ISSN 0029-3865

CNPq

CBPF - CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Notas de Física

CBPF-NF-005/93

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Rio de Janeiro
1993

ISSN 0029-3865

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A Statistical Model for Stock Exchange: Time Evolution and Stability

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CBPF-NF-005/93

Abstract

In the spirit of microfoundations of macroeconomic theory, we introduce a coupled map lattice model to describe time evolution of relevant quantities associated with stock exchange. In particular, computer simulations exhibit the stabilizing effect, on stock exchange, of dispersion of the external parameters which control the coupled map.

KEY-WORDS: Microfoundations of economy; Nonlinear dynamical system; Chaos; Coupled map.

The methods used in Statistical Physics have proved their efficiency in "extraphysical" areas such as Biogenesis ([1] and references therein), Immunology ([2] and references therein), Evolutionary Genetics ([3] and references therein), Neural Networks ([4] and references therein) and Economy ([5,6] and references therein). In what concerns the area of Economy, all the attempts of microfoundation of macroeconomic theory can be, in some sense, put into this category. This is, in particular, the case of L.Summers proposal for treating fluctuations in the stock market in terms of nonlinear dynamics. To do this, he introduced the concept of "noisy traders" and "sophisticated traders" and achieved some success. Nevertheless his model has not been considered fully convincing (page 248 of [5]). Within this type of philosophy, we propose here a simple theoretical model, based on coupled map lattices[7], for understanding some interesting phenomena currently occuring in stock exchanges, such as the influence of the massive use of computers by the brokers.

Let us assume N stock brokers(traders) buying ($X = 1$) or not buying ($X = 0$) stocks of an unique and big company. All the brokers are assumed to operate in competitive regime, and having free access to market information. We associate with each broker the following distribution law

$$P_i^{(t)}(X) = (1 - p_i^{(t)})\delta(X) + p_i^{(t)}\delta(X-1) \qquad (i = 1, 2, ..., N) \qquad (1)$$

where δ denotes Kroenecker's delta function ($\delta(X)$ equals 1 if $X = 0$, and vanishes otherwise), and $p_i^{(t)} \epsilon [0,1]$ represents the probability that he <u>wishes</u>.

at time t, to buy stocks (we refer to the "potential demand", not to be confused with the probability that he indeed buys the stocks). The $\{p_i^{(t)}\}$ are coupled as follows:

$$p_i^{(t+1)} = \alpha_i p_i^{(t)} + \frac{\beta_i}{N-1}\sum_{j\neq i} p_j^{(t)} - \frac{\gamma_i p_i^{(t)}}{N-1}\sum_{j\neq i} p_j^{(t)} \qquad (i = 1, 2, ..., N) \qquad (2)$$

where $\{\alpha_i, \beta_i, \gamma_i\}$ represent a set of positive external parameters, and $\sum_{i=1}^{N} p_i^{(t)} \epsilon$ [0,N]. The α_i-parameter measures the inertial persistency of the wish of the $i-th$ broker (if $\beta_i = \gamma_i = 0$, $p_i^{(t)}$ decreases with time if $0 < \alpha_i < 1$, remains constant if $\alpha_i = 1$, and increases with time if $\alpha_i > 1$). The β_i-parameter measures the influence, on the individual wish of a given broker, of the average wish of all the other brokers (if $\alpha_i = \gamma_i = 0$ and $\beta_i = \beta$, the average value $< p > (t) \equiv (\sum_{i=1}^{N} p_i^{(t)})/N$ (which somehow reflects the stock prize) decreases with time if $0 < \beta < 1$, remains constant if $\beta = 1$, and increases with time if $\beta > 1$). Finally, γ_i is associated with the tendency of a given broker not to buy stocks whenever most of the other brokers wish to buy (in the absence of such tendency, his individual profit will eventually be quite modest or impossible); by the way, let us recall the famous anecdote which states that Joseph Kennedy (President Kennedy's father) providentially escaped from the 1929 USA-wide stock exchange collapse because he sold his stocks as soon as he realized that his humble shoe-shine-boy was buying stocks!

It is clear that if $\{\alpha_i, \beta_i, \gamma_i\}$ are arbitrarily chosen, Eq. (2) does not automatically guarantee $0 \le p_i^{(t)} \le 1$ for all values of i and t, thus violating the definition of probability. This difficulty can, in priciple, be overcome through various procedures (e.g., conveniently introducing the hyperbolic tangent function which transforms the interval $(0, \infty)$ into $(0, 1)$). For simplicity, we adopt here the following prescription: whenever $p_i^{(t)}$ is above one (below zero) it is replaced by $p_i^{(t)} = 1$ ($p_i^{(t)} = 0$)). Let us also remark that $p_i^{(t)} = 0$ (for all values of i) is a fixed point of Eq. (2). Another fixed point is given, in the $N \to \infty$ limit, by $p_i = \beta_i p_{av}/(1 - \alpha_i + \gamma_i p_{av})$ ($\forall\, i$), where the average $p_{av} \equiv <p>(\infty)$ satisfies $\lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \beta_i/(1 - \alpha_i + \gamma_i p_{av}) = 1$ (in particular, $(\alpha_i, \beta_i, \gamma_i) \equiv (\alpha, \beta, \gamma)$ implies $p_i = p_{av} = (\alpha + \beta - 1)/\gamma,\ \forall\, i$).

An interesting property of Eq. (2) exists in the particular case $\alpha_i \equiv \alpha$, $\beta_i \equiv \beta$ and $\gamma_i \equiv \gamma$. If we apply $\frac{1}{N}\sum_{i=1}^{N}$ on both members of Eq. (2) we obtain

$$[<p>(t+1)] = (\alpha + \beta)[<p>(t)] - \gamma \frac{N}{N-1}[<p>(t)]^2 + \\ +\gamma \frac{1}{N(N-1)} \sum_{i=1}^{N} [p_i^{(t)}]^2 \tag{3}$$

hence, in the $N \to \infty$ limit,

$$[<p>(t+1)] = (\alpha + \beta)[<p>(t)] - \gamma[<p>(t)]^2. \tag{4}$$

By introducing now $Z(t) \equiv (\gamma[<p>(t)])/(\alpha+\beta)$, Eq. (4) becomes

$$Z(t+1) = (\alpha+\beta)Z(t)(1-Z(t)) \qquad (5)$$

which is the well known logistic equation (see [8] and references therein). Let us recall two remarkable properties associated with Eq. (5): (i) if $\alpha+\beta \leq 4$, $Z(t)$ automatically remains within the interval [0,1] if this was true at t=0, hence, the same occurs with $[<p>(t)]$ if $\alpha+\beta \leq \gamma$ (hence, $[<p>(t)] \epsilon [0,(\alpha+\beta)/\gamma]$; consequently, a realistic situation corresponds to $\alpha+\beta$ equal γ, or slightly below γ); (ii) if $\alpha+\beta \;\epsilon\; (3.57,4)$ the system can be chaotic (positive Lyapunov exponent).

Let us now go back to the more general case in which the values for $\{\alpha_i,\beta_i,\gamma_i\}$ are in principle different for each broker. We shall now assume that the set $\{\alpha_i\}$ is drawn from the distribution law $D_\alpha(\alpha_i)$, one and the same for all brokers; analogously, $\{\beta_i\}$ and $\{\gamma_i\}$ are respectively drawn from $D_\beta(\beta_i)$ and $D_\gamma(\gamma_i)$. These distributions are basically characterized by their mean values $\overline{\alpha}$, $\overline{\beta}$ and $\overline{\gamma}$, as well as by their widths W_α, W_β and W_γ. Human brokers typically correspond to large values for $(W_\alpha, W_\beta, W_\gamma)$, whereas computer brokers (i.e, brokers that blindly follow computer recommendations) correspond to small values for $(W_\alpha, W_\beta, W_\gamma)$ since the softwares they use are (most frequently) very similar.

One of the important effects we want to focus with the present model is that the risk of stock exchange collapse will be shown to monotonously decrease with increasing W_α, W_β and W_γ.

Two basically different models can be discussed. The first of them (referred to as quenched) consists in fixing once for ever $\{\alpha_i, \beta_i, \gamma_i\}$ (drawn respectively from $(D_\alpha, D_\beta, D_\gamma)$ at $t = 0$), i.e., each broker will evolve, at all times, under the same values for (α, β, γ), but these values might differ from broker to broker. The second model (referred to as annealed) consists in randomly choosing, at every new time step, $\{\alpha_i, \beta_i, \gamma_i\}$ (still drawn from $(D_\alpha, D_\beta, D_\gamma)$). Intuition suggests that the annealed model yields more randomized results with respect to those obtained from the quenched model. Our numerical results will confirm this intuition, and the quenched model with widths $(W_\alpha, W_\beta, W_\gamma)$ behaves very similarly to the annealed model with somewhat smaller widths. It suffices consequently to discuss the details of say the annealed model.

Another mechanism which increases randomness (note that Eq. (2) for the quenched model is strictly determinist) is the presence of a (centered) white-noise additive term $\eta_i^{(t)}$ in the second member of Eq. (2). No drastic influence is observed: if $\eta_i^{(t)}$ is not too large, the same phenomena occur, but in a smoother manner. In fact, a more general formulation of the present model demands the inclusion of terms of this type. It can be seen that a large variety of these coupled maps yield a value $<p>(t)$ which satisfies a logistic-like equation, thus exhibiting a sort of universal behavior.

Let us now discuss our numerical results for the mean value $<p>(t)$ and for the mean deviation

$$\sigma(t) \equiv \sqrt{\frac{1}{N}\sum_{i=1}^{N}[p_i^{(t)} - <p>(t)]^2}. \tag{6}$$

These quantities depend of course on the initial conditions $\{p_i^{(0)}\}$. We have used a great variety of them, drawn from both large and narrow random distributions in the interval [0,1]. In all the (generic) cases, after a short transient, the system "forgets" the initial distribution of $\{p_i^{(0)}\}$. So, for convenience, we have chosen to work sistematically with $\{p_i^{(0)}\}$ drawn from a white-noise distribution in the interval [0,1]. Clearly, $<p>(t)$ and $\sigma(t)$ also depend, in principle, on N. We have varied N in the range [30,3000] with no significative influence. We have consistently chosen to work with $N = 300$ (which, by the way, coincides with a reasonable order of magnitude for the number of brokers in a typical stock exchange). We have also verified that the particular shape of distributions $\{ D_\alpha(\alpha_i),\ D_\beta(\beta_i),\ D_\gamma(\gamma_i) \}$ is not very important, their influence being very well characterized through their mean value ($\overline{\alpha}$, $\overline{\beta}$, $\overline{\gamma}$) and their widths (W_α, W_β, W_γ). Consequently, for convenience, we have chosen to work with (α_i, β_i, γ_i) independently drawn from white-noise distributions in the intervals $\overline{\alpha} - \frac{W_\alpha}{2} \leq \alpha_i \leq \overline{\alpha} + \frac{W_\alpha}{2}$, $\overline{\beta} - \frac{W_\beta}{2} \leq \beta_i \leq \overline{\beta} + \frac{W_\beta}{2}$, $\overline{\gamma} - \frac{W_\gamma}{2} \leq \gamma_i \leq \overline{\gamma} + \frac{W_\gamma}{2}$ respectively. To illustrate the most interesting situation (i.e., chaos) we have mainly worked with $\overline{\alpha} = \overline{\beta} = \overline{\gamma}/2 \ \epsilon\ [1.78,2]$ and $W_\alpha = W_\beta = W_\gamma/2 \ \epsilon [0, 0.2]$. Finally, since the effects we want to emphasize are more clear-cut for the annealed model without noise, we have primarily addressed this case. In Fig. 1(a) we show a typical time-dependence of $< p > (t)$ (corresponding to $\overline{\alpha} = \overline{\beta} = \overline{\gamma}/2 = 1.85$); although $W_\alpha = W_\beta = W_\gamma/2 = 0.2$ was in fact used, the situation is practically the same for, say, $W_\alpha = W_\beta = W_\gamma = 0$. In Fig. 1(b) we represent, for the same two situations, the time evolution of $\sigma(t)$; we see now clearly the influence of the widths (W_α, W_β, W_γ).

It is now convenient to introduce the <u>stock collapse index</u> $\kappa(t)$ as follows:

$$\kappa(t) = (\frac{N_\uparrow^{(t)} - N_\downarrow^{(t)}}{N})^2 \quad \epsilon\ [0,1] \tag{7}$$

where $N_\uparrow^{(t)}$ ($N_\downarrow^{(t)}$) is the number of brokers whose $p_i^{(t)}$ is above (below) 1/2, i.e., who basically want to buy (not to buy) stocks. In practice, $N_{\uparrow}^{(t)} + N_{\downarrow}^{(t)} = N$ $(\forall t)$. $\kappa(t)$ vanishes if the brokers are equally distributed above and below $p_i^{(t)} = 1/2$; $\kappa(t)$ equals unity if all the brokers have simultaneously the same wish, either to buy or not to buy. We show in Fig. 2 a typical time evolution of $\kappa(t)$ (corresponding to $\overline{\alpha} = \overline{\beta} = \overline{\gamma}/2 = 1.9$ and $W_\alpha = W_\beta = W_\gamma/2 = 0.1$).

The time average $\overline{\kappa}$ of the stock collapse index is given by

$$\overline{\kappa} \equiv \frac{1}{T}\sum_{t=1}^{T}\kappa(t). \tag{8}$$

We have chosen $T = 100$ to be a typical value (which, in fact, correctly represents the $T \rightarrow \infty$ limit). We present, in Fig. 3, $\overline{\kappa}$ as a function of $W_\alpha = W_\beta = W_\gamma/2 \equiv W$ for a typical case ($\overline{\alpha} = \overline{\beta} = \overline{\gamma}/2 = 1.85$) and for all four cases, annealed or quenched, with or without noise. We clearly see that the risk of collapse of the stock market is smaller for human brokers (large W) than for computer brokers (small W).

To summarize the present work let us recall that the nonlinear dynamical model proposed through Eq. (2) (either in its quenched or its annealed version, with or without additive white-noise randomness) (i) presents the typical chaotic-like behaviour associated with the time evolution of stock exchanges, (ii) exhibits that computer brokers(narrow distributions of the external parameters) practically share, after a short transient, the same wishes (i.e., $p_i^{(t)} \approx <p>(t), \forall i$), and (iii) exhibits that computer brokers can more easily destabilize the stock market as compared to human brokers (large distributions of the external parameters).

Let us conclude by mentionning that the present simplified model could be quite naturally extended to more realistic ones along the following lines:

(i) A term like $\beta_i \frac{1}{N-1} \sum_{j\neq i}^N p_j$ could be generalized into $\frac{1}{N-1} \sum_{j\neq i}^N \beta_{ij} p_j$ (analogously for the γ-term);

(ii) Stocks of an arbitrary number of companies (and not only one) could be focused, and those companies could be not very big ones;

(iii) Distribution (1) could be generalized into

$$P_i^{(t)}(X) = q_i^{(t)} \delta(X+1) + (1 - p_i^{(t)} - q_i^{(t)}) \delta(X) + p_i^{(t)} \delta(X-1) \ .$$

being possible, for each broker, to buy stocks($X = 1$), to sell stocks($X = -1$), or not buy nor sell($X = 0$).

(iv) The broker population could be formed by, say, two constituents, for instance, both "computer" and "human" brokers. In this case, W would become a random variable itself determined by a distribution law such as $R(W) = (1-r)\delta(W - W_1) + r\delta(W - W_2)$, with, say, $0 \leq W_1 << W_2$ and

$0 \leq r \leq 1$;

(v) A term of the type $\delta_i \frac{(1-p_i)}{N-1} \sum_{j\neq i}^{N}(1-p_j)$ could be, as already mentionned, included in Eq. (2). This would turn the situation into a more symmetric one; one would, however, loose the $p_i = 0$ fixed point.

As a final remark, we might say that the present work gives quantitative substance to the well known Keynes' observation [9](on a similar phenomenon) "It is interesting that the stability of the system and its sensitiveness to changes in the quantity of money should be so dependent on the existence of a variety of opinion about what is uncertain. Best of all that we should know the future. But if not, then, if we are to control the activity of the economic system by changing the quantity of money, it is important that opinions should differ. Thus this method of control is more precarious in the United States, where everyone tends to hold the same opinion at the same time, than in England where differences of opinion are more usual".

We acknowledge very fruitful remarks from M.H.Simonsen, R.Guenzburger, P.Tafner and D.Ellis.

CAPTION FOR FIGURES

Figure 1 - $N = 300$ annealed model without noise for $\overline{\alpha} = \overline{\beta} = \overline{\gamma}/2 = 1.85$ and $W_\alpha = W_\beta = W_\gamma/2 \equiv W$: (a) $< p > (t)$ for $W = 0.2$; (b) $\sigma(t)$ for $W = 0.2$ (full circles) and $W = 0$(empty circles).

Figure 2 - $N = 300$ annealed model without noise for $\overline{\alpha} = \overline{\beta} = \overline{\gamma}/2 = 1.9$ and $W_\alpha = W_\beta = W_\gamma/2 = 0.1$: time evolution of the stock collapse index $\kappa(t)$.

Figure 3 - The time average stock collapse index $\overline{\kappa}$ as a function of the width $W_\alpha = W_\beta = W_\gamma/2 \equiv W$ for $N = 300$ and $\overline{\alpha} = \overline{\beta} = \overline{\gamma}/2 = 1.85$:

● annealed without noise, ○ annealed with noise ($|\eta_i^{(t)}| \leq 0.1$),

▲ quenched without noise, △ quenched with noise ($|\eta_i^{(t)}| \leq 0.1$).

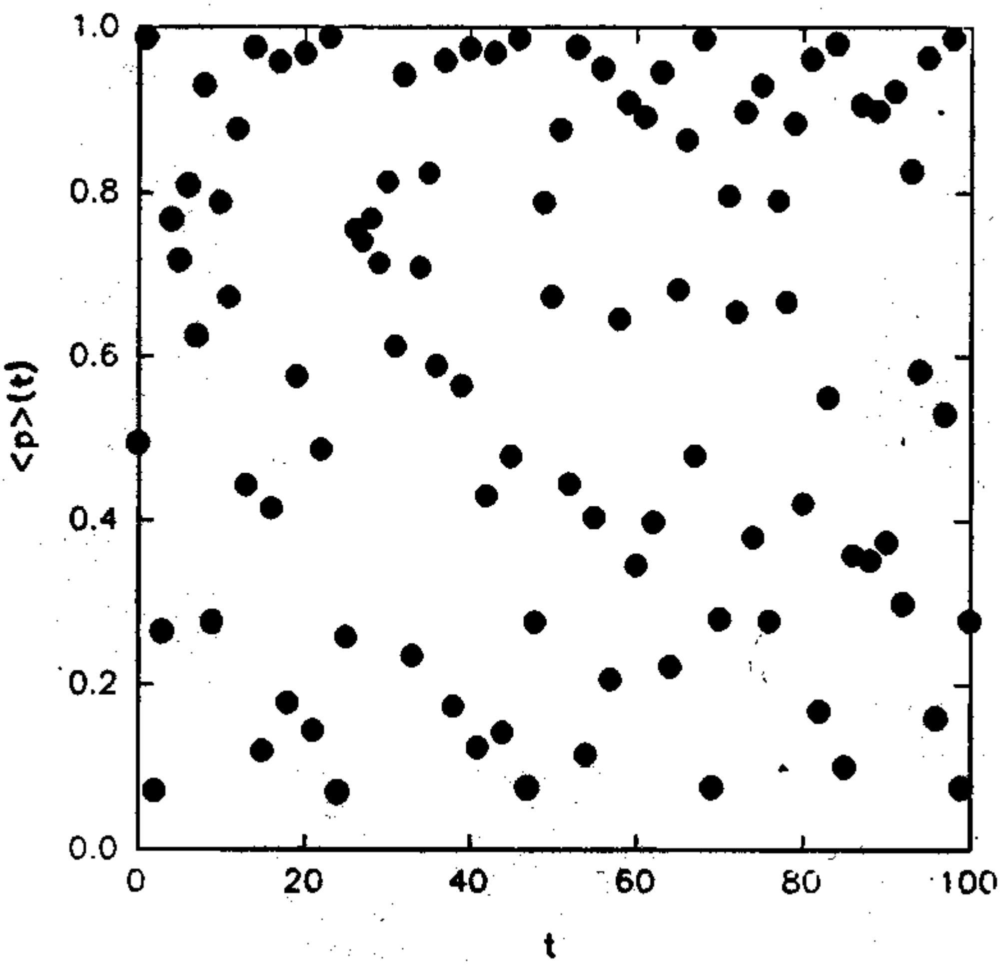

Figure 1(a)

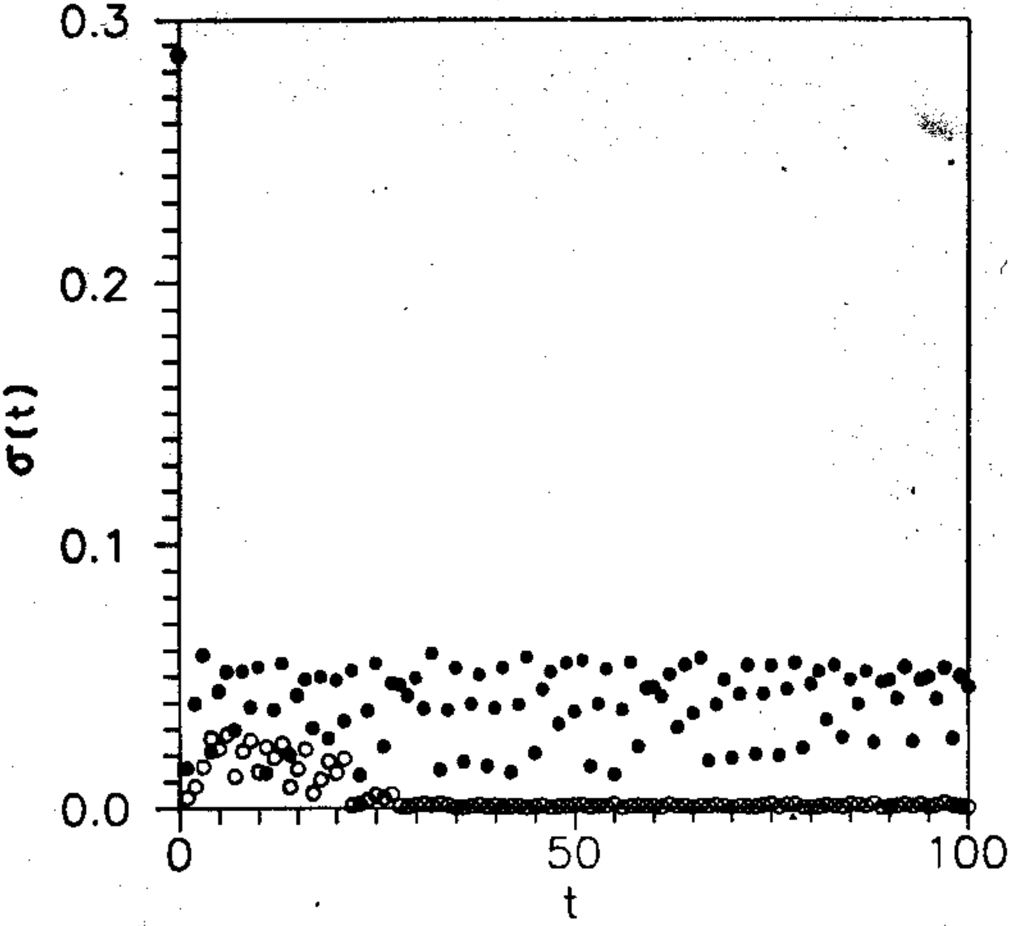

Figure 1(b)

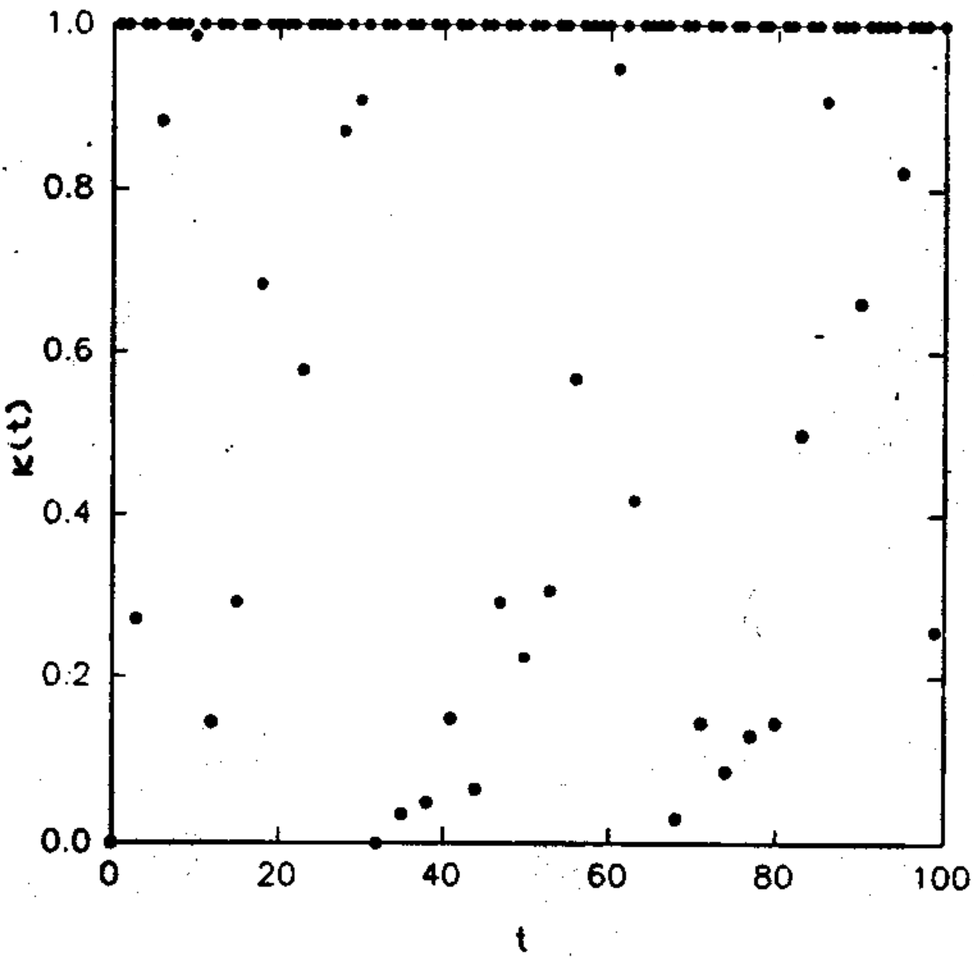

Figure 2

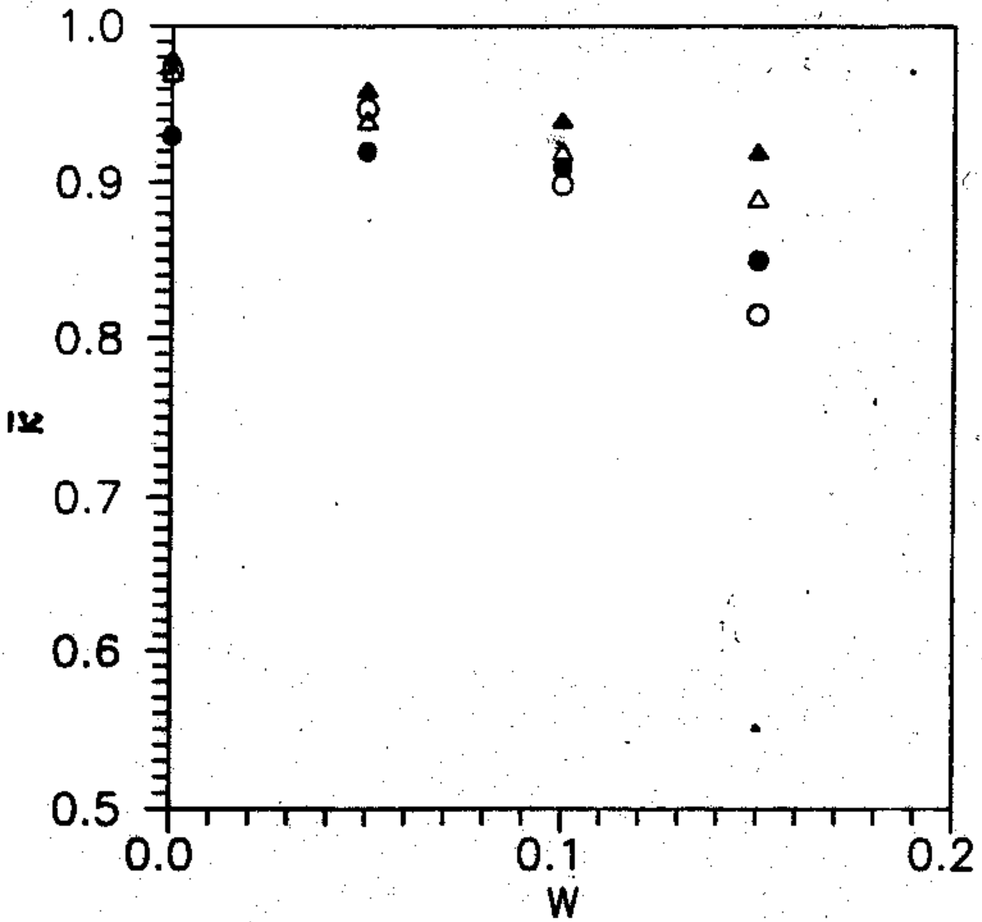

Figure 3

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