U_z = \tfrac{1}{2}\rangle + f_1^{3/2}(\alpha) | (U^{(1)} = \tfrac{1}{2},\ U^{(2)} = 1) U_{12} = \tfrac{3}{2},\ U^{(3)} = 1;$$
$$U = \tfrac{1}{2},\ U_z = \tfrac{1}{2}\rangle + f_0^{3/2}(\alpha) | (U^{(1)} = \tfrac{1}{2},\ U^{(2)} = 1) U_{12} = \tfrac{3}{2}, U^{(3)} = 0;$$
$$U = \tfrac{1}{2},\ U_z = \tfrac{1}{2}\rangle + \ldots\ .$$

The procedure of the preceding paragraphs leads us to the following set of relations among the amplitudes of (28):

$$\langle K^-(\pi^0+\sqrt{3}\eta)K^+ p|R|\gamma p\rangle - \langle \pi^-(\pi^0+\sqrt{3}\eta)\pi^+ p|R|\gamma p\rangle -$$

$$- \langle \pi^-(\pi^0+\sqrt{3}\eta)K^+\Sigma^+|R|\gamma p\rangle =$$

$$= 2\sqrt{2}\left\{\langle \pi^-\overline{K}^0K^+p|R|\gamma p\rangle + \langle K^-K^0\pi^+p|R|\gamma p\rangle + \langle K^-K^0K^+\Sigma^+|R|\gamma p\rangle\right\} ;$$

$$2\langle K^-(\pi^0+\sqrt{3}\eta)K^+ p|R|\gamma p\rangle + \langle \pi^-(\pi^0+\sqrt{3}\eta)\pi^+p|R|\gamma p\rangle +$$

$$+ \langle \pi^-(\pi^0+\sqrt{3}\eta)K^+\Sigma^+|R|\gamma p\rangle =$$

$$= \sqrt{2}\left\{-2\langle \pi^-\overline{K}^0K^+p|R|\gamma p\rangle + \langle K^-K^0\pi^+p|R|\gamma p\rangle + \langle K^-K^0K^+\Sigma^+|R|\gamma p\rangle\right\} ;$$

whence:

$$\langle K^-(\pi^0+\sqrt{3}\eta)K^+p|R|\gamma p\rangle = \sqrt{2}\left\{\langle K^-K^0\pi^+p|R|\gamma p\rangle + \langle K^-K^0K^+\Sigma^+|R|\gamma p\rangle\right\} \quad (29)$$

4. Single Photoproduction of Mesons on Deuteron

The U-spin of a nucleus with Z protons and A-Z neutrons is $U(A,Z) = A-Z/2$. For the deuteron, $U = 3/2$, $U_z = 3/2$.

Among the photodeuteron reactions which produce a meson, the following ones are of interest for the derivation of relations among their channel amplitudes

$$\gamma + d \longrightarrow \begin{pmatrix} p \\ \Sigma^+ \end{pmatrix} + \begin{pmatrix} n \\ \frac{1}{2}(\Sigma^0 + \sqrt{3}\Lambda) \\ \Xi^0 \end{pmatrix} + \begin{pmatrix} K^0 \\ \frac{1}{2}(\pi^0 + \sqrt{3}\eta) \\ \overline{K}^0 \end{pmatrix} ;$$

$$\longrightarrow \begin{pmatrix} K^+ \\ \pi^+ \end{pmatrix} + \begin{pmatrix} n \\ \frac{1}{2}(\Sigma^0 + \sqrt{3}\Lambda) \\ \Xi^0 \end{pmatrix} + \begin{pmatrix} n \\ \frac{1}{2}(\Sigma^0 + \sqrt{3}\Lambda) \\ \Xi^0 \end{pmatrix} .$$

The final state is a superposition of the form:

$$R|\gamma d\rangle = f_1^{3/2}\left\{-\sqrt{\tfrac{3}{5}}|pn\,\tfrac{1}{2}(\pi^0+\sqrt{3}\eta)\rangle - \sqrt{\tfrac{2}{5}}\left[-\sqrt{\tfrac{2}{3}}\,|p\,\tfrac{1}{2}(\Sigma^0+\sqrt{3}\Lambda)K^0\rangle + \right.\right.$$

$$+ \frac{1}{\sqrt{3}} \left|\Sigma^+ n\ K^o\right\rangle\Big]\Big\} + f_1^{\frac{1}{2}} \left\{\sqrt{\tfrac{1}{3}} \left|p\ \tfrac{1}{2}(\Sigma^o + \sqrt{3}\ \Lambda)K^o\right\rangle + \sqrt{\tfrac{2}{3}} \left|\Sigma^+ n\ K^o\right\rangle\right\} +$$

$$+ g_1^{3/2}\Big\{- \sqrt{\tfrac{3}{5}} \left|K^+ n\ \tfrac{1}{2}\ (\Sigma^o + \sqrt{3}\ \Lambda)\right\rangle - \sqrt{\tfrac{2}{5}} \Big[- \sqrt{\tfrac{2}{3}} \left|K^+\ \tfrac{1}{2}\ (\Sigma^o + \sqrt{3}\ \Lambda)\ n\right\rangle +$$

$$+ \frac{1}{\sqrt{3}} \left|\pi^+ n\ n\right\rangle\Big]\Big\} + g_1^{\frac{1}{2}} \left\{ \sqrt{\tfrac{1}{3}} \left|K^+\ \tfrac{1}{2}\ (\Sigma^o + \sqrt{3}\ \Lambda)n\right\rangle + \sqrt{\tfrac{2}{3}} \left|\pi^+ n\ n\right\rangle\right\} + \ldots$$

from which the following equations are deduced:

$$\left\langle\Big[K^o(\Sigma^o + \sqrt{3}\ \Lambda) + (\pi^o + \sqrt{3}\,\eta)n\Big]p\,|R|\gamma d\right\rangle = \sqrt{2}\left\langle K^o\ n\ \Sigma^+|R|\gamma d\right\rangle ; \qquad (30)$$

$$\left\langle\Big[n(\Sigma^o + \sqrt{3}\ \Lambda) + (\Sigma^o + \sqrt{3}\ \Lambda)n\Big]K^+|R|\gamma d\right\rangle = \sqrt{2}\left\langle n\ n\ \pi^+|R|\gamma d\right\rangle . \qquad (30a)$$

The wave function $\frac{1}{\sqrt{2}}\ |n\ Z_o + Z_o\ n\rangle$, $Z_o = \frac{1}{2}\ (\Sigma^o + \sqrt{3}\,\Lambda)$, is a $U = 2$, $U_z = 1$ state, symmetric in the U-spin variables; it will be antisymmetric in the space and spin variables of n and Z_o, as for the wave function $|nn\rangle$.

There are other photodeuteron reactions in which the produced meson is accompanied of the baryon quartet. Thus:

$$\gamma + d \rightarrow \begin{pmatrix} N^{*-} \\ Y_1^{*-} \\ \Xi^{*-} \\ \Omega^- \end{pmatrix} + \left(\tfrac{1}{2}\left(\begin{matrix} K^o \\ \pi^o \\ \bar{K}^o \end{matrix} + \sqrt{3}\,\eta\right)\right) + N^{*++}$$

and we have:

$$R|\gamma d\rangle = \ldots + f\ \left\{ \sqrt{\tfrac{3}{5}}\ \left|\left(U^{(1)} = \tfrac{3}{2},\ \ U^{(2)} = 0\right)\ U_{12} = \tfrac{3}{2},\ U^{(3)} = 1;\right.\right.$$

$$\left.(U_{12})_z = \tfrac{3}{2},\ U_z^{(3)} = 0\right\rangle - \sqrt{\tfrac{2}{5}}\ \left|\left(U^{(1)} = \tfrac{3}{2},\ U^{(2)} = 0\right)\ U_{12} = \tfrac{3}{2},\ U^{(3)} = 1;\right.$$

$$\left.(U_{12})_z = \tfrac{1}{2},\ U_z^{(3)} = 1\right\rangle + \ldots\ .$$

We thus obtain the ratio:

$$\frac{\langle(\pi^0+\sqrt{3}\eta)\ N^{*++}\ N^{*-}|R|\gamma d\rangle}{\langle K^0\ N^{*++}\ Y_1^{*-}|R|\gamma d\rangle} = \sqrt{6}\ . \tag{31}$$

Finally for the channel:

$$\gamma + d \longrightarrow \begin{pmatrix} N^{*-} \\ Y_1^{*-} \\ \Xi^{*-} \\ \Omega^{-} \end{pmatrix} + \begin{pmatrix} p \\ \Sigma^+ \end{pmatrix} + \begin{pmatrix} K^* \\ \pi^+ \end{pmatrix}$$

One obtains:

$$\frac{\langle N^{*-}(p\ \pi^+ + \Sigma^+\ K^+)|R|\gamma d\rangle}{\langle Y_1^{*-}\ p\ K^+|R|\gamma d\rangle} = \sqrt{3}\ . \tag{32}$$

5. Photoproduction of Mesons on Helium

The photoproduction of mesons by heavier nuclei may yield information on the interaction of hyperons with nuclei leading to the formation of hypernuclei.

Thus, the photodisintegration of helium may proceed through several channels. If the unitary symmetry is good, those channels for which the final nucleus is formed of, say, three baryons, belong to certain U-spin multiplets. Consider, as an example, the following reactions:

$$\gamma + He^4 \longrightarrow \begin{pmatrix} He^3 \\ \circ\circ\circ \\ \circ\circ\circ \\ \circ\circ\circ \\ \circ\circ\circ \end{pmatrix} + \begin{pmatrix} n \\ \frac{1}{2}\left(\Sigma^0 + \sqrt{3}\ \eta\right) \\ \Xi^0 \end{pmatrix} + \begin{pmatrix} K^0 \\ \frac{1}{2}\left(\pi^0 + \sqrt{3}\ \eta\right) \\ \bar{K}^0 \end{pmatrix} \tag{33}$$

and

$$\gamma + He^4 \longrightarrow \begin{pmatrix} H^3 \\ \circ\circ\circ \\ \circ\circ\circ \\ \circ\circ\circ \\ \circ\circ\circ \\ \circ\circ\circ \end{pmatrix} + \begin{pmatrix} p \\ \Sigma^+ \end{pmatrix} + \begin{pmatrix} K^0 \\ \frac{1}{2}\left(\pi^0 + \sqrt{3}\ \eta\right) \\ \bar{K}^0 \end{pmatrix}. \tag{34}$$

In reaction (33), the U-spin of the initial state is equal to that of He^4, namely, $U=3$, $U_z=3$. The multiplet which He^3 belongs to is a quintet. Tritium belongs to a sextet. The quintet results from the coupling of the deuteron quartet:

$$\begin{pmatrix} d = -|(pn)\rangle \\ \sqrt{\frac{2}{3}}\,|p\,Z^o\rangle - \frac{1}{\sqrt{3}}\,|\Sigma^+\,n\rangle \\ \frac{1}{\sqrt{3}}\,|p\,\Xi^o\rangle + \sqrt{\frac{2}{3}}\,|\Sigma^+\,Z^o\rangle \\ |\Sigma^+\,\Xi^o\rangle \end{pmatrix}$$

with $\begin{pmatrix} p \\ \Sigma^+ \end{pmatrix}$. The state $|2,2\rangle$ is He^3. The state $|2,1\rangle$ is:

$$|2,1\rangle = -|(n\,p\,\Sigma^+)\rangle + \frac{\sqrt{2}}{2}\,|p\,p\,Z^o)\rangle\ , \quad Z^o = \frac{1}{2}\,(\Sigma^o + \sqrt{3}\,\Lambda)\ ,$$

where the wave function $|(n\,p\,\Sigma^+)\rangle$ is symmetrical in the U-spin variables of p and Σ^+:

$$|(n\,p\,\Sigma^+)\rangle = |n\,\tfrac{1}{2}\,(p\,\Sigma^+ + \Sigma^+\,p)\rangle$$

Call:

$$|_{\Sigma^+}He^3\rangle = -\,|(n\,p\,\Sigma^+)\rangle\ , \quad |_{Z^o}He^3\rangle = |(p\,p\,Z^o)\rangle\ . \tag{35}$$

The reaction (33):

$$\gamma + He^4 \longrightarrow \begin{pmatrix} He^3 \\ {}_{\Sigma^+}He^3 + \frac{\sqrt{2}}{2}\ {}_{Z^o}He^3 \\ \cdots\cdots \\ \cdots\cdots \\ \cdots\cdots \end{pmatrix} + \begin{pmatrix} n \\ Z^o \\ \Xi^o \end{pmatrix} + \begin{pmatrix} K^o \\ \frac{1}{2}\,(\pi^o + \sqrt{3}\,\eta) \\ \bar{K}^o \end{pmatrix}$$

gives the following contribution to the final state $R|\gamma\,He^4\rangle$:

$$-(\sqrt{3}/2)\, f_1 |He^3\, n\, \tfrac{1}{2}\,(\pi^0 + \sqrt{3}\,\eta)\rangle - \left(\sqrt{\tfrac{2}{3}}\, f_2 - \tfrac{\sqrt{3}}{6}\, f_1\right) |He^3\, Z^0\, K^0\rangle -$$

$$- \frac{\sqrt{3}}{3}\left(f_2 + \frac{\sqrt{2}}{2}\, f_1\right) |_{\Sigma^+}\, He^3\, n\, K^0\rangle - \frac{\sqrt{3}}{6}\,(f_1 + \sqrt{2}\, f_2)|_{Z^0} He^3\, n\, K^0\rangle$$

from which one obtains:

$$\langle He^3\, Z^0\, K^0|R|\gamma\, He^4\rangle + \sqrt{2}\, \langle_{\Sigma^+} He^3\, n\, K^0|R|\gamma\, He^4\rangle -$$

$$- 4\langle_{Z^0} He^3\, n\, K^0|R|\gamma\, He^4\rangle = -\langle He^3\, n\, \tfrac{1}{2}\,(\pi^0 + \sqrt{3}\,\eta)|R|\gamma\, He^4\rangle. \qquad (36)$$

In the same way we find for the channel (34):

$$\gamma + He^4 \rightarrow \begin{pmatrix} H^3 \\ -\sqrt{\tfrac{2}{5}}\; {}_{Z^0}H^3 + \tfrac{1}{\sqrt{5}}\; {}_{\Sigma^+}H^3 \\ \cdots\cdots \\ \cdots\cdots \\ \cdots\cdots \\ \cdots\cdots \end{pmatrix} + \begin{pmatrix} p \\ \Sigma^+ \end{pmatrix} + \begin{pmatrix} K^0 \\ \tfrac{1}{2}\,(\pi^0 + \sqrt{3}\,\eta) \\ \bar{K}^0 \end{pmatrix}$$

where:

$$Z^0 = \tfrac{1}{2}\,(\Sigma^0 + \sqrt{3}\,\Lambda),\quad |_{Z^0}\, H^3\rangle = |p(n\, Z^0 + Z^0\, n)\rangle,\quad |_{\Sigma^+}\, H^3\rangle = |(\Sigma^+ n\, n)\rangle, \qquad (37)$$

the following equations:

$$\langle_{\Sigma^+}\, H^3\, p\, K^0|R|\gamma\, He^4\rangle = -\frac{1}{\sqrt{2}}\, \langle_{Z^0}\, H^3\, p\, K^0|R|\gamma\, He^4\rangle,$$

$$\langle H^3\, \Sigma^+\, K^0|R|\gamma\, He^4\rangle + 5\langle_{\Sigma^+}\, H^3\, p\, K^0|R|\gamma\, He^4\rangle =$$

$$= \sqrt{2}\,\langle H^3 p\, \tfrac{1}{2}\,(\pi^0 + \sqrt{3}\,\eta)|R|\gamma\, He^4\rangle.$$

According to the definitions (35) and (36), the wave functions of those hypernuclei are antisymmetric in the spin and space variables of two of the baryons - in virtue of the symmetry in the corresponding U-spin variables.

Table I

Baryon U-spin multiplets

Wave function $\|U, U_z\rangle$	Electric charge		
	+	0	-
Singlet $\|0,0\rangle$		$\frac{1}{2}(\sqrt{3}\|\Sigma^0\rangle - \|\Lambda\rangle)$	
Doublets $\begin{pmatrix} \|\frac{1}{2}, \frac{1}{2}\rangle \\ \|\frac{1}{2}, -\frac{1}{2}\rangle \end{pmatrix}$	$\begin{pmatrix} \|p\rangle \\ \|\Sigma^+\rangle \end{pmatrix}$		$\begin{pmatrix} \|\Sigma^-\rangle \\ \|\Xi^-\rangle \end{pmatrix}$
Triplets $\begin{pmatrix} \|1,1\rangle \\ \|1,0\rangle \\ \|1,-1\rangle \end{pmatrix}$		$\begin{pmatrix} -\|n\rangle \\ \frac{1}{2}(\|\Sigma^0\rangle + \sqrt{3}\|\Lambda\rangle) \\ \|\Xi^0\rangle \end{pmatrix}$	

Table II

Pseudoscalar meson U-spin multiplets

Wave function $\|U, U_z\rangle$	Electric charge		
	+	0	-
Singlet $\|0,0\rangle$		$\frac{1}{2}(\sqrt{3}\|\pi^0\rangle - \|\eta\rangle)$	
Doublets $\begin{pmatrix} \|\frac{1}{2}, \frac{1}{2}\rangle \\ \|\frac{1}{2}, -\frac{1}{2}\rangle \end{pmatrix}$	$\begin{pmatrix} \|K^+\rangle \\ \|\pi^+\rangle \end{pmatrix}$		$\begin{pmatrix} \|\pi^-\rangle \\ \|K^-\rangle \end{pmatrix}$
Triplets $\begin{pmatrix} \|1,1\rangle \\ \|1,0\rangle \\ \|1,-1\rangle \end{pmatrix}$		$\begin{pmatrix} -\|K^0\rangle \\ \frac{1}{2}(\|\pi^0\rangle + \sqrt{3}\|\eta\rangle) \\ \|\bar{K}^0\rangle \end{pmatrix}$	

Table III

Baryon resonance U-spin multiplets

Wave function $\|U, U_z\rangle$	Electric charge ++	+	0	-
Singlet $\|0,0\rangle$	$\|N^{*++}\rangle$			
Doublet $\begin{pmatrix} \|\frac{1}{2}, \frac{1}{2}\rangle \\ \|\frac{1}{2}, -\frac{1}{2}\rangle \end{pmatrix}$		$\|N^{*+}\rangle$ $\|Y_1^{*+}\rangle$		
Triplet $\begin{pmatrix} \|1, 1\rangle \\ \|1, 0\rangle \\ \|1, -1\rangle \end{pmatrix}$			$\begin{pmatrix} -\|N^{*o}\rangle \\ \|Y^{*o}\rangle \\ \|\Xi^{*o}\rangle \end{pmatrix}$	
Quartet $\begin{pmatrix} \|\frac{3}{2}, \frac{3}{2}\rangle \\ \|\frac{3}{2}, \frac{1}{2}\rangle \\ \|\frac{3}{2}, -\frac{1}{2}\rangle \\ \|\frac{3}{2}, -\frac{3}{2}\rangle \end{pmatrix}$				$\begin{pmatrix} \|N^{*-}\rangle \\ \|Y_1^{*-}\rangle \\ \|\Xi^{*-}\rangle \\ \|\Omega^{-}\rangle \end{pmatrix}$

Table IV

Vector meson U-spin multiplets

Wave function $\|U, U_z\rangle$	Electric charge +	0	-
Singlet $\|0,0\rangle$		$\frac{1}{2}(\sqrt{3}\|\rho^o\rangle - \|\omega_8\rangle)$	
Doublets $\begin{pmatrix} \|\frac{1}{2}, \frac{1}{2}\rangle \\ \|\frac{1}{2}, -\frac{1}{2}\rangle \end{pmatrix}$	$\begin{pmatrix} \|K^{*+}\rangle \\ \|\rho^{+}\rangle \end{pmatrix}$		$\begin{pmatrix} \|\rho^{-}\rangle \\ \|K^{*-}\rangle \end{pmatrix}$
Triplet $\begin{pmatrix} \|1,1\rangle \\ \|1,0\rangle \\ \|1, -1\rangle \end{pmatrix}$		$\begin{pmatrix} -\|K^{*o}\rangle \\ \frac{1}{2}(\|\rho^o\rangle + \sqrt{3}\|\omega_8\rangle) \\ \|\bar{K}^{o*}\rangle \end{pmatrix}$	

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