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$K_{\mu 3}$ AND K_{e3} DECAY AND THE NATURE OF STRANGENESS VIOLATING WEAK INTERACTIONS

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ABSTRACT. There is at present only partial and incomplete information on the nature of strangeness violating weak interactions. From the theoretical point of view the confirmation of universal vector-axial vector interaction is of the foremost importance. The best source of the required information is to be found in the analysis of the spectra of the decay products of $K_{\mu 3}$ and K_{e3}. A theoretical discussion of these spectra is carried out in this paper. It is shown that there exists an angular correlation $\frac{dT}{d \cos \alpha}$ between the direction of the neutrino momentum in the center of mass system of the lepton pair and the direction of the pion. This angular correlation is quadratic in $\cos \alpha$ independently of the structure of the form factors and provides the best means of identification of the type of coupling. Additional information concerning particularly to the form factor structure can be learned from the pion spectrum. A quantitative test, independent of the form factors, of the universality of the muon and electron interaction is suggested. Some quantitative properties of the electron spectrum in K_{e3}-decay, independing of the form factor structure, have been established. They are characteristic of the type of coupling. It is shown how one can obtain the form factor from the knowledge of the electron spectrum if there is only one type of interaction.

INTRODUCTION.

The attractive hypothesis that all leptonic decays proceed via an interaction with a leptonic vector current is based on the decay of strong interacting particles without strangeness, (β--decay, pion decay) and the muon decay. On the other hand in the decay of strange particles this hypothesis has not been verified. The information from $K_{\mu 2}$-decay together with the relative absence of the K_{e2} mode is that there must exist a strangeness violating (axial-vector) with the leptonic current and no pseudo-scalar coupling. But scalar, vector and tensor couplings do not contribute to these decay modes. Hence, either the existence of vector coupling or the inexistence of scalar and tensor has not been experimentally established for strangeness violating interactions. Since, actually, all theories of weak interactions[1-4] assume the hypothesis of a universal vector-axial vector interaction it is a question of fundamental importance to verify the validity of this hypothesis for strangeness violating processes. This kind of information has to be obtained from three body leptonic decay of strange particles of which $K_{\mu 3}$ and K_{e3} decays provide the best examples. Contrariwise to $K_{\mu 2}$ and K_{e2}, these decay modes proceed only via S, V or T coupling. The importance of the $K_{\mu 3}$ and K_{e3} decays arises from the fact that they can give direct information on the non-leptonic currents themselves. In addition to the type of coupling, one obtains information on the isospin character of the currents and on the validity of the selection rule $\Delta S = \Delta Q$. With respect to the strangeness violating currents they play the same role that β-decay represented for the knowledge of the strangeness conserving currents. Like in β-decay the experiments required to investigate the type of coupling can be classified in three groups:

1) Determination of the muon, electron[5], neutrino or pion[6] energy spectrum. This type of experiment does not require a complete knowledge of the kinematics of the events. Some data on the muon and positron spectrum in K^+ decay has already been obtained[7-10]. The determination of these spectra is, no doubt, the simplest experiment with charged K-mesons. For K^o-mesons, on the other hand, the neutrino spectrum can be obtained from decay in flight, as pointed out by Furuichi[11]. If the production of the K^o is observed, one could determine the pion spectrum, which is of primary interest.

2) Angular correlation measurements: pion-lepton[12] and lepton-lepton angular correlation. They require a complete determination of the kinematics of the events. This is now, a feasible bubble chamber experiment with K^o-mesons. The most complete information will of course be obtained when sufficient statistics of such events is accumulated, that a two dimensional analysis of a Dalitz plot becomes possible.

3) Polarization measurements. The simplest experiment of this kind is the measurement of the longitudinal polarization of μ-mesons in $K^+_{\mu 3}$[13-16] decay with the observation of the muon decay.

In the first two kinds of experiments the identification of the type of coupling is neater with K_{e3} rather than with $K_{\mu 3}$. However $K_{\mu 3}$-decay will be useful in the investigation of form factors. The polarization measurements would give clear indication of the type of coupling as well as information on the form factors, and time reversal invariance[17-18].

In this paper we shall discuss some theoretical questions related to the identification of the type of coupling by means of the first two kinds of experiments.

In section II the transition rate is written in terms of the total energy of the lepton pair in its center of mass system and the angle α between the center of mass momentum of the neutrino and the direction of the pion. It is shown that ($d\Gamma/d \cos \alpha$) is in general a quadratic form in $\cos \alpha$, which in K_{e3}-decay becomes a constant for pure scalar coupling, $\cos^2 \alpha$ and $\sin^2 \alpha$ respectively for tensor and vector. Together with this angular distribution, the pion spectrum is the best thing to look at, for the identification of the type of coupling. But the first is pratically independent of the structure of the form factors, whereas the second one can give all the information about them. A test of the identity of the muon and electron interactions is proposed, independing of the structure of the form factors. A test of time reversal invariance is also discussed. In all this section it is assumed that the kinematics of the events is completely determined.

Section III is devoted to the energy spectrum of electrons. The theoretical analysis becomes much simplified for the electron spectrum when the electron mass is neglected. It is then possible to establish some quantitative and qualitative properties of the spectrum, which are independent of the structure of the form factors, but characterize the type of coupling. The choice of a proper variable is, for this purpose, quite a relevant question. Thus, for an unambiguous test of vector coupling, the distribution of events should be plotted against the variable $\eta = E(1 - E/m_K)$, and the function $(dT/d\eta)$ against the variable $w=2E(W_e-E)/(m_K-2E)$. The maximum of $(dT/d\eta)$ occurs at a definite value of η(114 MeV) independently of the form factor structure. For (V,S.) and (V,T.) combinations the absolute maximum of $(dT/d\eta)$ cannot occur below η = 114 MeV.

Finally we consider the problem of determining the form factors from the electron energy spectrum. The solution is given for the cases of a pure scalar or vector interaction.

TRANSITION RATE AND KINEMATICS.

We consider the decay processes:

$$K_{\mu 3} \longrightarrow \pi + \mu + \nu$$
$$K_{e3} \longrightarrow \pi + e + \nu$$

assuming that the lepton pair is locally produced, which, in terms of Feynman diagrams, means that they are produced at the same vertex, via an interaction with scalar, vector or tensor coupling. Let E_π, E, E_ν be the energies of the final particles for decay at rest, satisfying the condition $E_\pi + E + E_\nu = m_K$. The maximum energy of the pion and the lepton (ℓ) are respectively:

$$W_\pi = (m_K^2 + m_\pi^2 - m_\ell^2)/2\, m_K \qquad (1)$$

$$W = (m_K^2 - m_\pi^2 + m_\ell^2)/2\, m_K$$

We shall introduce new variables, the total energy W of the lepton pair in its centre of mass system and the angle α between the direction of the center of mass momentum of the neutrino and the direction of the pion. These variables are related to E_π, E, E_ν by:

$$W^2 = (p_K - p_\pi)^2 = m_K^2 + m_\pi^2 - 2m_K\, E_\pi \qquad (2)$$

$$\cos\,\alpha = \left[(E - E_\nu)\;\; W^2 - m_\ell^2\,(m_K - E_\pi)\right]/(W^2 - m_\ell^2)\; p_\pi \qquad (3)$$

We also have:

$$2\, m_K(W_\pi - E_\pi) = W^2 - m_\ell^2 \qquad (4)$$

$$p_\pi = \left[(W^2 - m_K^2 - m_\pi^2)^2 - 4\, m_\pi^2\;\, m_K^2\right]^{\frac{1}{2}}/2\, m_K$$

For the electron, by neglecting the electron mass, (3) reduces to:

$$\cos\,\alpha = (E - E_\nu)/p_\pi \qquad (5)$$

The matrix element for the decay has the form

$$M = (2\pi)^4\,\delta^4(p_K - p_\pi - p - p_\nu)\,\frac{1}{m_K}\,\Sigma\;\bar{u}_\nu\, O_j\; u_\ell\; A^j \qquad (6)$$

where the O_j's are Dirac matrices corresponding to the type of coupling and the A^j's are the following covariant functions.

a) Scalar $(O_j = I)$; $A^j = g_S$

b) Vector $(O_j = \gamma_\alpha)$; $A^j = \frac{1}{m_K}\,(g_V\; p_K^\alpha + g_{V'}\; p_\pi^\alpha)$ (7)

c) Tensor $(O_j = i\sigma_{\alpha\beta})$; $A^j = \frac{1}{m_K{}^2}\, g_T\; p_K^\alpha\;\; p_\pi{}^\beta$

The g's are dimensionless form factors which depend on the pion energy alone, as a result of the assumption of local production of the lepton pair. The elementary transition rate for processes with a given final configuration can be written in the form:

$$dT = \frac{1}{(4\pi\, m_K)^3}\, \rho(W^2, \cos\alpha)\, \frac{p_\pi}{m_K}\, \frac{W^2 - m_\ell^2}{W^2}\, d\,W^2\, d\cos\alpha \tag{8}$$

The most general expression for ρ has been given in a previous paper[6], together with a discussion of the Dalitz plot in the variables E_π and E. In terms of W^2 and $\cos\alpha$ one obtains:

$$\rho = \frac{W^2 - m_\ell^2}{4W^2} \left\{ W^2 \left[(g_s + \frac{m_\ell}{m_K} f_o) + (g_T - \frac{m_\ell\, m_K}{W^2} f_1) \frac{p_\pi}{m_K} \cos\alpha \right]^2 + p_\pi^2 \sin^2\alpha \left| f_1 - \frac{m_\ell}{m_K} g_T \right|^2 \right\} \tag{9}$$

where f_o and f_1 are the scalar and vector form factors for vector coupling:[19]

$$f_o = \tfrac{1}{2}(g_V - g_{V'}) + \tfrac{1}{2}(g_V + g_{V'}) \frac{m_K^2 - m_\pi^2}{W^2} \tag{10}$$

$$f_1 = g_V + g_{V'} \,. \tag{11}$$

The angular correlation in α has a simple explanation: in its center of mass system the lepton pair is produced in the singlet state for scalar coupling g_s and f_o, and in a triplet state if the coupling is vector f_1 and tensor g_T. In K_{e3} the component of the total spin along the neutrino momentum is 1 and 0 respectively for vector and tensor coupling.

In the forthcoming discussion we shall examine the implications of two general hypothesis:

1) Invariance under time reversal. As a consequence of this as-

sumption the form factors can be taken to be real.

2) Universality of the weak interaction, in the sense that the couplings with muons and electrons, hence the form factors, are identical.

In the approximation where the electron mass is neglected the transition rate is independent of the form factor f_o. The other ones g_s, f_1, g_T can be determined from the K_{e3} distribution if assumption 1) is valid.

Similarly, from the $K_{\mu 3}$ distribution one can determine $g_s + \frac{m_\ell}{m_K} f_o$, g_T and f_1. If assumption 2) holds, then one can determine f_o and the overdetermination of f_1 and g_T provides a test of both assumptions. Let us write:

$$\frac{dT}{d\,W^2\,d\cos\alpha} = \frac{1}{(4\pi\,m_K)^3}\left[a_o(W^2) + \sqrt{2}\,a_1(W^2)\cos\alpha + a_2(W^2)\cos 2\alpha\right] \tag{12}$$

where

$$a_o\,(W^2) = \frac{p_\pi}{m_K}\left(\frac{W^2 - m_\ell^2}{W^2}\right)^2 \frac{1}{4}\left\{W^2\left|g_s + \frac{m_\ell}{m_K} f_o\right|^2 + \tfrac{1}{2}\,p_\pi^2 \times \left[\frac{W^2}{m_K^{\,2}}\left|g_T - \frac{m_\ell\,m_K}{W^2} f_1\right|^2 + \left|f_1 - \frac{m_\ell}{m_K} g_T\right|^2\right]\right\} \tag{13}$$

$$a_1(W^2) = \frac{p_\pi}{m_K}\left(\frac{W^2 - m_\ell^2}{W^2}\right)^2 \frac{\sqrt{2}\,p_\pi}{4m_K}\,W^2\,\mathrm{Re}\left(g_s + \frac{m_\ell}{m_K} f_o\right)\left(g_T - \frac{m_\ell\,m_K}{W^2} f_1\right)^* \tag{14}$$

$$a_2(W^2) = \frac{p_\pi}{m_K}\left(\frac{W^2 - m_\ell^2}{W^2}\right)^2 \frac{1}{8}\,p_\pi^2\left[\frac{W^2}{m_K^{\,2}}\left|g_T - \frac{m_\ell\,m_K}{W^2} f_1\right|^2 - \left|f_1 - \frac{m_\ell}{m_K} g_T\right|^2\right] =$$

$$= \frac{p_\pi}{m_K}\left(\frac{W^2 - m_\ell^2}{W^2}\right)^3 \frac{1}{8}\,p_\pi^2\left[\frac{W^2}{m_k^{\,2}}\,|g_T|^2 - |f_1|^2\right] \tag{15}$$

The coefficient a_o is positive definite, larger than (or equal to) the magnitudes of a_1 and a_2, and for all W^2 the condition $a_o + a_2 \geqslant \sqrt{2}\,|a_1|$ must hold. The last expression for $a_2(W^2)$ shows that, if the weak couplings with muons and electrons are identical, one obtains (for $m_\mu^2 < W^2 \leqslant (m_K - m_\pi)^2$)

$$(W^2 - m_\mu^2)^{-3}\, a_{2\mu}\,(W^2) \equiv (W^2 - m_e^2)^{-3}\, a_{2e}\,(W^2) \qquad (16)$$

independently of the validity of 1). If the above identity is verified, the $K_{\mu 3}$ and K_{e3} distributions taken together will provide five independent equations to determine the form factors. Now, if time reversal invariance holds, there will be four real quantities to be determined. The compatibility of the system of equations give a check on the validity of assumption 1). It is clear that this test is much weaker than the test on assumption 2).

The transition rate, (12) integrated over the pion energy, gives the pion-neutrino angular correlation:

$$\frac{dT}{d\cos\alpha} = \frac{1}{(4\pi\, m_K)^3}\,(A_o + \sqrt{2}\,A_1 \cos\alpha + A_2 \cos 2\alpha) \qquad (17)$$

or

$$\frac{dT}{d\cos\alpha} = \frac{A_o}{(4\pi\, m_K)^3}\,(1 + \sqrt{2}\,\lambda_1 \cos\alpha + \lambda_2 \cos 2\alpha) \qquad (18)$$

where

$$A_n = \int_{m_\ell^2}^{(m_K - m_\pi)^2} a_n(W^2)\, d\,W^2 \qquad (19)$$

The coefficients A_n satisfy the same innequalities as the a_n's. Hence:

$$|\lambda_1| \leqslant 1,\ |\lambda_2| \leqslant 1,\ 1 + \lambda_2 \geqslant \sqrt{2}\,|\lambda_1| \qquad (20)$$

This angular correlation will be extremely helpful in identifying the type of coupling. In particular in K_{e3}-decay it gives, in most cases, clear-cut discrimination of the nature of the interaction, independently of the structure of the form factors. A summary of the values of λ_1 and λ_2 for the possible combinations of S,V, and T coupling is given in Table 1. The mass of the electron is being neglected.

TABLE 1

	V	S	T	(V,S)	(V,T)	(S,T)
λ_1	0	0	0	0	0	$\neq 0$
λ_2	-1	0	1	< 0	$\neq 0$	> 0

There will be ambiguity only between the groups (V,S) and (V,T) if $\lambda_1 = 0$ and $\lambda_2 < 0$, and in distringuishing (S,T) from (S,T,V) if $\lambda_1 \neq 0$ and $\lambda_2 > 0$. The best way to differentiate between the two alternatives in both cases, is to look at the pion spectrum in K_{e3}--decay. We have:

$$\frac{dT}{dW^2} = \frac{1}{(4\pi\ m_K)^3}\ \frac{p_\pi}{2m_K}\left(W^2\left|g_S\right|^2 + \frac{p_\pi^2}{3m_K^2}\ W^2\ \left|g_T\right|^2 + \frac{2}{3}\ p_\pi^2\ \left|f_1\right|^2\right) \quad (21)$$

Thus the absence of events at the far end of the pion spectrum ($W^2 = 0$) rules out vector coupling. On the other hand, the inexistence of low energy pion events ($p_\pi^2 \approx 0$) excludes S-coupling. The electron-neutrino angular correlation also allows the differentiation between V and (S,T) couplings. In the case of vector coupling the pair goes off preferentially in the same direction whereas for scalar or tensor they tend to go in opposite direction

A convenient analysis of K_{e3} events in terms of the variables E_π and $Q = E - E_\nu$, has been proposed by Kobzarev[20]. The distribution is given by:

$$\frac{dT}{d\,E_\pi\,dQ} = \frac{1}{(4\pi\,m_K)^3}\left(m_K(W_\pi - E_\pi)\left|g_s + \frac{Q}{m_K}\,g_T\right|^2 + \tfrac{1}{2}(E_\pi^2 - m_\pi^2 - Q^2)\left|f_1\right|^2 \right) \tag{22}$$

The boundary curves are:

$$\begin{aligned} W_\pi - E_\pi &= 0 \\ E_\pi^2 - m_\pi^2 - Q^2 &= 0 \end{aligned} \tag{23}$$

Hence, for pure vector coupling no events near the hyperbola should be found; for (S,T) combinations no events are to be found close to the straight line; for a pure tensor coupling the population of events around the vertical axis should be scarce. The distribution for pure scalar is uniform along the horizontal lines. The distribution will be symmetrical with respect to the vertical axis, unless there is a mixture of scalar and tensor interactions. As pointed out by Okun'[20], the symmetry of the distribution is sufficient to ensure that the number of electrons with energy $E \leqslant W_e/2$ is less than half the total number of electrons. This condition will obtain for any combination of scalar and vector, or tensor and vector couplings[21].

THE ELECTRON ENERGY SPECTRUM.*

If the non-leptonic current is a pure $T = \frac{1}{2}$ isospinor the

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* After the completion of this manuscript a paper was published by M. Bolsterli and D. A. Geffen (Phys. Rev. Letters 7, 203 (1961)) using the same approach of this section and obtaining the results found here for vector coupling.

distributions of the decay products of charged and neutral K-mesons must be identical except for small corrections due to mesonic mass differences within the multiplets. A pure $T = 3/2$ current also gives, for the decay of neutral K-mesons satisfying the selection rule $\Delta S = \Delta Q$, the same spectrum as for the decay of the charged meson. However, a mixture of $T = \frac{1}{2}$ and $T = 3/2$ currents will, in general, give different distributions for the K^+ and K^0 decays. Therefore, an investigation of the spectra of both charged and neutral mesons must be carried out. For the charged K-mesons it is not so easy to determine the complete kinematics of the events. It would be necessary to see the conversion of both γ-rays from the π^0-decay. The energy spectrum of the charged lepton is then the most accessible to experimental determination.

In this section we shall discuss the question of extracting from the lepton spectrum alone definite information on the type of coupling. An analisys of the spectrum with this same purpose has been carried out by Furuichi et al[21]. We have to find out properties of the lepton spectrum which are characteristic of the type of coupling and independent of the structure of the form factors. For this purpose the electron spectrum is much more convenient than the muon one, because by neglecting the electron mass, one achieves a considerable simplification in the expressions for the energy spectrum. The electron spectrum is then given by:

$$\frac{dT}{dE} = \frac{4}{(4\pi\, m_K)^3} \int_0^w (\rho_S + \rho_V + \rho_T + \rho_{ST})\, d\left(\frac{W^2}{2\, m_K}\right) \qquad (24)$$

where

$$w = 2E(W_e - E)/(m_K - 2E)$$

The energy E is a double valued function of w:

$$E = \tfrac{1}{2}\,(W_e + w \pm \sqrt{D}) \tag{26}$$

where

$$D(w) = (W_\pi - w)^2 - m_\pi^2 \tag{27}$$

In the physical region w is positive; the end points of the spectrum correspond to w = 0; the maximum value of w is $w_o = \frac{1}{2m_K}(m_K - m_\pi)^2$ corresponding to the energy $E_o = \frac{1}{2}(m_K - m_\pi) = 180$ MeV.

The expressions for the ρ's in terms of E and $w' = W^2/2\,m_K$ are:

$$\rho_S(E,w') = \tfrac{1}{2}\left|g_S(w')\right|^2 m_K\, w' \tag{28}$$

$$\rho_V(E,w') = \tfrac{1}{2}\left|f_1(w')\right|^2 (m_K - 2E)(w - w') \tag{29}$$

$$\rho_T(E,w') = \tfrac{1}{2}\left|g_T(w')\right|^2 (2E - W_e - w')^2\,\frac{w'}{m_K}$$

$$= \tfrac{1}{2}\left|g_T(w')\right|^2 (w - w' \pm \sqrt{D})^2\,\frac{w'}{m_K} \tag{30}$$

$$\rho_{ST}(E,w') = \mathrm{Re}(g_S g_T^*)\,(2E - We - w')\,w'$$

$$= \mathrm{Re}(g_S g_T^*)\,(w - w' \pm \sqrt{D})\,w' \tag{31}$$

Now:

$$\frac{d}{dE}\left(\frac{dT}{dE}\right) = \frac{dw}{dE}\,\rho(E,\,w) + \int_0^w \frac{\partial\rho}{\partial E}\,dw' \tag{32}$$

where:

$$\frac{dw}{dE} = 1 - \frac{m_\pi^2}{(m_K - 2E)^2} = \mp\,2\sqrt{D}\,/\,(m_K - 2E) \tag{33}$$

The slope of the spectrum at E_o, for the different types of coupling, is given by:

$$\frac{d}{dE}\left(\frac{dT_S}{dE}\right) = 0$$

$$\frac{d}{dE}\left(\frac{dT_V}{dE}\right) = -\frac{4}{(4\pi\, m_K)^3}\int_0^{w_o} |f_1(w')|^2\,(w_o - w')\,dw' < 0 \qquad (34)$$

$$\frac{d}{dE}\left(\frac{dT_T}{dE}\right) = \frac{8}{(4\pi\, m_K)^3}\int_0^{w_o} |g_T(w')|^2\,(w_o - w')\,\frac{w'}{m_K}\,dw' > 0$$

One arives immediately at the following results: The energy spectrum of electrons has a maximum:

a) at the energy E_o, if the coupling is scalar.

b) below E_o, if the coupling is vector.

c) above E_o, if the coupling is tensor.

For scalar coupling $\frac{\partial \rho_S}{\partial E} = 0$ and the curve has only one maximum at $E = E_o$. If the form factor $g_s(w)$ vanishes for some value of w in the physical region $(0, w_o)$, the curve has points of inflexion with zero slope on the left and right of E_o at the energies corresponding to that value of w.

For vector coupling $\rho_V(E, w) = 0$, and:

$$\frac{\partial \rho_V}{\partial E} = -(w - w' \pm \sqrt{D})\,|f_1(w')|^2 \qquad (35)$$

Below $E = W_e/2$ one has $w < \sqrt{D}$ and $\frac{\partial \rho_V}{\partial E} > w'|f_1(w')|^2 > 0$.

Hence, for vector coupling the maximum of the spectrum is located in the interval ($W_e/2$, E_o).

In the case of tensor coupling one can see from the last expression for ρ_T that the peak above E_o is an absolute maximum. However further details on the shape of the spectrum cannot be foretold without some knowledge about the form factors.

These general features of the spectrum can best be exhibited in a plot against the variable w. The curves shown in Fig. 1 have two branches corresponding to the intervals ($0, E_o$) and (E_o, W_e). They all start with zero slope, like w^2 for small w. If the coupling is scalar the two branches shall coincide Fig. 1a. For vector coupling and (S,V) combination, Fig. 1b (tensor coupling Fig. 1c) the branch for $E < E_o (E > E_o)$ stays above the other branch $E > E_o (E < E_o)$ and has an absolute maximum. The two branches join smoothly at the turning point $w = w_o$. Thus, the plot of $\frac{dT}{dE}$ against w provides a quantitative test of pure scalar coupling.

One can obtain a similar test for vector coupling. We introduce the variable $\eta = E(1 - E/m_K)$; the spectrum $\frac{dT}{d\eta}$ of η, has its maximum at $\eta_o = E_o(1 - E_o/m_K) = \frac{W_e}{2}$. Therefore the distribution of events plotted against η, peaks at a definite point $W_e/2$ independently of the form factor. Moreover the two branches $E < E_o$ and $E > E_o$ of the function $(dt/d\eta)$ plotted against w, must coincide if the coupling is pure vector. For pure scalar or tensor couplings the branch of $(dT/d\eta)$ for $\eta > \eta_o$ stays always above the branch $\eta < \eta_o$. Therefore, the same will happen for any combination (V,S) or (V,T). Moreover the distribution of events against η has an absolute maximum at $\eta > \eta_o$ for both tensor and scalar coupling. Hence, for the combinations (V,S) and (V,T) the absolute maximum of $(dT/d\eta)$ must occur above η_o. In order to obtain the absolute maximum of $(dT/d\eta)$ below η_o, a mixture of scalar and tensor interactions is required.

In the case of tensor coupling there is no simple test analogous to the precedings ones. The function which exhibit such symmetry in w, contains first and second derivatives of the energy spectrum. This function is:

$$\left(D(w) \frac{d^2}{dw^2} - 2 \frac{d}{dw} \frac{m_\pi^2}{m_K - 2E} \right) \left(D(w)^{-\frac{1}{2}} \frac{dT}{dE} \right) \tag{36}$$

Since an extremely precise determination of the spectrum would be required to obtain this function, its practical use is rather limited.

Finally we consider the question of determining the form factors from the knowledge of the spectrum. For pure scalar and vector couplings they would be given by:

$$|g_s(w)|^2 = \frac{(4\pi\ m_K)^3}{m_K} \frac{d}{d\ w^2} \left(\frac{dT}{dE} \right) \tag{37}$$

$$|f_1(w)|^2 = \frac{(4\pi\ m_K)^3}{2\ m_K} \frac{d^2}{dw^2} \left(\frac{dT}{d\eta} \right) \tag{38}$$

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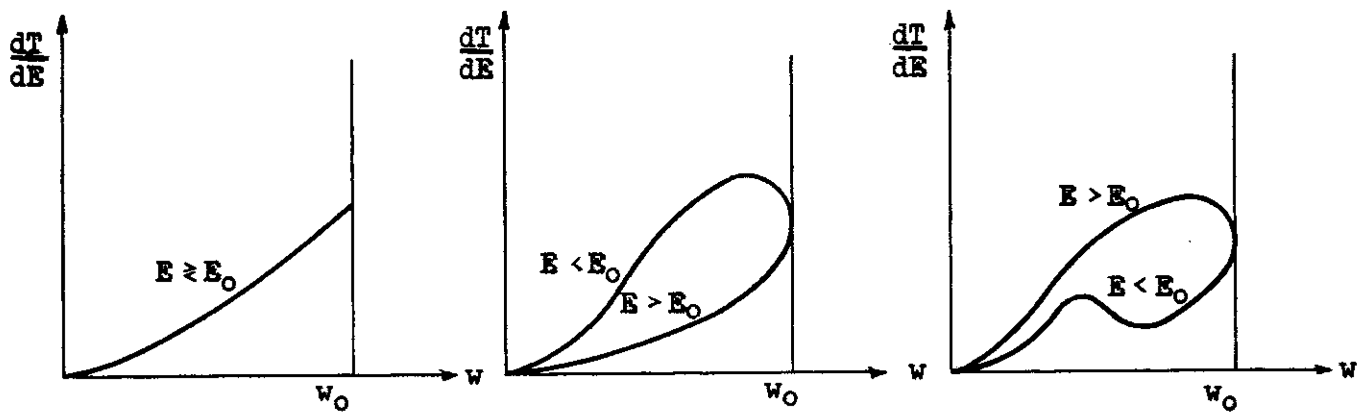

Fig. 1 - General shape of the spectrum $\frac{dT}{dE}$ plotted against the variable w. The curve has in general two branches corresponding respectively to energies $E < E_o$ and $E > E_o$. For pure scalar interaction, Fig. 1a, the branches coincide. Pure vector or tensor coupling would give a curve like in Fig. 1b and Fig. 1c respectively.